Calculation of the energy density parameter ρ in the cosmological model ΛCDM and through experimental data

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Abstracs

We have calculated the energy density parameter, which appears in the cosmological model ΛCDM in the Friedman equation, using cosmological data values from recent experimental results reported in the literature. A statistical calculation has been performed to determine its uncertainty. Finally, the result is commented linking it with the possible existence of dark energy.

Keywords

Dark energy, ΛCDM model, general relativity.

1.- Equation to solve

According to the Friedman equation, the Hubble parameter H, the Gaussian curvature of the space-time K, and the energy density parameter ρ_m, are related in the ΛCDM cosmological model as follows [1].

$H^2 = (a'/a)^2 = (8\pi G\rho_m + \Lambda c^2)/3 - Kc^2$ (1)

being "a" the scale factor of the universe, G the universal gravitational constant, Λ the cosmological constant. The energy density parameter ρ_m is commonly called ρ in the literature.

The critical density ρ_c is called the energy density that results when in the Friedmann equation the cosmological constant and the curvature are made equal to zero. Thus, according to equation (1) this critical density is given by:

$H^2 = 8\pi G\rho_c/3$

$\rho_c = 3H^2/8\pi G = 0{,}92.10^{-26}$ Kg/m³

We transform equation (1) in the following way,

$H^2 = (a'/a)^2 = 8\pi G(\rho_m + \rho_\Lambda +\text{-}\rho_k)/3$

where ρ_m is the energy density related to mass, ρ_Λ is the energy density of vacuum space, related to the cosmological constant by the equation $\rho_\Lambda = \Lambda c^2/8\pi G$ and ρ_k is the energy density related to the curvature of space-time $\rho_k = Kc^2/8\pi G$.

We propose to calculate ρ_m from recent experimental data:

To determine ρ_k we use, among others, data from the Planck mission [3], there the parameter $\Omega_k = Kc^2/H^2$ was measured as curvature

Substituting this expression in equation (1) and dividing both sides of the equation by H^2, results in the equation that we are going to use to calculate the energy density ρ_m:

$$\mathbf{1 = 8\pi G/3H^2 \, (\rho_m + \rho_\Lambda) + \Omega_k} \qquad (2)$$

2- Experimental data to use

The energy density of the vacuum space ρ_Λ is referred to [2]:

$\rho_\Lambda = (0.603 \pm 0.013) \times 10^{-26}$ kg/m^3

The curvature of space-time is referred to [3]:

$\Omega_k = 0{,}001 \pm 0{,}002$

$-0{,}001 \leq \Omega_k \leq 0{,}003$

The Hubble parameter, is referred to [4]:

$H_0 = 73{,}52 \pm 1{,}62$ km/s por mega parsec, igual a $2{,}35.10^{-18}$ m/s por metro

3,- Calculation of the parameter ρ_m

According to equation (2)

$1 = 8\pi G/3H^2 \, (\rho_m + \rho_\Lambda) + \Omega_k$

doing:

$G = 6{,}67.10^{-11}$ N.m^2/Kg^2

result:

$\boldsymbol{\rho_m = 0{,}3848.10^{-26}}$ **Kg/m^3**

4.- Calculation of the associated uncertainty

We calculate the uncertainty associated with the energy density parameter ρ_m in accordance with the regulatory recommendations contained in [5]:

X = X ±Kσ	Experimental data	Associated uncertainty
K=2	$\rho_\Lambda = (0.603 \pm 0.013) \times 10^{-26}$ kg/m^3	$\sigma_\Lambda = 1{,}1\%$
K=2	$H_0 = 73{,}52 \pm 1{,}62$ sec^{-1}	$\sigma_H = 1{,}1\%$
K=2	$\Omega_k = 0{,}001 \pm 0{,}002$; $\Omega_k = - 0{,}001$; $\rho = 0{,}3867$ $\Omega_k = -0{,}003$; $\rho = 0{,}3828$ $\sigma = 0{,}0019$	$\sigma_\Omega = 0{,}5\%$
		$\sigma_\rho^2 = (0{,}011)^2 + (0{,}011)^2 + (0{,}005)^2$ $\sigma_\rho = \sigma = 1{,}6\ \%$

Result:

$\rho = \rho \pm 2\sigma$ **$\rho = \rho_m = (0{,}385 \pm 0{,}012).10^{-26}$ Kg/m^3**	

5. Conclusions

In the ΛCDM model, this parameter ρ_m represents the value of the energy density in space due to the existence of gravitational mass. It is therefore associated with the mass of the cosmos (baryonic mass and dark matter). The first thing that surprises us about our result is its lower value than the experimental value of the vacuum energy density of $0.603.10^{-26}$ Kg/m^3. Our calculation of the parameter ρ_m, with a value of $0.385.10^{-26}$ Kg/m^3 seems to indicate that in addition to the energy due to mass, there is another type of energy in the cosmos that could well be dark energy and that it does contribute to the density of real energy measured experimentally in the value of ρ_Λ According to our results, this dark energy density would be at least of the order of the difference between ρ_Λ and ρ_m, that is, $0.218.10^{-26}$ Kg/m^3.

Thus, the result of our calculation leads us to the need for the existence of a necessary energy beyond that corresponding to that created by the gravitational masses, the existence of dark energy responds to this need.

Referencias

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