How to Receive Continuous Gravitational Waves

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The Kepler space telescope and the Transiting Exoplanet Survey Satellite (TESS) have discovered numerous multiple star systems emitting GW. Some of these have planets that phase modulate the GW and allow the planetary mass to be determined. The detection techniques are explained in detail using examples.

1 Introduction

Binary stars radiate energy in the form of continuous gravitational waves (GW). Their modulation allows the identification of planets and system properties such as mass exchange between the partners. For about 20 years, unsuccessful attempts have been made to detect continuous GW, although huge antennas are being built and operated for this purpose. Is it because the antennas are too short? From electromagnetic waves it is known that the antenna length should be about $\lambda/2$. LIGO/Virgo interferometers are 4000 m long and could therefore be good antennas for frequencies around 40 kHz – if GW travels at the speed of light (this has never been checked). It is unclear whether GW sources exist that emit such high-frequency continuous signals.

Procedures are described below for receiving signals in the μHz range, because there are numerous double stars with orbital periods around 40 hours. These powerful GW sources radiate at precisely known frequencies $f_{GW} = 2f_{orbit}$ and can therefore be reliably identified. A good antenna should be several AU long – that is hardly feasible. Whether the earth can serve as a replacement antenna can only be answered experimentally and not philosophically. Experiments show that the atmosphere is more suitable than the earth itself because it is not affected by earthquakes. The study of natural resonances of the earth shows that long-term recordings of barometers contain signals of unknown origin [1]. In the records of weather stations that are far apart, one finds identical spectral lines with the properties of gravitational waves. This study assumes that the atmosphere – just like any other object – responds to GW and is therefore a suitable antenna for receiving GW.

Examples from radio technology (GPS) show that the low signal amplitude of "impossible" antennas can be compensated by a highly developed receiver technology – if one knows the modulation of the signal. That is the topic of this paper.

2 Boundary conditions

Antennas deliver an analogue signal mixture, for example variable electrical voltages. In the early years of radio technology, useful signals were isolated and decoded with amplifiers and resonant circuits. Current technology works differently: An ADC measures the signal voltage of the antenna (sampling) in specified time steps and saves the result as a digital value. This sequence of numbers can be saved without errors and analyzed as often as you like. There is no connection between the frequency examined and the speed of digital data processing. Communications engineering knows best practices to check

- whether there is a signal in the noise,
- how big is the frequency and whether it changes (drift),
- whether and how the signal is modulated,
- how to decipher existing modulations.

There is only one method of detecting weak signals: filtering with the smallest possible bandwidth. A windowed-sinc filter with 10^6 taps and 0.8 nHz bandwidth delivers good results. Other methods such as IIR filters are unsuitable because they produce phase distortions. It rarely makes sense to choose the bandwidth narrower than the *natural line width*. If the bandwidth is too large, the inevitable noise can overwhelm the signal. The line width Δf of a signal is determined by the measurement time T and checked by spectral decomposition (FFT). This applies according to Küpfmüller [4]

$$T \cdot \Delta f \geq 0.5. \tag{1}$$

If one wants to detect GW in the frequency range around 10 μHz, the frequency resolution Δf should be better than 1 nHz. In order to achieve this, data must be collected over a period of at least 16 years. The German weather service DWD stores air pressure data [2], which can be used after some preparatory work. In order to reduce the influence of local peculiarities and data gaps from individual weather stations, the measured values of as many barometers as possible, which are distributed all over Germany, are added. Because the wavelength of the searched GW is at least a factor of 10^6 higher than the mutual distances between the barometers, all instruments react in phase to the GW. This coherent addition significantly improves the S/N of the signals sought and makes visible spectral lines that disappear in the noise when analyzing a single data chain.

Nearby celestial bodies stimulate the earth and atmosphere to vibrate, the frequencies of which can be found in the tidal potential catalog HW95 [3]. Every suspected GW line must be checked to ensure that it is not generated here in the solar system.

3 Pretreatment of the received data

A receiver (comparable to an FM radio) and an antenna that supplies signals are all that is needed to measure GW. This is not about discussing the differences in quality of antenna designs, but about the detection and analysis of weak AC voltage signals of known frequencies. Measured against the effort for LIGO and VIRGO, air pressure data is an extremely inexpensive method of receiving GW. All designs suffer from interference from the restless earth and probably have a certain directivity. In order to improve the S/N, one can run the receiver at full gain only when the signal amplitude is good. There are no empirical values for any antenna design, so it is worth experimenting: If a GW deforms the atmosphere of the earth in the rhythm of its frequency, areas of different pressure are created. Since the earth rotates under this variable mass distribution, corresponding fluctuations in air pressure should be measurable. Then the atmosphere is a receiving antenna. The segment caused by a specific GW can be isolated based on the known frequency f_{GW}. The method has one disadvantage: it is an amplitude modulation with an oscillation period of 24 hours. This prevents discovering planets around other suns with a comparable orbital period.

4 Phase Modulation (PM)

Presumably no continuous GW is amplitude modulated. Verification is difficult because the S/N of a GW is rarely sufficient to detect and demodulate AM.

Each GW is phase modulated with at least one frequency because the antenna orbits the sun during the measurement period. The detection of a PM with $f_{orbit} = 31.7$ nHz is the necessary confirmation that the GW is *not* generated in the solar system. The MSH method can significantly increase the amplitude of the spectral line at f_{GW} and therefore also detect signals with very low S/N. The star system Kepler-47 is a good example: at least three planets ensure that the binary system near the center orbits around the center of mass of the entire system [5]. The radiated GW is therefore phase-modulated several times with the modulation frequencies $1/P_A$, $1/P_B$ and $1/P_C$. The maximum frequency shift Δf of the GW each planet causes also depends on its mass and orbital radius.

Let's consider the "middle" planet with $P_C = 187$ days. Measurements of the Doppler effect show [6] that it wobbles f_{GW} periodically by an amount Δf_C. The modulation index $a = P_C \cdot \Delta f_C \approx 2.4$ is calculated from this. The calculation of the spectrum to be expected based on the PM laws produces a surprise: Using this value of a, the amplitude of the carrier frequency $f_{GW} = 3.1078$ μHz is approximately zero and the total energy of the GW is contained in the surrounding sidebands. This may explain the amazing spectrum of the raw antenna data (Figure 1).

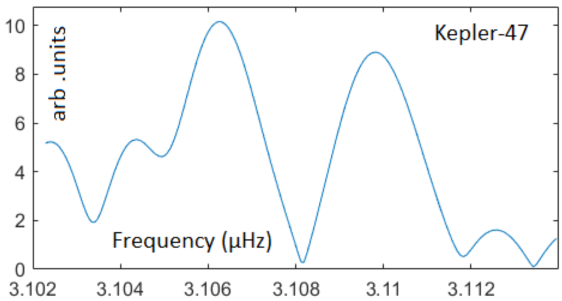

Figure 1): *Spectrum of the mean air pressure in Germany. Where one expects f_{GW} from Kepler-47, the amplitude is particularly small. The almost symmetrical structure could be generated by a PM with $f \approx 1.9$ nHz. Verification requires demodulation of the PM.*

With the MSH method, three characteristic values of a GW can be determined: frequency, frequency drift and all PM. Each PM has three parameters: modulation frequency f_{mod}, maximum frequency deviation Δf and phase shift. The MSH method facilitates the search for GW: it compensates for the PM and ensures that the entire energy of the GW is concentrated in a single spectral line f_{GW}. The amplitude increases and the S/N improves significantly.

Figure 2 shows the result: The MSH method reduces the frequency of the GW emitted by the Kepler-47 binary system to $f_{ZF} = 1/(400 \cdot 3600\ s) \approx 694$nHz.

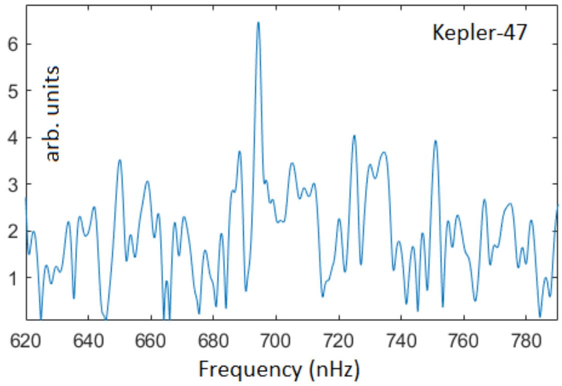

Figure 2): *Spectrum around* f_{ZF}. *The MSH method reduces the value of* f_{GW} *and only evaluates the narrow range* (694 ± 1) *nHz and ignores the neighboring lines. Possibly these are generated by other binary systems.*

It is possible that previous attempts to detect GW failed *because* of an attempt to circumvent the detection and elimination of the PM with $f_{mod} = 31.7$ nHz: The raw data provided by the antenna are usually converted as if the antenna were located at the center of gravity of the solar system. It is assumed without proof that $v_{GW} = c$ holds. Previous measurements in the μHz range deliver reproducible results that can only be explained with the assumption $v_{GW} < c$. This question will not be discussed in this paper. Here we are only talking about the receiver technology.

A binary star with planets emits a phase-modulated continuous GW because the GW source moves around the common center of gravity. Known orbital periods are between a few minutes and many years, and the search range for possible modulation frequencies is correspondingly large. Each PM produces sidebands that must be processed to learn what type of modulation is present. Sidebands are groups of spectral lines spaced f_{mod} above and below f_{GW}.

- At a very low modulation frequency (far away planet) the sideband lines are close together and can hardly be separated from f_{GW} even with good filters.
- Planets with a short orbital period produce a high modulation frequency, the mutual distance between the sideband frequencies is large. The gaps contain annoying noise and should be suppressed or ignored. In order to avoid signal distortions, all lines must be processed within the sufficiently large Carson bandwidth, which is often significantly larger than the maximum frequency change Δf.

These cases require different procedures. The measurement method for a depends on whether Δf is larger or smaller than the linewidth of f_{GW}.

4.1 Distant Planets

If the planet has much less mass than the binary star and orbits at a distance greater than about 0.1 AU, the GW source rotates very close and with low orbital speed around the common center of gravity. Then the maximum value Δf of the periodic Doppler shift is comparable to the natural line width and the bandwidth of the signal processing of about 0.8 nHz. A distinction between a short section of the sinusoidal Doppler shift and the linear frequency drift is only possible with a high computational effort.

It is important to determine and compensate for this very slow PM and the drift before further analysis steps follow, so that f_{ZF} is constant. The MSH method is very well suited, Figure 3 shows the two-stage signal processing. In this example, f_{GW} is first decreased by 9 μHz because the subsequent decimation speeds up the calculations. The MSH method follows: If all modulations of f_{osz2} are identical to the low-frequency modulations of f_{GW}, f_{ZF} is unmodulated. The high modulation frequencies of f_{GW} are suppressed by a band filter for f_{ZF}.

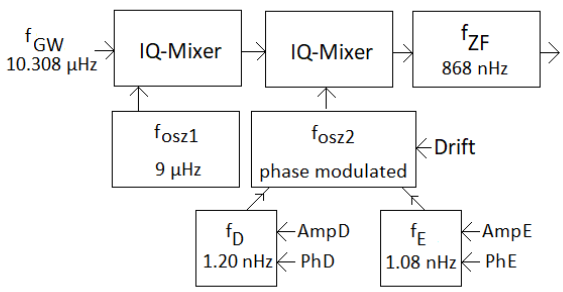

Figure 3): *The MSH method: f_{osz1} reduces the signal frequency without affecting the modulation. f_{osz2} mimics the low frequency characteristics of the signal to keep f_{ZF} constant. f_D and f_E are the modulation frequencies. The period of oscillation of f_{ZF} is determined at the output at the top right.*

An oscillator with the starting value $f_D \approx 1.20$ nHz phase-modulates oscillator 2 of the heterodyne method (f_{osz2}). Its parameters f_D, a_D and ϕ_D are iterated with the aim of making all slow frequency fluctuations (also the frequency drift) of f_{GW} disappear. This is checked by counting the period of f_{ZF}. The accuracy increases if you choose the value $f_{sampling}/8$ or $f_{sampling}/10$ for f_{ZF}. If it turns out that f_{GW} is modulated with two different low frequencies, calculate the frequency of Oscillator-2 using the formula

$$f_{osz2} = f_{GW} - f_{osz1} + f_{ZF} + t \cdot \dot{f}_{GW} + a_D \cdot sin(2\pi t f_D + \phi_D) + a_E \cdot sin(2\pi t f_E + \phi_E) \tag{2}$$

The parameters have the following meaning:
f_{GW} is the frequency of the suspected GW.
f_{ZF} is basically arbitrary. A special value is chosen here, where the oscillation period is an integer in relation to the sampling period.
$\dot{f}_{GW}$ is the frequency drift of f_{GW} (initial value=0 Hz/s)
a_D is the modulation index (section 5) of the PM assuming that the planet D describes a circular orbit around the GW source.
f_D is the orbital frequency of the suspected planet D.

From the phase ϕ_D it follows when D is in front of or behind the GW source from the point of view of the earth.
a_E, f_E and ϕ_E apply accordingly to the assumed planet E. Addends for additional planets may be added.

The aim is to determine the values of the parameters in equation (2) in such a way that f_{ZF} is constant (figure 4). Then the intermediate frequency is unmodulated. The signal amplitude is irrelevant. Astronomical information such as the orbital period of the planet may be calculated from the parameters.

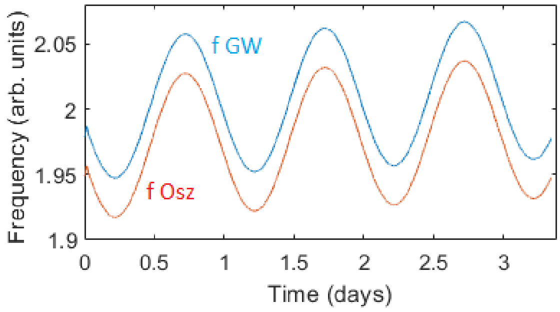

Figure 4): *The idea behind the MSH method: The frequency of the GW oscillates around an average value that slowly increases. If it is possible to generate an auxiliary frequency f_{Osz} with identical modulation, the difference $f_{GW} - f_{Osz} = f_{ZF}$ (= vertical distance between the two curves) is constant.*

4.2 Nearby planets

When the distance between the binary star and the planet is small, the high orbital frequency causes the sidebands to consist of spectral lines that are far apart. Example: If a planet has an orbital period of 52 days, f_{GW} is phase-modulated with $f_{mod} = 223$ nHz. With the estimated value $a \approx 2.5$ the Carson bandwidth $2f_C \cdot (a + 2) \approx 2000$ nHz is calculated.

If you process the signal with this enormous bandwidth, the S/N drops and it is hopeless to discover and identify the many lines in the noise. The MSH method allows analysis with a much smaller bandwidth (BW$\approx$ 1 nHz) without distorting the PM. This improves the S/N by a factor of 2000.

In addition, the energy of the GW is distributed over about nine lines with different amplitudes. The MSH method does *not* search for these lines in the noise in order to reconstruct a single spectral line at f_{GW}. With MSH, an auxiliary frequency is generated that has as many properties as possible of the GW. The smaller the differences, the larger the amplitude of the intermediate frequency f_{ZF} because there are no longer any energy-consuming sidebands.

The big advantage over other methods: Every GW signal is accompanied by strong noise, the mean value of which is close to zero. In the long run ($T \approx 20$ years) the noise cannot systematically change the amplitude of the GW signal.

The modulation index is unknown. The PM generated by different planets superimpose linearly. This means that each PM can be analyzed separately.

5 The Modulation Index *a*

Each antenna rotates around the sun with orbital velocity v_{orbit} and approaches the GW source for a certain period of time. Then the reception frequency is higher than the transmission frequency because of the Doppler effect. Six months later the antenna is moving away at the same rate and we are receiving a reduced frequency. Accordingly, one can also measure whether the GW source moves periodically. The highest frequency deviation from the mean is called the frequency deviation Δf.

Example: A GW source lies in the ecliptic plane and emits a GW of frequency 10 μHz. The earth moves with the maximum speed of 30 km/s in the radiation field. Since one orbit lasts one year, $f_{mod} = 31.688$ nHz. Assuming that GW propagate at the speed of light, we get

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{orbit}}{c - v_{orbit}}} - 1 \right) \approx f_{GW} \cdot \frac{v_{orbit}}{c} \approx f_{GW} \cdot 10^{-4}. \tag{3}$$

This limits the modulation index to the maximum value

$$a = \frac{\Delta f}{f_{mod}} = \frac{10 \times 10^{-6} \cdot 10^{-4}}{31.688 \times 10^{-9}} = 0.032 \tag{4}$$

However, measurements at $f_{GW} \approx 10$ μHz usually deliver values [7] that are about 100 times higher. PM is so easy to demodulate that measurement errors can be ruled out. With increasing frequency, a [8] decreases and at $f_{GW} \approx 60$ Hz (crab pulsar) values $\Delta f \approx f_{GW} \cdot 10^{-4}$ are obtained. Does v_{GW} depend on the frequency?

6 Summary

From a communications point of view, decoding the phase modulations of f_{GW} is a standard task if the signal has a good S/N. The opposite is true for the interpretation of the results from an astronomical point of view: the high values for the frequency deviation (Δf) can only be explained by the assumption that gravitational waves at low frequencies around 8 μHz do not propagate at the speed of light, but considerably more slowly.

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