Inhomogeneous distribution of the universe's matter density as a physical basis for MOND's acceleration a_0

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Abstract

One of the most effective theories for dark matter is Milgrom's Modified Newtonian Dynamics, where a modified law of gravity based on a fixed acceleration scale a_0 is postulated that provides a correct description of the gravitational fields in galaxies. However, the significance of a_0 is unknown, and the whole theory is generally viewed as a phenomenological description of the observations. Based on Newton's gravitational law as applied to a uniform continuous mass we posit a non-homogeneous distribution of mass at cosmological scales that would gives rise to a constant acceleration and agrees with MOND's a_0. The implications for MOND as a viable theory of dark matter and for the problem of dark energy are discussed.

Modified Newtonian Dymanics (MOND) is a Newtonian-derived hypothetical model of gravity proposed 40 years ago by Mordehai Milgrom to explain the multiple gravitational anomalies observed in galaxies and galaxy clusters [1-3]. They are summarized and conventionally explained through the existence Dark Matter, an elusive new form of matter that interacts only gravitationally and is not included in the Standard Model of Particle Physics. While no such particles have yet been found, the search goes on and MOND usually plays a secondary role in the list of candidate explanations for dark matter. One of the reasons is that a_0, the distinctive feature of MOND, does not correspond to any physical entity, and –it is argued- was postulated solely as a means to obtain a gravitational law that fits the observations. It is sometimes called a phenomenological explanation.

While a_0 agrees to within one order of magnitude with the acceleration calculated at the border regions of the observable universe from the simple Newtonian gravitational formula and is also

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found to relate to Hubble's constant and to the square root of the cosmological constant Λ, in both cases scaled by the speed of light *c*, no physical representation of such an acceleration has been devised, and most physicists would agree that it represents another constant of nature, whose role would be to relate fundamental gravitational phenomena in the low-acceleration regime, implying probably some modification of the laws of gravity.

The Newtonian ball model of gravity

A generally accepted assumption of all current astrophysical models is the Cosmological Principle, the idea that the universe at large scales is both homogeneous and isotropic. While it may still be isotropic and strong constraints have been set on the range of variation in matter density, the homogeneity condition has little theoretical supporting evidence. Based on original ideas of Isaac Newton, we shall argue that the universe can be modelled as a nearly homogeneous continuous distribution of mass that obeys simple dynamics embodied in the Universal Law of Gravitation. As Newton amazingly found in the late 1600s [5], when a continuous distribution of mass with constant density is allowed to evolve according to such law, an acceleration appears that is null at the center and increases outwards in linear proportion to radial distance until it reaches, for a distance equal to the radius of the ball, the exact same value as predicted by conventional Newtonian gravity.

$$F_B = G\,M\,m\,r\,/\,R^3$$

as opposed to a point-mass gravitational field:

$$F_N = G\,M\,m\,/\,R^2$$

where F_B (force in the Newtonian ball model) and F_N (Newton's conventional point-mass gravitational force) are the force on a test particle with mass m placed at a distance r from the center of the R-ball, or at a distance R from the central point-mass M, respectively. The acceleration for the ball with mass M is then

$$Acc_B = G\,M\,r\,/\,R^3$$

and solving for G

$$G = Acc_B\,R^3\,/\,M\,r$$

We now define G' as $4\pi G$ and substitute it for G above. The resulting expression is mathematically equivalent, though it may facilitate the visualization of upcoming considerations.

$$G' = (Acc_B\,4\pi\,R^3)\,/\,(M\,r) \qquad\qquad [G' := 4\pi G]$$

And multiplying both parts of the right-hand quotient by a factor of three,

$$G' = 3 \, Acc_B \, 4/3 \, \pi \, R^3 \, / \, M \, r$$

and since $4/3 \, \pi \, R^3 \, / \, M$ is the inverse of the matter density for the spherical volume,

$$G' = 3 \, (Acc_B \, / \, r) \cdot (1/\rho)$$

$$G' = 3 \, Acc_B \, / \, (r \cdot \rho) \tag{1}$$

where ρ is now the average, not necessarily constant matter density of the universe.

Looking at equation (1) we see that in such a ball model of the universe, if ρ is constant, then the quotient ($Acc_B \, / \, r$) must be constant, which agrees with the Newtonian view but does not help us understand the existence of a constant acceleration pervading the whole universe that at the same time agrees with the Newtonian acceleration at its border regions, as MOND postulates and available evidence strongly suggests.

We therefore let ρ vary with radial distance, however small the constant of proportionality may be, and assume that it is the product in the denominator of Equation (1) ($r \cdot \rho$) that is constant. In other words, we let density to decay as the inverse of radial distance. We immediately see that since both G' and the product ($r\cdot\rho$) are constant, so must be Acc_B, and this acceleration agrees with MOND's universal acceleration a_0 and with the calculated Newtonian acceleration at the border regions of the ball to whithin one order of magnitude, as can be easily checked. Indeed, feeding in the accepted values for the mass of the observable universe (10^{53} Kg), radial distance (10^{26} m) and G, it turns out that the acceleration perceived at the border regions of the observable universe is about $3.4 \cdot 10^{-10}$ m·s^{-2}, quite close to the reported value for a_0 ($1.2\cdot10^{-10}$). And according to the Newtonian ball model, assuming $r\cdot\rho$ constant, this same aceleration would be present as a background curvature in the whole universe.

The range of variation in mass density that could be expected depends on how far we are from the central region of the universe, and can be approximately estimated:

From Eq (1), taking $Acc_B = a_0 = 1.2\cdot10^{-10}$ ms^{-2}; $R_U = 4.4\cdot10^{26}$ m; $G' = 8.38\cdot10^{-10}$ m^3·Kg^{-1}·s^{-2}, we have

$$r\cdot\rho = 0.4295 \, Kg\cdot m^{-2}$$

$$\rho = 0.4295 \, / \, r$$

Assuming we are in a mid-radius region, $R_0 = 2.2 \cdot 10^{26}$ m and making dr = 1 Mpc = $3.1 \cdot 10^{22}$ m, it turns out that the expected decrease in density per Mpc at a radius half the universe's radius would be:

$d\rho = -0.4295 \cdot R_0^{-2} \cdot dr$

$d\rho = 6.29 \cdot 10^{-30}$ Kg/m³/Mpc

This is aproximately 1% of the accepted baryonic mass density of the universe (4.6% of the critical density 10^{-26} Kg/m³, or $4.6 \cdot 10^{-28}$ Kg/m³). For regions closer to the center, the predicted relative variations are larger, in more external regions they become much smaller and practically unmeasurable.

Observational evidence for the distribution of mass density in the universe is scant. The large-scale average density of the universe, known as the cosmic density parameter, Ω, depends on its composition and, according to the ΛCDM model, is very close to the critical mass density Ω_c, the one required to make the universe flat. The density of matter, including dark matter would amount to about 28% of the global density ($\Omega_m = 0.28$), while the density of baryonic matter is though to comprise a bare 4.6% of the total density. Distribution of average density as a function of distance is generally assumed to follow the general trend of decreasing as the radius increases, reflecting the overall dilution of matter on larger scales, but observations are dominated by a complex hierarchical structure, the so-called cosmic web, that hinders any precise average estimates. Therefore, no reliable data are currently available.

Several authors [6 - 10] notably Peebles, Karachentsev, Nuza and others have probed into the mass distribution in the vicinity of our Milky Way and found that, on average, its density is significantly lower than the average for the whole universe. We would thus be in a local region of low density, the Local Void, which makes the observations not representative of the universe. The interpretation of the results is also compounded by the influence of dark matter and structure formation, two processes of which we know little.
In two important studies [6, 7] the authors examined the distribution of the mean density of matter in spheres of various radii in our Local Universe and found that matter density up to about 50 Mpc decays with distance. The authors conclude that density is on average lower that the global density for the universe ($\Omega_{m,local} = 0.08$ vs $\Omega_m = 0.28$) and tends to an asymptotic minimum value. However, looking at the data in the figures, we speculate that they might also be consistent with a 1/r decay in that range. However, as the authors point out, larger scale distances are needed to avoid local variations, probably 100 Mpc at least. In the papers, uncertainties in the range up to 90 Mpc seem too large to draw a conclusion. As we have seen, a reliable measurerment of the average baryonic-mass density around the Milky Way could be used to gauge our proximity to the center of the universe.

Another interesting observation is the striking ressemblance of equation (1) with the Friedmann equation. For flat space (k = 0), the Friedman equation can be expressed as

$G' = 4\pi G = (3/2) \cdot H^2 / \rho$ which certainly reminds us of Eq 1:

$G' = 3 \cdot Acc_B / (r \cdot \rho)$ and since dimensions of Accel / r are $1/T^2$, we have

G' = 3· $(1 / t)^2$ · $(1 / \rho)$

If we then interpret 1/t as the constant rate of expansion H,

G' = 3 · H^2 / ρ

which differs from the Friedman equation by a factor of 2. The reason for the discrepancy we ignore, but it has happened before in astrophysics that a classical, non-relativistic approach has been later superseded by the appropriate relativistic version that differs from it by a factor of two, e.g., in the old pre-Einstein estimation of the lensing of light from Newtonian gravity by Johann Soldner in 1804.

Thus, the hypothesis of a decreasing matter density that scales inversely with distance seems a reasonable one and, from Newtonian mechanics, this would lead to a constant background cosmic acceleration that agrees with MOND's a_0 and would account for the rotation curves in galaxies. The observed galactic accelerations below a certain threshold also agree with MOND and turn out to be the geometric average of the Newtonian and the background a_0. This would now be understood as a real physical phenomenon related to the interaction of two competing accelerations, not only a mathematical construct.

We cannot discuss here the other predictions of MOND related to dark matter. We would rather refer the reader to the works of the original author [1-3]. We should point out however that the discrepancies of MOND with the observations in galaxy clusters might be addressed by an averaging of the gravitational fields due to the cluster itself and to nearby galaxies [4]. We are not aware of any consistent explanation for the accelerations observed in colliding clusters like the Bullet. As for the CMB, we have argued elsewhere [4] that our understanding of the CMB has some problems that limit its ability to be used as the gold standard to adjudicate prospective fundamental theories.

We therefore conclude that

1.   In a modified Newtonian ball model of the universe, a continuously decreasing matter density that scales as 1/r, as opposed to the uniform distribution from the Cosmological Principle, would give rise to a constant universal physical acceleration that agrees with MOND's a_0.

2.   This could provide a physical basis for MOND and support it as a viable interpretation of the dark matter problem.

3.   The resulting matter-density distribution may be hard to verify experimentally, for the densities involved, as well as the variations incurred are very low. A variation in mass density around 1% per Mpc is expected.

Cosmological acceleration as a basis for the universe's expansion

We now turn our attention to the mysterious empirical relation observed between a_0 and the parameters that reflect the universe's expansion, H_0 and Λ.

Indeed, the numerical value of MOND's a_0 has been found to be approximately

$$a_0 \sim (c / 2\pi) \cdot H_0 \sim (c^2 / 2\pi) \cdot \mathrm{SQRT}(\Lambda/3)$$

Why is that? What is the intimate relation of a_0 to the accelerated expansion of the universe?

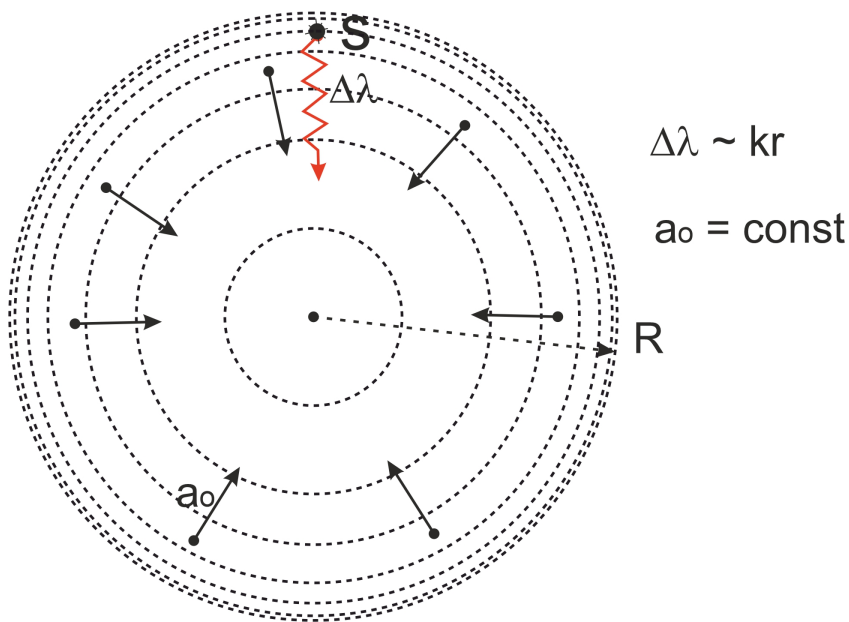Observable Universe with a_0

Fig 1. A non-homogeneous universe with matter density that decays as $1/r$ would generate a constant background acceleration a_0 that pervades the whole universe. This would give rise to a redshit corresponding to velocities that increase more than linearly with distance. No expansion is needed. Varying intervals of elementary space layers representing acceleration are greatly exaggerated.
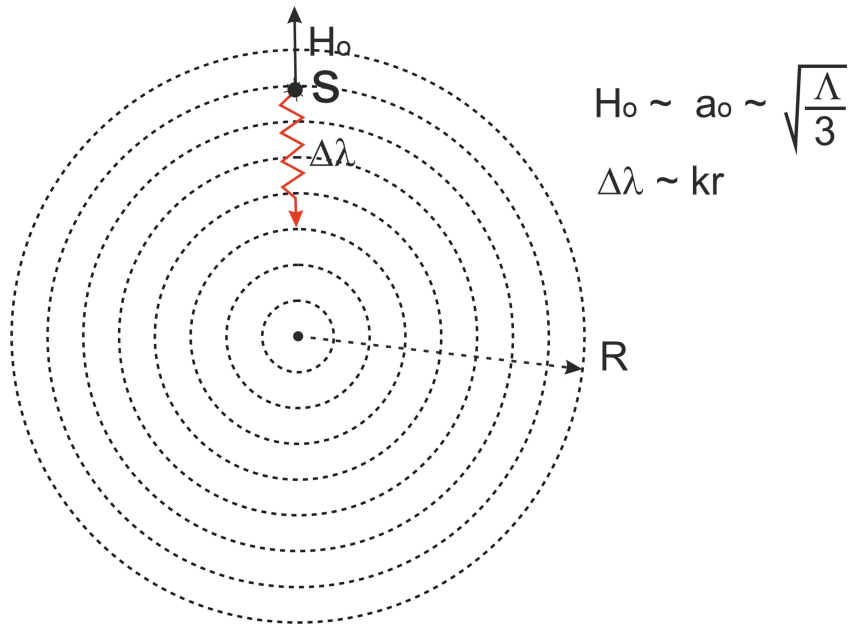
Observable Flat Universe with Λ

$$H_o \sim a_o \sim \sqrt{\frac{\Lambda}{3}}$$

$$\Delta\lambda \sim kr$$

Fig 2. Current models of the universe postulate a constant acceleration for all unbound stellar bodies that agrees with a_0, H, and Λ. Recessional velocities and redshift that increase with distance indicate that an expansion is taking place.

Let's take a look at the modified Newtonian ball model of gravity as applied to the whole universe. The postulated real universe (Fig 1) consists of a spacetime network with a constant acceleration (a_0) where the separation between neighbouring layers of the network decays linearly with radial distance, i.e., there is a constant gradient of the deformation in the radial direction that corresponds to the constant acceleration a_0. In our current cosmological models in contrast, flat space is assumed (Fig 2) and the intervals between neighboring nodes and layers of the network are constant, there is no definite acceleration in space. But we then observe that faraway galaxies seem to recede with a constant acceleration that turns out to be the same or very close to a_0 (Fig 2). From Hubble's Law and previous observations on the redshift of moving stellar bodies, recessional velocities that increased linearly with distance was interpreted as the cumulative effect of a universe that was expanding. More precise observations in 1998 showed that the speeds were actually accelerating, as if a constant energy pervading all the space was driving them apart.

From general Relativity's Equivalence Principle, the scenario of a flat universe and a cosmic accelerated expansion (Fig 2) is completely indistinguishable from a static universe with a constant gravitational field pervading it (Fig 1). All gravity-related phenomena –including redshift- must be present equally in both situations. A decision as to which is happening must come from external observations or reasoning. For any observer bound to a galaxy and placed a large distance away from us, it would be impossible to tell which one of these is happening:

1.   The galaxy is accelerating away from the center of the universe by a_0 in flat space. (Fig 2). And since all neighboring galaxies are reporting the same, with velocities that increase with distance at a constant rate, the universe must be expanding all around them at an accelerated rate. Or else,

2.   The galaxy is rotationally bound but static, immersed in a field with constant acceleration a_0 (Fig 1). And since all other galaxies are experiencing the same and the field does not decay as the inverse square of distance but remains constant, the conclusion would be that a constant acceleration due to a new form of gravity is pervading the whole universe.

It is worth noting that redshift would not allow them to tell which one is true. Of course, if there is evidence suggesting a non-homogeneous distribution of matter, or if we lack any ideas about the origin of the energy that drives the expansion, this would favor the conclusion that the accelerated outward motion is apparent and the real thing is a constant gravitational field that generates a redshift proportional to recessional velocity. Moreover, any interpretation that relies on option 1 must remain silent about the relation of a_0 with the Hubble parameter and the cosmological constant Λ. According to option 2, these are but two equivalent ways to represent the same phenomenon.

We notice that this does not invalidate redshift as an accurate indicator of both velocity and distance for luminous stellar bodies. That redshift increases linearly with recessional velocity has been confirmed in multiple studies. In reality, galaxies are either accelerating away from us with a very small acceleration a_0 (option 1, Fig 2) or they stay separated from us by a space with constant curvature and acceleration (option 2, Fig 1). Since a_0 is so small, redshift at shorter ranges increases approximately linear with distance, and this allows us to use it as a robust indicator of distance. It is only with very precise measurements at large cosmic scales that its true behavior is exposed. This is what the groups of Riess, Perlmutter and Schmidt accomplished in 1998, establishing a solid relation of redshift to velocities that are slowly increasing at a rate that scales with Λ and a_0.

Discussion and Q&A

The first comment that comes to mind is how plausible a non-uniform distribution of matter is, given the fundamental character of homogeneity and isotropy in modern cosmology. Even assuming that inhomogeneities in the mass distribution are constrained by the CMB and the wide field observations of distant galaxies, small inhomogeneities like the ones predicted here cannot be currenlty ruled out. As for the theoretical arguments, given that we can only record with certainty a minute fraction of the matter assumed as present in the universe, this highlights our limitations to determine theoretical estimates and boundaries for the mass density distribution.

On the other hand, a central concentration of mass density would intuitively make sense when considering masses governed only by gravity. Some authors have reported decreasing densities with distance in the local universe, although these are not representative of the whole.

Moreover, an agreement with observations without the need for dark matter or dark energy might count as partial supporting evidence. But a definitive answer must come from direct observations. Measuring a gradient in the mass density across the universe seems the most promising, if challenging path. The expected rate of decrease in mass density (about 1% per Mpc) might be within reach of our present technology.

Q. -Assuming that mass density varies with radial distance in a model that was meant to describe uniformly distributed masses, does it not disprove the argument ?

    A. The Newtonian model for gravity in solid spheres is valid not only for spheres with uniform density, but for any sphere in which density depends only on radial distance, i.e, for any spherically symmetrical distribution of matter.

Q. -How well does the model support MOND as an effective theory for dark matter ?

    A. MOND has been considered by most authors either as a modification of the laws of gravity awaiting proper justification, or as a mere mathematical description based on the introduction of a new free variable a_0, to fit the observations. We would claim that it is neither.
The main drawback of MOND has been so far its speculative nature and the arbitrary splitting of the gravitational law in two domains, corresponding to accelerations higher and lower than a_0. The view that a_0 is a real acceleration based on a plausible distribution of matter and on the Newtonian laws of gravity makes it more plausible. It would explain not only the value of a_0, but also the fact that MOND kicks in at a definite threshold acceleration. MOND asserts that the observed accelerations in MOND regime are the geometric average of Newtonian acceleration and a_0, and this can now be understood as the influence of a constant, backgrond acceleration playing its role only when the Newtonian gravitational field is comparable or weaker than a_0.

Q. -How does this model affect other parts of the current ΛCDM cosmological framework such as dark energy and the Big Bang?

    A. It is currently difficult to forsee the impact that a confirmation of the physical nature of a_0 and a non-homogeneous distribution of matter would bring about, but a revision of dark energy and the concept of expansion would certainly be among the first consequences. The idea that the observed expansion of the universe is an apparent phenomenon due to a constant cosmological acceleration seems a disturbing hypothesis that is almost guaranteed to earn strong rejection. And yet, most of our observations would remain valid. Our fundamental theories, in particular General Relativity would not only be unaffected but would be further confirmed, driving away recent suspicions that it might need modification. It would mean just a change in perspective, an indeed a more effective one in terms of explanatory power vs complexity. Perhaps it would also make Einstein happy, for he could finally do away with the cosmological constant, that dreaded parameter that he called the worst blunder of his life.

Q. -If the universe has been found to be approximately flat from multiple observational evidences, how can it contain a constant acceleration that behaves as a positive curvature?

A. The curvature implied in the Newtonian ball model is subsumed and interpreted in the FLRW model by the idea of expansion. It was argued that both concepts are equivalent and interchangeable, according to the Equivalence Principle. The only thing needed to change the framework is to put away the assumption of homogeneity and to show that a physical constant acceleration exists. A cosmological acceleration a_0 is then substituted for the cosmological constant, and an equivalent universe without dark energy nor expansion results. In ΛCDM as it stands, this is not possible, for both ΛCDM and the Friedmann equations depend on a uniform and isotropic universe as their fundamental starting point.

Q. -Why don't astrophysical observations support the view that there is a center of the universe? Why is redshift approximately the same in all directions?

A. Unless we are very close to the actual center of the universe –which seems unlikely though not impossible- there should be ways to tell where the center of the universe is, or at least in what direction it lies. Masses and distances for faraway galaxies are hard to assess by means other than redshift. Uncertainties grow at larger distances, and current technology has limited power to resolve them at cosmic scales. On the other hand, it seems that redshift should provide useful clues. Unfortunately, light reaching us from the center regions of the universe is also expected to be redshifted, just as when it comes from the periphery. The same reasoning applies as above. When looking towards the center of the universe, a constant gravitational field is present, with us in the high-potential side of it. The situation is equivalent to us moving away from the center, which in turn is equivalent to the observed stars in the center moving away from us. Thus, light received from stationary galaxies in the central regions of the universe would be also redshifted. (Incidently, this is probably the main reason why the universe was viewed as isotropic from the very start by cosmologists). However, when looking in the centripetal direction, the number of galaxies and the mass density as a function of redshift should increase as we approach the center, followed by a steep decrease when looking through into the other side. Conversely, when looking in the other direction away from the center, the number of galaxies and matter density should always decrease, as does the average density in the theoretical model. The precision needed would still be a challenge, the expected variations in matter density being of the order of 10^{-28} Kg/m^3/Mpc, or around 1% per Mpc for the mid-radius regions of the universe.

Lastly, it can be argued that this discussion is limited to the non-relativistic case, where time is absolute and space is treated quite naively. The observation would be pertinent, but we are not all that sure that absolute space is not a real feature of the universe. Special and General Relativity are the only correct descriptions of the universe we live in, but this might be only as long as we do not take actual deformations of spacetime into account. Einstein himself warned us in his famous Leiden address of 1920 that spacetime might in the end turn out to be a real entity, albeit possibly an unmeasurable one. And perhaps by real, in this context, he meant

absolute. At any rate, the present discussion seeks to adress very real problems in the astrophysical realm that for the most part take place at sub-relativistic speeds. The approach is classical, intuitive and Newtonian because this is the only way our imagination can be put to work, but we expect that a formulation that respects Lorentz invariance and the postulates of the Relativity should be eventually worked out.

Two developments are expected for the future that would help in solving these issues: On the one hand, accurate measurements of average mass-densities at sufficiently large scales. On the other, an understanding of the deeep mechanisms underlying gravity, based on the fundamental structure of spacetime, a goal that both Newton and Einstein longed for unsuccessfully.

Acknowledgements

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