No Singularities in Schwarzschild Black Hole and Big Bang

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Abstract: We prove no singularities in Schwarzschild black hole (SBH) and Big Bang considering the quantum effect. We find the equation of gravitational energy density inside SBH; derive that the gravitational energy density is proportional to the square effective temperature far from the event horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; obtain their entropic density is direct proportion with the effective temperature. Then we assume that the center of SBH and Big Bang being in the minimum entropy state, which value is equal to the Boltzmann constant; propose and prove the SBH uncertainty relation (UR) and Big Bang UR by the generalized relational expression (GRE), which suggest no singularity in them.

1. Introduction

S.W. Hawking and R. Penrose proved the theory of singularities [1-3]. It shows that the singularities are in the black holes and the universe originated the Big Bang singularity. Many literatures discussed no singularity in black holes and Big Bang with the quantum effect; please refer to [4-18]. Moreover, M. Planck considered the reduced Planck constant ħ being the minimum action [19]. R. Penrose considered the Big Bang being in the minimum entropy equal to zero [20], which is the initial condition of Big Bang probably. Similarly, we propose the center of Schwarzschild black hole (SBH) and Big Bang being in the minimum entropy, but the minimum entropy doesn't equal to zero. Then we can prove the uncertainty relation (UR) of Schwarzschild black hole (SBH) and Big Bang UR which suggest no singularity in them [21].

This paper is organized as follows. In Sec. 2, we find that the gravitational energy density is proportional to the square effective temperature far from the horizon inside SBH, and its entropic density is direct proportion with the effective temperature. In Sec. 3, we propose the SBH UR and Big Bang UR by the generalized relational expression (GRE), and prove them. We conclude in Sec. 4.

2. Relations for Coulomb-like Gravitational Fields and Wave-like Ones

In this section, we review [22] briefly; find that the gravitational energy density is proportional to the square effective temperature far from the event horizon inside SBH, and its entropic density is direct proportion with the effective one.

2.1 Relations for Coulomb-like gravitational fields

First let us review [22] briefly. Gravitational fields can be classified two types: Coulomb-like gravitational fields and wave-like ones. In general, they are mixed. For the Coulomb-like gravitational fields

$$8\pi\rho_{grav} = 2\alpha\sqrt{2W/3} \text{ and } p_{grav} = 0 \tag{1}$$

where ρ_{grav} is the gravitational energy density, α is a constant, $W = T_{abcd}u^a u^b u^c u^d$, T_{abcd} is the Weyl tensor, u^a, u^b, u^c, u^d are the timelike unit vectors, and p_{grav} is the isotropic pressure. The Schwarzschild geometry can be written in Gullstrand– Painlevé coordinates as

$$ds^2 = -[1-(2m/r)]\, c^2dt^2 - 2\sqrt{2m/r}\,dr\,c\,dt + dr^2 + r^2 d\Omega^2 \tag{2}$$

where $m = GM/c^2$ is the constant mass parameter, G the gravitational constant, and c the speed of light in vacuum. Gravitational energy density and temperature is given at each point in the region $r < 2m$ by

$$\rho_{grav} = 2\alpha mc^4/8\pi r^3 \tag{3}$$

$$T_{grav} = \hbar c m/2\pi\kappa r^2\sqrt{|1-(2m/r)|} \tag{4}$$

where T_{grav} is the effective temperature and κ the Boltzmann constant.

Taking (4) to (3), we find

$$\rho_{grav} = \alpha\pi[2-(r/m)]\kappa^2c^2T_{grav}^2/\hbar^2G \tag{5}$$

That is the equation concerning the gravitational energy density and effective temperature in the region $r < 2m$. Note that the isotropic pressure is zero. When $r \ll 2m$, we derive

$$\rho_{grav} = 2\alpha\pi\kappa^2c^2T_{grav}^2/\hbar^2G \tag{6}$$

So the gravitational energy density is proportional to the square effective temperature far from the horizon inside SBH. It includes two regions far from horizon inside SBH: singularity and vacuum. In (5) when $r \to 0$, we gain (6) also,

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that is the gravitational energy density being proportional to the square effective temperature in the singularity and near the one.

2.2 Relations for the wave-like gravitational fields

In [22], for the wave-like gravitational fields

$$8\pi\rho_{grav}=\beta\sqrt{4W} \text{ and } p_{grav}=\rho_{grav}/3 \quad (7)$$

where β is a constant. For the SBH, the gravitational energy density is given at each point in region $r < 2m$ by

$$\rho_{grav}=\sqrt{6}\beta mc^4/r^3 \quad (8)$$

Taking (4) to (8), we find

$$\rho_{grav}=\sqrt{6}\beta\pi[2-(r/m)]\kappa^2c^2T_{grav}^2/\hbar^2G \quad (9)$$

When $r << 2m$ or $r \to 0$, we obtain

$$\rho_{grav}=2\sqrt{6}\beta\pi\kappa^2c^2T_{grav}^2/\hbar^2G \quad (10)$$

Above is very similar to (6). Therefore, the gravitational energy density is proportional to the square effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

2.3 Relations of entropic density

For the Coulomb-like gravitational fields, its entropic density s_{grav} is [22]

$$\delta s_{grav}=\delta(\rho_{grav}v)/\kappa T_{grav} \quad (11)$$

where $v = z^a\eta_{abcd}dx^bdx^cdx^d$ and we can set an arbitrary constant to zero.

Substituting (5) into (11) and integral, we get

$$s_{grav}=\alpha\pi[2-(r/m)]\kappa^2c^2T_{grav}/\hbar^2G \quad (12)$$

That is the equation concerning the entropic density and effective temperature in the region $r < 2m$. When $r << 2m$ or $r \to 0$, we derive

$$s_{grav}=2\alpha\pi\kappa^2c^2T_{grav}/\hbar^2G \quad (13)$$

So the entropic density is direct proportion to the effective temperature far from the horizon inside SBH.

For the wave-like gravitational fields, substituting (9) into (11) and integral, we obtain

$$s_{grav}=\sqrt{6}\beta\pi[2-(r/m)]\kappa^2c^2T_{grav}/\hbar^2G \quad (14)$$

When $r << 2m$ or $r \to 0$, we derive

$$s_{grav}=2\sqrt{6}\beta\pi\kappa^2c^2T_{grav}/\hbar^2G \quad (15)$$

Therefore, the entropic density is direct proportion to the effective temperature far from the horizon inside SBH, whether the gravitational fields are Coulomb-like or wave-like.

3. No Singularity in SBH and Big Bang

In this section, we review [21] briefly, propose the SBH UR and Big Bang UR by the GRE, assume that the SBH center and Big Bang are in the minimum entropy being equal to the Boltzmann constant, find the temperature of SBH center and its volume having the inversely-proportional relationship, prove the SBH UR and the Big Bang UR.

3.1 Basic relationship and GRE

Basic relationship [21] is

$$A \sim A_P =[\hbar^{(\alpha+\beta+\gamma+\delta)}\ G^{(\alpha-\beta+\gamma-\delta)}\ c^{-(3\alpha-\beta+5\gamma-5\delta)}\ \kappa^{-2\delta}\ e^{2\varepsilon}]^{1/2} \quad (16)$$

Where A is any physical quantity, $[A] = [L]^\alpha[M]^\beta[t]^\gamma[T]^\delta[Q]^\varepsilon$ its dimensions, L, M, t, T and Q are the dimensions of length, mass, time, temperature and electric charge separately (here we use the LMTΘQ units [21]), A_P the corresponding Planck scale of A, α, β, γ, δ and $\varepsilon$the real number, ħ, G, c, κ and e the reduced Planck constant, gravitational constant, speed of light in vacuum, Boltzmann constant and elementary charge separately.

GRE [21] is

$$\prod_{i=1}^{n} A_i^{\alpha_i} \sim \prod_{i=1}^{n} A_{iP}^{\alpha_i};\ i = 1, 2, 3\ldots n \quad (17)$$

where A_i is the physical quantity, α_i the real number, and A_{iP} the corresponding Planck scale.

3.2 Big Bang UR

S.W. Hawking and R. Penrose proved that the universe originated the Big Bang singularity [23]. Many literatures discussed no singularity at the Big Bang and black holes with the quantum effect, please refer to [24] [9-29]. One of the characteristic of Big Bang singularity is zero volume and limitless high temperature.

Then we can find the relationship of Big Bang temperature and its volume by the GRE (17) [21]

$$T_BV_B \sim T_PV_P=T_PL_P^3=\hbar^2G/\kappa c^2 \quad (18)$$

where T_B is the Big Bang temperature, V_B its volume, T_P the Planck temperature, $V_P= L_P^3$ the Planck volume, and $L_P =\sqrt{\hbar G/c^3}$ the Planck length. Above is the Big Bang UR. That is impossible to measure the Big Bang temperature and its volume simultaneously. When $\hbar \to 0$, we obtain

$$T_BV_B \sim 0 \quad (19)$$

Because $T_B > 0$[30], we gain $V_B \sim 0$, the Big Bang volume is zero, thus the Big Bang singularity appears without the quantum effect. We suggest no singularity at the Big Bang with quantum effect.

3.3 SBH UR

Similarly considering the SBH mass and its volume, we find [21]

$$M_H V_H \sim \mathrm{M_P V_P} = \mathrm{M_P L_P^3} = \hbar^2 \mathrm{G/c^4} \quad (20)$$

where M_H is the SBH mass, V_H its volume, and the $\mathrm{M_P}$ Planck. Above is the SBH UR. Also that is impossible to measure the SBH mass and volume simultaneously. When $\hbar \to 0$, we obtain

$$M_H V_H \sim 0 \quad (21)$$

Because $M_H > 0$, we have $V_H \sim 0$, the volume is zero, the SBH singularity appears without quantum effect also. We also suggest no singularity in SBH with quantum effect. Taking $M = \rho V$ to (20), we gain

$$M_H^2/\rho_H \sim \hbar^2 \mathrm{G/c^4},\ \rho_H V_H^2 \sim \hbar^2 \mathrm{G/c^4} \quad (22)$$

where ρ_H is the mass density of SBH. Above are the URs for the mass density of SBH and its mass or volume.

3.4 Proving SBH UR

We prove (20) now. For the Coulomb-like gravitational fields, from (13) and $S_{grav} = \int_V s_{grav}$[22], we obtain

$$S_{grav} = 2\alpha\pi\kappa^2 \mathrm{c}^2 T_H V_H/\hbar^2 \mathrm{G} \quad (23)$$

where S_{grav} is the entropy, and V the spatial volume. For the SBH center, $V \to V_H$ and $T_{grav} \to T_H$, where T_H is the temperature of center.

We assume $S_{grav} \sim \kappa$, which is SBH center being in the minimum entropy equal to Boltzmann constant, and the Boltzmann constant being the minimum entropy, resembling ħ. Then we find

$$T_H V_H \sim \hbar^2 \mathrm{G}/2\alpha\pi\kappa \mathrm{c}^2 \quad (24)$$

Therefore, the temperature of SBH center and its volume has the inversely-proportional relationship.

From the gravitational analogue of the fundamental law of thermodynamics in the form [22]

$$T_{grav} \mathrm{d}S_{grav} = \mathrm{d}U_{grav} + p_{grav} \mathrm{d}V \quad (25)$$

where U_{grav} and p_{grav} denote the internal energy and isotropic pressure of the free gravitational field, respectively. Taking (24), $p_{grav} = 0$ [22], and $\mathrm{d}U_{grav} \approx \mathrm{d}(M_H \mathrm{c}^2)$ to (25), we give

$$M_H V_H \sim \hbar^2 \mathrm{G}/2\alpha\pi \mathrm{c}^4 \quad (26)$$

Similarly for the wave-like gravitational fields, $p_{grav} = \rho_{grav}/3$ [22], we obtain

$$M_H V_H \sim 3\hbar^2 \mathrm{G}/8\sqrt{6}\beta\pi \mathrm{c}^4 \quad (27)$$

Then we prove (20).

3.5 Gravitational fields near SBH center

Substituting $M_H = \rho_{grav} V_H/\mathrm{c}^2$ and $V_H = 4\pi R^3/3$ to (26), we get

$$\rho_{grav} \sim 9\hbar^2 \mathrm{G}/32\alpha\pi^3 \mathrm{c}^2 R^6 \quad (28)$$

where R is the SBH radius. So the energy density of Coulomb-like gravitational fields is inversely proportional to sextic radius.

Similarity

$$\rho_{grav} \sim 27\hbar^2 \mathrm{G}/128\sqrt{6}\beta\pi^3 \mathrm{c}^2 R^6 \quad (29)$$

The energy density of wave-like gravitational fields is inversely proportional to sextic radius.

Taking $a \sim \mathrm{c}^2/R$ [31] to (28), we obtain

$$\rho_{grav} \sim 9\hbar^2 \mathrm{G} a^6/32\alpha\pi^3 \mathrm{c}^{14}$$

$$\to a \sim \mathrm{c}^2 \sqrt[6]{32\alpha\pi^3 \rho_{grav} \mathrm{c}^2/9\hbar^2 \mathrm{G}} \quad (30)$$

It is the gravitational acceleration for Coulomb-like gravitational fields near SBH center.

Substituting $M_H^2/\rho_{grav} \sim \hbar^2 \mathrm{G}/2\alpha\pi \mathrm{c}^6$ to (30), we get

$$a \sim 2\mathrm{c}^3 \sqrt[3]{\alpha\pi^2 M_H \mathrm{c}/3\hbar^2 \mathrm{G}} \quad (31)$$

Thus gravitational acceleration for Coulomb-like gravitational fields near SBH center is direct proportion to the third SBH mass.

Similarity

$$\rho_{grav} \sim 27\hbar^2 \mathrm{G} a^6/128\sqrt{6}\beta\pi^3 \mathrm{c}^{14}$$

$$\to a \sim 2\mathrm{c}^2 \sqrt[6]{2\sqrt{6}\beta\pi^3 \rho_{grav} \mathrm{c}^2/27\hbar^2 \mathrm{G}} \quad (32)$$

This is the gravitational acceleration for wave-like gravitational fields near SBH center.

Similarity

$$a \sim 2\mathrm{c}^3 \sqrt[3]{4\sqrt{6}\beta\pi^2 M_H \mathrm{c}/9\hbar^2 \mathrm{G}} \quad (33)$$

The gravitational acceleration for wave-like gravitational fields near SBH center is direct proportion to the third SBH mass.

Taking (6) to (30), we gave

$$T_{grav} \sim 3\ \hbar^2 \mathrm{G} a^3/8\alpha\pi^2 \mathrm{kc}^8 \quad (34)$$

Therefore, the effective temperature is direct proportion to cube gravitational acceleration for Coulomb-like gravitational fields near SBH center, like Unruh formula [32].

Substituting (33) to (34), or $S_{grav} \sim \kappa$, $p_{grav} = 0$ [22], and $\mathrm{d}U_{grav} \approx \mathrm{d}(M_H \mathrm{c}^2)$ to (25), we obtain

$$T_{grav} \sim M_H \mathrm{c}^2/\mathrm{k} \quad (35)$$

So the effective temperature is direct proportion to SBH mass for Coulomb-like gravitational fields. Because $M_H > M_\mathrm{P}$, we get $T_{grav} > T_\mathrm{P}$, that is the temperature near SBH center being higher than Planck one.

Similarity

$$T_{grav} \sim 3\sqrt{3}\hbar^2 \mathrm{G} a^3/16\sqrt{6}\beta\pi^2 \mathrm{kc}^8 \quad (36)$$

The effective temperature is direct proportion to cube gravitational acceleration for wave-like gravitational fields near SBH center.

And

$$T_{grav} \sim 2\sqrt{3}M_H c^2/3\mathrm{k} \quad (37)$$

Thus the effective temperature is direct proportion to SBH mass for wave-like gravitational fields. Also the temperature near SBH center is higher than Planck one.

3.6 Proving Big Bang UR

For a spatially flat Robertson–Walker geometry with scalar perturbations in a longitudinal gauge, such that the line-element can be written [22]

$$\mathrm{d}s^2 = a^2(\tau)[-c^2(1+2\Phi)\mathrm{d}\tau^2 + (1-2\Phi)(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)] \quad (39)$$

$$u^a = [(1-\Phi)/a;\ u^i],\ z^a = (0;\ \nabla_i\Phi)/a\,|\,\nabla\Phi\,| \quad (40)$$

$$S_{grav} \sim \mathrm{k}t^{5/3} \quad (41)$$

where a is the scale factor, u^a the timelike unit vector, z^a a spacelike unit vector, and $t = \int a(\tau)d\tau$ the proper time of comoving observers.

When $t \to 0$, $S_{grav} \to 0$, so R. Penrose considered the Big Bang being in the minimum entropy equal to zero [20]. But we propose

$$S_{grav} \sim \mathrm{k}t^{5/3} + S_{grav0} \quad (42)$$

where $S_{grav0} \geq 0$ is the minimum entropy. When $t \to 0$, $S_{grav} \to S_{grav0}$, $V \to V_B$, and $T_{grav} \to T_B$, Substituting them, $\mathrm{d}U_{grav} = \mathrm{d}(\rho_{grav}V)$, and $p = \omega\rho_{grav}$ into (25), we obtain

$$T_B\mathrm{d}S_{grav0} \sim V_B\mathrm{d}\rho_{grav} + (1+\omega)\rho_{grav}\mathrm{d}V_B \quad (43)$$

where ω is the coefficient of state.

From [22]

$$8\pi\mathrm{G}\rho_{grav} = \alpha\,|\,(a^4\dot{u}_{<i,j>})z^i z^j\,|\,/a^3 \quad (44)$$

$$T_{grav} = \hbar\,|\,H\,|\,/2\pi\mathrm{k}c \quad (45)$$

where i, j are spatial indices, we gain

$$\rho_{grav} \sim \kappa^2 c^2 T_{grav}^2/\hbar^2\mathrm{G} \quad (46)$$

Taking it to (43), we give

$$\mathrm{d}S_{grav0} \sim \kappa^2 c^2[2V_B\mathrm{d}T_B + (1+\omega)T_B\mathrm{d}V_B]/\hbar^2\mathrm{G} \quad (47)$$

Ordering $\omega = -1$, integrating, afterwards assuming $S_{grav0} \sim \kappa$, that is Big Bang being in the minimum entropy equal to Boltzmann constant also, we find

$$T_B V_B \sim \hbar^2\mathrm{G}/2\kappa c^2 \quad (48)$$

Hence we prove (18). Note (24) and (48), they are analogous, but their physical meaning aren't same.

4. Conclusion

In this paper, we found the equation concerning the gravitational energy density and effective temperature in the region $r < 2m$ [22] inside SBH; derived that the gravitational energy density is proportional to the square effective temperature far from the horizon in SBH interior, whether the gravitational fields of SBH are Coulomb-like or wave-like; obtained that their entropic density are direct proportion with the effective one. These equations are true in the singularity and near the one of SBH also; proposed the SBH UR and Big Bang UR by the GRE which suggests no singularity in them; assumed the center of SBH being in the minimum entropy equal to the Boltzmann constant κ, found the temperature of center and its volume having the inversely-proportional relationship; proved the SBH UR; derived energy density is inversely proportional to sextic radius and direct proportion sextic gravitational acceleration; got effective temperature is direct proportion to cube gravitational acceleration, like Unruh formula [32]; obtained effective temperature is direct proportion to SBH mass and higher than Planck temperature; proposed the Big Bang being in the minimum entropy equal to κ also, and proved the Big Bang UR.

From the original definition of entropy $S = Q/T$, for the Big Bang, the heat quantity Q is tremendous but finite, if $S \to 0$, the temperature $T \to \infty$, that is infinite. Above is the classical solution. Considering the quantum effect, T is impossibly infinite, so $S \neq 0$. Then we proposed the Big Bang being in the minimum entropy equal to κ. Similarity for the center of SBH, $Q \approx M_H c^2$, the temperature of center isn't infinite with quantum effect. Note here we only consider the center of SBH, not the total SBH, it isn't against principle of entropy increase of black holes.

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