A natural explanation of cosmological acceleration

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Abstract

The problem of cosmological acceleration (PCA) is usually considered in the framework of General Relativity and here the main uncertainty is how the background space is treated. In the approaches where it is flat, PCA is usually treated as a manifestation of dark energy and (as acknowledged in the literature) currently its nature is a mystery. On the other hand, if the background space is curved then a problem arises why the observed value of the cosmological constant is as is. Following the results of our publications, we show that the solution of PCA does not contain uncertainties because cosmological acceleration is an inevitable kinematical consequence of quantum theory in semiclassical approximation. In this approach, background space and its geometry (metric and connection) are not used and the cosmological constant problem does not arise.

Keywords: quantum de Sitter symmetry; cosmological acceleration; irreducible representations

In the problem of cosmological acceleration (PCA), only nonrelativistic macroscopic bodies are involved, and one might think that here there is no need to involve quantum theory. However, ideally, the results for every classical (i.e., non-quantum) problem should be obtained from quantum theory in semiclassical approximation. We will see that, considering PCA from the point of view of quantum theory sheds a new light on understanding this problem.

In PCA, it is assumed that the bodies are located at large (cosmological) distances from each other. Therefore, interactions between them can be neglected and we can consider quantum theory of N free bodies. It is also reasonable to assume that the sizes of the bodies are much less than distances between them. Therefore, the internal degrees of freedom of those bodies can be neglected and, from the formal point of view, the description of our system is the same as the description of N spinless elementary particles.

In the literature, symmetry in Quantum Field Theory (QFT) is usually explained as follows. Since Poincare group is the group of motions of Minkowski space, the system under consideration should be described by unitary representations of this group. This approach is in the spirit of the Erlangen Program proposed by Felix Klein.

However, Minkowski space is only a classical concept. In particle theory, transformations from Poincare group are not used because, according to the Heisenberg S-matrix program, it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$. In this theory, systems are described by observable physical quantities — momenta and angular momenta. So, in fact, symmetry at the quantum level is defined not by a background space but by a representation of a Lie algebra A by self-adjoint operators (see [1, 2] for more details).

Then each elementary particle is described by an irreducible representation (IR) of A and a system of N noninteracting particles is described by the tensor product of the corresponding IRs. This implies that, for the system as a whole, each momentum operator is a sum of the corresponding single-particle momenta, each angular momentum operator is a sum of the corresponding single-particle angular momenta, and *this is the most complete possible description of this system*. In particular, nonrelativistic symmetry implies that A is the Galilei algebra, relativistic symmetry implies that A is the Poincare algebra, the de Sitter (dS) symmetry implies that A is the dS algebra so(1,4) and the de anti-Sitter (AdS) symmetry implies that A is the AdS algebra so(2,3).

In his famous paper "Missed Opportunities" [3] Dyson notes that:

- a) Relativistic quantum theories are more general (fundamental) than nonrelativistic quantum theories even from pure mathematical considerations because Poincare group is more symmetric than Galilei one: the latter can be obtained from the former by contraction $c \to \infty$.

- b) dS and AdS quantum theories are more general (fundamental) than relativistic quantum theories even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one: the latter can be obtained from the former by contraction $R \to \infty$ where R is a parameter with the dimension *length*, and the meaning of this parameter will be explained below.

- c) At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

As noted above, symmetry at the quantum level should be defined by a Lie algebra, and in [2], the statements a)-c) have been reformulated in terms of the corresponding Lie algebras. It has also been shown that the fact that quantum theory is more general (fundamental) than classical theory follows even from pure mathematical considerations because formally the classical symmetry algebra can be obtained from the symmetry algebra in quantum theory by contraction $\hbar \to 0$. *For these reasons, the most general consideration of PCA should be carried out in terms of dS or AdS symmetries.*

The definition of quantum dS and AdS symmetries is as follows. If M^{ab} ($a, b = 0, 1, 2, 3, 4$, $M^{ab} = -M^{ba}$) are the angular momentum operators for the system under consideration, they should satisfy the commutation relations:

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \qquad (1)$$

where $\eta^{ab} = 0$ if $a \neq b$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{44} = \mp 1$ for the dS and AdS symmetries, respectively.

Although the dS and AdS groups are the groups of motions of dS and AdS spaces, respectively, the description in terms of relations (1) does not involve those spaces at all, and *those relations can be treated as a definition of dS and AdS symmetries at the quantum level* (see the discussion in [1, 2]).

In QFT, *Minkowski, dS and AdS spaces are auxiliary mathematical concept for describing interacting fields*. However, since we consider only noninteracting bodies, we don't need to use these spaces.

The procedure of contraction from dS or AdS symmetry to Poincare one is defined as follows. If we *define* the momentum operators P^μ as $P^\mu = M^{4\mu}/R$ ($\mu = 0, 1, 2, 3$) then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (1) become the commutation relations for the Poincare algebra (see e.g., [1, 2]). Here R

is a parameter which has nothing to do with the relation between the Minkowski and dS/AdS spaces.

As seen from Eqs. (1), quantum dS and AdS theories do not involve the dimensionful parameters $(c, \hbar, R)$ at all. In other words, one can say that Eqs. (1) are written in units $(c = \hbar = R = 1)$. The parameters (kg, m, s) are meaningful only at the macroscopic level.

In particle theories, the quantities c and $\hbar$ typically are not used and it is said that the units $(c = \hbar = 1)$ are used. Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8 m/s$ and $\hbar \approx 1.054 \cdot 10^{-34} kg \cdot m^2/s$ and those values are as are simply because people want to describe velocities in m/s and angular momenta in $kg \cdot m^2/s$. At the same time, physicists usually believe that physics should derive the value of the cosmological constant Λ and that the solution to the dark energy problem depends on this value.

The cosmological constant has a physical meaning only at the classical level. At this level, Λ is the curvature of the background space and equals $3/R^2$ where R is the radius of this space. As noted below, in semiclassical approximation, R is the same as the parameter R in quantum theory where this parameter is only the coefficient of proportionality between $M^{4\mu}$ and P^μ. As follows from the above discussion, at the quantum level, the quantity R is fundamental to the same extents as c and $\hbar$. Here the question why R is as is does not arise simply because the answer is: because people want to describe distances in meters. There is no guaranty that the values of $(c, \hbar, R)$ in (kg, m, s) will be the same during the whole history of the universe.

Standard particle theories involve IRs of the Poincare algebra by self-adjoint operators. They are described even in textbooks and do not involve Minkowski space. Therefore, when Poincare symmetry is replaced by more general dS or AdS one, dS and AdS particle theories should be based on IRs of the dS or AdS algebras by self-adjoint operators, respectively. However, physicists usually are not familiar with such IRs because they believe that dS and AdS quantum theories necessarily involve quantum fields on dS or AdS spaces, respectively.

The important observation is that, at this stage, we have no spatial coordinates yet. For describing the motion of particles in terms of spatial coordinates, we must define the position operator. A question: is there a law defining this operator?

The postulate that the coordinate and momentum representations are related by the Fourier transform was taken at the dawn of quantum theory by analogy with classical electrodynamics, where the coordinate and wave vector representations are related by this transform. But the postulate has not been derived from anywhere, and there is no experimental confirmation of the postulate beyond the nonrelativistic semiclassical approximation. Heisenberg, Dirac, and others argued in favor of this postulate but, for example, in the problem of describing photons from distant stars, the connection between the coordinate and momentum representations should be not through the Fourier transform, but as shown in [2]. However, since, PAC involves only nonrelativistic bodies then, as follows from the above remarks, the position operator in momentum representation can be defined as usual, i.e., as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$ where $\mathbf{p}$ is the momentum.

The mathematical literature on unitary IRs of the dS group is wide but there are only a few papers where such IRs are described for physicists. For example, the excellent Mensky's book [4] exists only in Russian. At the same time, to the best of our knowledge, IRs of the dS algebras by self-adjoint operators have been described only in [2, 5, 6].

The explicit derivation in Chap. 3 of [2] (see also [7]) gives that, for the dS case in semiclassical approximation, for each pair of bodies in the N-body system, the relative acceleration is given by

$$\mathbf{a} = \mathbf{r}c^2/R^2 \tag{2}$$

where $\mathbf{a}$ and $\mathbf{r}$ are the relative acceleration and relative radius vector of the bodies, respectively. An analogous calculation using the results of Chap. 8 of [2] on IRs of the AdS algebra gives that in the AdS case, $\mathbf{a} = -\mathbf{r}c^2/R^2$, i.e., we have attraction instead of repulsion. The experimental facts that the bodies repel each other show that in PCA, dS symmetry is more relevant than AdS one. The fact that the relative acceleration of noninteracting bodies is not zero does not contradict the law of inertia, because this law is valid only in the case of Galilei and Poincare symmetries, and in the formal limit $R \to \infty$, $\mathbf{a}$ becomes zero as it should be.

Relative accelerations given by Eq. (2) are the same as those derived from General Relativity (GR) if the curvature of dS space equals $\Lambda = 3/R^2$, where R is the radius of this space. *However, the crucial difference between our results and the results of GR is as follows. While in GR, R is the radius of the dS space and can be arbitrary, in* *quantum theory, as noted above, R is the coefficient of proportionality between $M^{4\mu}$ and P^μ, this coefficient is fundamental to the same extent as c and $\hbar$, and a question why R is as is does not arise.*

In GR, the result (2) does not depend on how Λ is interpreted, as the curvature of empty space or as the manifestation of dark energy. However, in quantum theory, there is no freedom of interpretation. Here R is the parameter of contraction from the dS Lie algebra to the Poincare one, it has nothing to do with the radius of the background space and with dark energy and it must be finite because dS symmetry is more general than Poincare one.

Attempts to derive the value of Λ have been made in the problem of cosmological constant which starts from Poincare invariant QFT of gravity on Minkowski space. This theory contains only one phenomenological parameter — the gravitational constant G, and Λ is defined by the vacuum expectation value of the energy-momentum tensor. The theory contains strong divergencies which cannot be eliminated because the theory is not renormalizable. The results can be made finite only with a choice of the cutoff parameter. Since G is the only parameter in the theory, the usual choice of the cutoff parameter in momentum space is $\hbar/l_P$ where l_P is the Plank length. Then, if $\hbar = c = 1$, G has the dimension $1/length^2$ and Λ is of the order of $1/G$. However, this value is more than 120 orders of magnitude greater than the experimental one.

As explained above, in quantum theory, Poincare symmetry is a special degenerate case of dS symmetry in the formal limit $R \to \infty$, R is a parameter of contraction from dS algebra to Poincare one, this parameter has nothing to do with the relation between Poincare and dS spaces and the problem why R is as is does not arise by analogy with the problem why c and $\hbar$ are as are. Therefore, the cosmological constant problem and the problem why the cosmological constant is as is do not arise.

Therefore, the phenomenon of cosmological acceleration has nothing to do with dark energy or other artificial reasons. This phenomenon is an inevitable kinematical consequence of quantum theory in semiclassical approximation and the problem of cosmological constant does not arise.

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