

The Elemental Property of Primes and Small Gaps Between Primes

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Abstract

The solution to the Twin Prime Conjecture lies in the elemental property of primes. We construct a sequence of consecutive primes, analyzing and handling them by the combination of the elemental property of primes and the Statistics theory reveal that Twin Prime Conjecture is true.

Keywords. Twin Prime Conjecture, Statistics theory

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1 Introduction

One of the most famous problems in mathematics is Twin Prime Conjecture. Up to now, Y. Zhang [1], Terence Tao, James Maynard and dozens of mathematicians [2] have succeeded in making dramatic new progress. Unfortunately, there is a key limitations inherent in standard sieve methods. This conjecture is still very much open and very significant new ideas are required for the final proof.

The solution lies in Twin Primes themselves.

2 The small gap between primes

Elemental property of primes *Every prime greater than 3 must be of either the form “ $6N+1$ ” or the form “ $6N-1$ ” (N , integer ≥ 1)*

We construct and consider a number sequence:

$$\{p_n, p_{n+1}\}$$

where p_n, p_{n+1} are consecutive primes, p_n is the n -th prime.

By **Elemental property of primes**, p_n and p_{n+1} must be of either the form “ $6N+1$ ” or the

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form “ $6N-1$ ”.

Thus, the number sequence must be only three cases:

- ① $\{p_n = 6N - 1, p_{n+1} \geq 6N + 5\}$
- ② $\{p_n = 6N - 1, p_{n+1} = 6N + 1\}$
- ③ $\{p_n = 6N + 1, p_{n+1} \geq 6N + 5\}$

For every n and every p_n , the number sequence must be one of the three cases. By Statistics theory: as $n \rightarrow \infty$, every case \rightarrow infinitely often.

Hence,

$$(\text{ case } \textcircled{2}) \quad \liminf_{n \rightarrow \infty} (p_{n+1} - p_n) = 2$$

$$(\text{ case } \textcircled{1} \text{ and } \textcircled{3}) \quad \liminf_{n \rightarrow \infty} (p_{n+1} - p_n) > 2$$

Thus, Twin Prime Conjecture is true.

Terence Tao and James Maynard have proved that there are infinitely many consecutive primes with a distance of 246 at most. Therefore:

$$(\text{ case } \textcircled{1} \text{ and } \textcircled{3}) \quad 2 < \liminf_{n \rightarrow \infty} (p_{n+1} - p_n) \leq 246$$

3 Conclusion

Wir müssen wissen

Wir werden wissen (D. Hilbert)

In 1900, D. Hilbert listed Twin Prime Conjecture as one of the 8th mathematical problems at the International Mathematical Conference. Today, it is proved by way of the combination of the elemental property of primes and the Statistics theory.

References

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