Sequences of prime numbers

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Abstract. By combining two theorems of the theory "the octets of odd numbers" we obtain an algorithm for finding prime numbers.

1. Introduction

"The octets of the odd numbers" theory categorizes odd numbers based on the form of the octet they belong to or produce (see [7]). An algorithm for finding prime numbers results from this categorization. This algorithm does not have the limitations of the so far known formulas for calculating prime numbers (see [1-6] and [8-15]).

2. The sequences W, Θ, U, Φ

"The octets of the odd numbers" theory categorizes odd numbers into four categories (see [7], equations (5.7), (5.8)):

$$D_1 = 11 + 8m' = 3 + 8m, m \in \mathbb{N} \tag{1}$$

$$Q_1 = 13 + 8m' = 5 + 8m, m \in \mathbb{N} \tag{2}$$

$$D_2 = 7 + 8m, m \in \mathbb{N} \tag{3}$$

$$Q_2 = 9 + 8m' = 1 + 8m, m \in \mathbb{N} \ . \tag{4}$$

Based on these equations we define the sequences W, Θ, U, Φ:

Definition 1. We define as sequences W, Θ, U, Φ the following four functions of N and n:

$$\begin{aligned} W_n &= 1 + 8(N+n) \\ \Theta_n &= 3 + 8(N+n) \\ U_n &= 5 + 8(N+n) \ . \\ \Phi_n &= 7 + 8(N+n) \\ N, n &\in \mathbb{N} \end{aligned} \tag{5}$$

For each pair (N, n) the sequences W, Θ, U, Φ give four consecutive odd numbers. When these numbers are composite, their factors cannot have the same form (see [7], section 3, corollary 2). Therefore we have four consecutive odd numbers with factors of different form. The different structure of these numbers gives an algorithm for finding prime numbers.

3. The algorithm

In equations (5) we randomly choose N and give values $n = 0, 1, 2, 3, \ldots$. Through a primality or factorization test we ask for terms of the sequences W, Θ, U, Φ that are prime numbers or the product of a large prime with smaller ones. The sequences W, Θ, U, Φ maintain this pattern, for all levels of values of N: as the algorithm runs, these numbers appear with great frequency.

4. The application of the algorithm

For small values of N the algorithm always has a high efficiency. Thus we apply the algorithm for large values of N.

We apply the algorithm by randomly choosing $N(145 digits) =$
676788754321074332198453289754398765321809853782212349847651209165397452197064363218360098994322134543677648897532134456765456539809087540845321 (144 digits).
We run the algorithm for values of n from $n = N$ to $n = N + 160$:

$n = N + 3$. $W =$ 11 × 13^2 × 2912 485225 695855 114355 904420 675196 408055 125782 817481 871318 563568 221183 226238 039217 722905 213531 241030 849607 956525 151735 155526 440192 830930 783627 (142 digits).
$\Theta =$ 3 × 5 × 29 × 89 × 139 850446 456634 241446 148167 842830 688946 777187 918318 450181 614611 218472 933425 713932 732710 339453 818341 117123 109677 909261 930882 697980 588213 879033 (141 digits).
$n = N + 4$. $\Theta =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762603 (145 digits) **is prime**.
$n = N + 8$. $\Theta =$ 5 × 23 × 47080 956822 335605 718153 272330 740783 674560 685480 501728 685053 997159 331996 674578 390484 755485 147431 105011 734097 314610 931092 644553 498421 501741 971849 (143 digits).
$n = N + 11$. $\Phi =$ 4 317701 × 1 253979 845887 567170 025813 811108 085094 955504 985235 823138 003583 312814 662158 768408 212089 461681 564929 363184 116081 956637 820834 310586 193548 997563 (139 digits).
$n = N + 14$. $\Phi =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762687 (145 digits) **is prime**.
$n = N + 15$. $U =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762693 (145 digits) **is prime**.
$n = N + 18$. $W =$ 7 × 773472 862081 227808 226803 759719 312874 653496 975751 099828 397315 667617 597088 225216 415106 697255 993511 010907 060170 168608 153664 874807 474067 528618 108959 (144 digits).
$U =$ 139 × 2659 × 14 649067 601463 726173 867566 153866 440086 943700 991765 982231 598966 651397 533062 888127 753298 505122 969302 237681 773564 412047 250018 597493 833817 279517 (140 digits).
$n = N + 20$. $\Theta =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762731 (145 digits) **is prime**.

$U =$ 3 × 2143 × 842 169860 719955 616361 428887 546304 265449 444521 738637 237327 921865 503683 250517 423379 335336 878512 144513 353464 176571 824090 162408 407577 923891 791377 (141 digits).

$n = N + 25$. $\Phi =$ 3 × 5^2 × 13 × 83 × 66 905283 096306 390578 778205 968924 190578 615740 874361 430939 526841 808133 205036 472226 206325 372776 701601 190601 435788 449268 775460 286095 995955 518403 (140 digits).

$n = N + 27$. $\Phi =$ 11 × 492210 003142 599514 326147 847094 108192 961316 257296 154436 252837 243029 379965 234228 627795 170981 086779 734213 583744 652750 643241 283968 392588 427302 432981 (144 digits).

$n = N + 30$. $\Theta =$ 3 × 7 × 32983 × 7 816884 072413 342309 945565 490498 265517 119899 905517 992383 928242 504902 496116 435905 518061 802100 006175 008408 980082 351596 819218 737000 617161 655177 (139 digits).

$n = N + 31$. $\Theta =$ 23 × 235404 784111 678028 590766 361653 703918 372803 427402 508643 425269 985796 659983 372891 952423 777425 737155 525058 670486 573054 655463 222767 492107 508709 859253 (144 digits).

$n = N + 35$. $W =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762849 (145 digits) **is prime**.

$n = N + 36$. $\Phi =$ 347 × 421 × 37 062230 277633 154610 524046 068679 554803 469705 245899 353116 849614 772862 606649 301545 693640 644218 544956 610440 499094 240124 553547 705492 743862 905849 (140 digits).

$n = N + 40$. $W =$ 3^2 × 17 × 1427 × 24 798631 594086 935238 640533 492885 527582 315286 561494 697495 001670 277345 771409 357878 202119 171313 073164 490381 215636 717905 728706 063968 554500 736619 (140 digits).

$n = N + 52$. $U =$ 11 × 19 × 2113 × 12 260193 866107 044469 727447 806663 217499 721430 176505 204280 589763 694158 466765 492530 644759 782327 117337 141345 150189 372820 963989 438025 072568 991517 (140 digits).

$n = N + 63$. $\Theta =$ 3 × 5^2 × 51517 × 1 401300 550515 647173 447632 514324 770495 842661 342924 393578 503202 094667 308427 011540 502681 155292 933614 684175 300371 056869 790723 870735 826613 273373 (139 digits).

$n = N + 65$. $\Phi =$ 5 × 23 × 3847 × 12 238356 335413 466524 084552 204507 612080 728018 060957 038909 553937 395199 375272 830358 847090 066323 740864 312902 026855 890546 163931 518975 414999 153099 (140 digits).

$n = N + 89$. $W =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763281 (145 digits) **is prime**.

$n = N + 90$. $\Phi =$ 5 × 13 × 83297 077454 901456 270578 866431 310617 270376 597388 579981 519710 918051 125840 270407 921626 875089 106993 493482 298787 556619 339625 448056 189514 964620 411743 (143 digits).

$n = N + 91$. $\Theta =$ 60353 × 89 710702 609126 218375 020733 319556 444958 402711 219950 935310 278025 505329 968975 469569 130728 891553 933973 064295 415160 476812 679636 863988 840201 818083 (140 digits).

$n = N + 92$. $\Phi =$ 31 × 173 × 881 × 1 145933 499146 651121 239896 418545 956333 539086 990559 754300 609191 467522 175975 924607 842008 837361 463446 640283 080837 694239 754684 824669 371076 995237 (139 digits).

$n = N + 93$. $W =$ 883 × 6131 721443 452542 081073 189488 148573 185248 560396 667835 559208 617976 583442 375511 341909 113115 279676 757730 860046 649128 263958 838192 131730 999660 619211 (142 digits).

$n = N + 94$. $\Theta =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763323 (145 digits) **is prime**.

$n = N + 102$. $U =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763389 (145 digits) **is prime**.

$n = N + 104$. $\Theta =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763403 (145 digits) **is prime**.

$n = N + 109$. $\Phi = 3^2 \times 314453 \times$ 1 913131 704391 292059 398958 515275 446612 432975 791915 802573 138896 812109 062621 821425 673487 640368 779569 275447 071295 650350 529563 020247 264817 423811 (139 digits).

$n = N + 110$. $\Phi = 5 \times 7 \times$ 154694 572416 245561 645360 751943 862574 930699 395150 219965 679463 133523 519417 645043 283021 339451 198702 202181 412034 033721 630732 974961 494813 505723 621813 (144 digits).

$n = N + 112$. $U = 7^2 \times$ 110496 123154 461115 460971 965674 187553 521928 139393 014261 199616 523945 371012 603602 345015 242465 141930 144415 294310 024086 879094 982115 353438 218374 015581 (144 digits).

$n = N + 117$. $U = 23 \times 73 \times$ 3224 723070 022986 693024 196734 982245 457161 690786 335734 841442 054599 954246 347573 862361 969553 777221 308562 447540 911959 652814 564695 445097 363133 011771 (142 digits).

$n = N + 122$. $W = 5 \times$ 1 082862 006913 718931 517525 263607 038024 514895 766051 539759 756241 934664 635923 515302 981149 376158 390915 415269 884238 236051 415130 824730 463694 540065 352709 (145 digits).

$n = N + 127$. $W = 3 \times 5 \times$ 360954 002304 572977 172508 421202 346008 171631 922017 179919 918747 311554 878641 171767 660383 125386 130305 138423 294746 078683 805043 608243 487898 180021 784239 (144 digits).

$n = N + 128$. $\Phi = 139439 \times$ 38 829237 405378 657747 026486 980222 105168 385307 053677 226592 138567 210917 889669 149340 613077 265269 792361 364822 045419 002266 766500 933399 683536 889441 (140 digits).

$n = N + 130$. $W = 3^2 \times 7 \times$ 85941 429120 136423 136311 528857 701430 517055 219527 899980 933035 074179 733009 802801 823900 744139 554834 556767 451130 018734 239296 097200 830451 947624 234343 (143 digits).

$n = N + 133$. $\Phi = 3 \times 29 \times$ 62233 448673 202237 443535 934690 059656 581315 848623 651710 330818 501992 220455 374442 700066 056101 056949 161797 119783 806669 621559 242800 601361 755176 169697 (143 digits).

$n = N + 146$. $\Phi = 97 \times$ 55817 629222 356645 954511 611526 135980 645097 719899 563905 142074 326529 104944 511098 091811 823513 319119 351302 571352 486394 402841 795089 199159 512374 502719 (143 digits).

$n = N + 151$. $\Phi = 3 \times 1217 \times$ 1482 966320 068089 470716 961467 552777 354854 691544 852834 510759 027574 177808 714756 646098 533793 698152 445104 450676 853240 278574 542351 041445 760805 348333 (142 digits).

$n = N + 154$. $W = 3 \times 73 \times 2953 \times$ 8 372122 204597 436949 944296 749587 046564 478935 329689 795840 745824 110954 697595 010591 977119 283990 979805 501331 238398 811605 681791 172280 984236 602243 (139 digits).

$\Theta = 11 \times$ 492210 003142 599514 326147 847094 108192 961316 257296 154436 252837 243029 379965 234228 627795 170981 086779 734213 583744 652750 643241 283968 392588 427302 433073 (144 digits).

$n = N + 158$. $\Phi =$ 5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763839 (145 digits) **is prime**.

The algorithm has already shown its two main characteristics:

1. It gives prime numbers with great frequency. In the app we did he gave the prime numbers

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762603

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762687

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762693

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762731

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 762849

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763281

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763323

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763403

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763403

5 414310 034568 594657 587626 318035 190122 574478 830257 698798 781209 673323 179617 576514 905746 880791 954577 076349 421191 180257 075654 123652 318472 700326 763839

in a small range of n from $n = N$ to $n + N + 160$. For the value of N we chose (145 digits) these numbers are probably consecutive primes. If this is confirmed for very large values of N, this will mean that the algorithm gives the set of prime numbers.

2. The idea on which the construction of the algorithm was based is confirmed. As the value of n increases, the number of digits of the sequences W, Θ, U, Φ increases and decreases periodically, following a certain pattern.

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