New Consideration of Electromagnetic Induction

Faraday's Law of Electromagnetic Induction vs. Lorentz's Magnetic Field Force Theorem

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Abstract: Faraday's law of electromagnetic induction reveals that the induction electromotive force generated in a metal coil is proportional to the change rate of the magnetic flux passing through the coil. Lorentz's magnetic field force theorem reveals that an electric charge moving in a magnetic field is affected by the Lorentz magnetic field force. Lorentz's magnetic field force theorem is the microscopic physical essence of the induction electromotive force. An induction electromotive force will be generated between the two ends of a metal wire moving in a magnetic field. In this study, calculation formulas of the electromotive force of metal wires were separately derived based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem. When a metal wire moves at a uniform speed in a magnetic field, the calculation formulas derived from both of them are the same. When a metal wire moves back and forth sinusoidally in a magnetic field, the electromotive forces of the wire derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem are different. Lorentz's magnetic force theorem is a universal fundamental electromagnetic theorem. Therefore, Faraday's law of electromagnetic induction is an engineering approximation formula. This study proposes the electron motion resistance force theorem: When an electron moves in the metal wire, it will be affected by the motion resistance force, and the electron motion resistance force is proportional to the speed of the electron. An electric charge moving in a uniform magnetic field is affected by the Lorentz magnetic field force, which is the microscopic physical essence of the motional electromotive force. An electric charge at rest in a changing magnetic field wave is also affected by the Lorentz magnetic field force, which is the microscopic physical essence of the induced electromotive force. The electromotive force in metal wires and coils is essentially the result of the counter-potential movement of electric charges under the action of the Lorentz magnetic field force. This study reveals that Faraday's law of electromagnetic induction is an engineering approximation formula, which is a great challenge for Maxwell's equations and the fundamental electromagnetic theorems.

Keywords: Faraday's Law of Electromagnetic Induction, Lorentz's Magnetic Field Force Theorem, Electron Motion Resistance Force Theorem, Induction Electromotive Force, Motional Electromotive Force, Induced Electromotive Force, Maxwell's Equations.

1. Introduction

In 1820, the Danish physicist Oster found that an electric current could produce a magnetic field. Many scientists began to explore and study whether a magnetic field could also produce an electric current. In 1831, Faraday revealed for the first time through experiments that a changing magnetic flux in a metal coil could produce induced current and voltage in the metal coil.

As shown in Figure 1.1, a metal coil was connected in series with voltmeter V. When magnetic rod B was inserted or pulled out of metal coil C, coil C generated an induction electromotive force, and the pointer of voltmeter V deflected. The faster the insertion or withdrawal was, the greater the induction electromotive force generated by coil C. When the magnetic rod was stopped in the coil, there was no induction electromotive force in the coil, and the pointer of voltmeter V did not deflect.

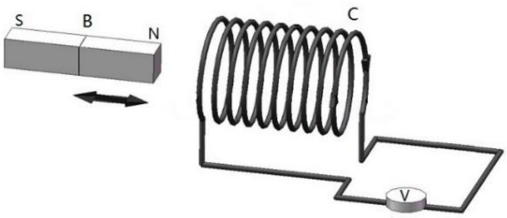

Figure 1.1 Electromagnetic induction in a metal coil

Through a large number of experiments [1] [2] [3], Faraday revealed that the induction electromotive force generated in a metal coil was proportional to the change rate of the magnetic flux passing through the coil. This conclusion is called Faraday's law of electromagnetic induction. The induction electromotive force is expressed in Faraday's law as:

$$\varepsilon = - d\Phi/dt \qquad (1\text{-}1)$$

where "-" indicates the direction of the induction electromotive force, which is determined by Lenz's law.

Faraday's law of electromagnetic induction states that whenever the magnetic flux passing through the coil changes, an induction electromotive force is generated in the coil. There are two ways in which the magnetic flux can change. The first is when the magnetic field intensity remains constant while the whole or part of the metal loop moves in the magnetic field. The electromotive force generated in this way is called the motional electromotive force. The second is when no part of the metal loop moves while the space magnetic field changes. The electromotive force generated in this way is called the induced electromotive force.

In Figure 1.1 above, the electromotive force generated by coil C is the induced electromotive force. Below is an example to further analyze and illustrate the motional electromotive force.

As shown in Figure 1.2, which is cited from a physics textbook [4], in uniform magnetic field B, metal wireframe ABCD is placed. The wireframe has two parts. The fixed U-shaped part of the wireframe is composed of metal wires AD, AB and BC. The metal wire CD can slide left and right, and its length is L_{CD}.

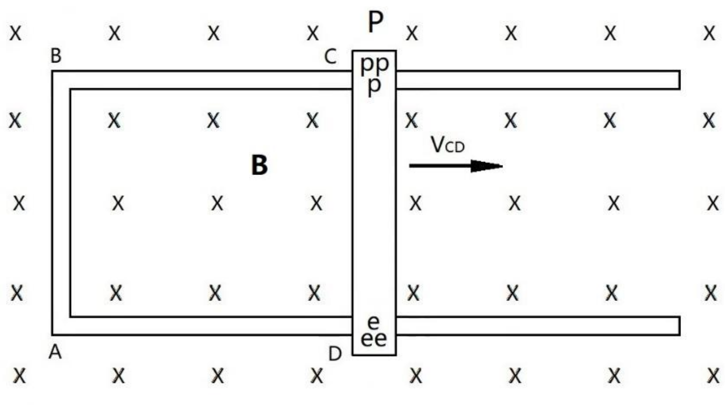

Figure 1.2 Motional electromotive force

When metal wire CD is not in contact with the fixed U-shaped part of the wireframe, that is, the circuit is open, and metal wire CD moves to the right at speed V_{CD}, the magnetic flux increase in wireframe ABCD within dt is:

$$d\Phi = \mathbf{V}_{CD} L_{CD} B\, dt \quad (1\text{-}2)$$

Then, the change rate of the magnetic flux passing through wireframe ABCD is:

$$d\Phi/dt = \mathbf{V}_{CD} B L_{CD} \quad (1\text{-}3)$$

According to equations (1-1) and (1-3), the electromotive force of metal wire CD is:

$$\varepsilon_{CDO} = \mathbf{V}_{CD} B L_{CD} \quad (1\text{-}4)$$

According to equation (1-4), the electromotive force ε_{CDO} is proportional to the speed V_{CD} of the metal wire CD, that is, the change rate of the magnetic flux.

The metal wire CD is equivalent to a battery. The C end is positive, and the D end is negative. A chemical battery is essentially the result of the counter-potential movement of electric charges under the action of electrochemical forces. Similarly, the electromotive force of metal wire CD is essentially the result of the counter-potential movement of electric charges under the action of the Lorentz magnetic field force.

The microscopic physical essence of the motional electromotive force is that the charges within the wire move relative to a magnetic field, and the charges are affected by the Lorentz magnetic field force. Based on the Lorentz magnetic field force, we will derive the motional electromotive force of metal wire CD below.

When metal wire CD is not in contact with the fixed U-shaped part of the wireframe and moves right at speed V_{CD}, the free electrons e in the metal wire CD move along the wire from the C end to D end under the action of the Lorentz magnetic field force such that the negative electrons e accumulate at the D end and the positive charges p accumulate at the C end, thus generating an electromotive force ε_{CDO} from the C end to D end. The charges on metal wire CD are affected by both the Lorentz magnetic field force and the electric field force. When the Lorentz magnetic field force and the electric field force are balanced, the accumulation of charges stops, as shown in Figure 1.3.

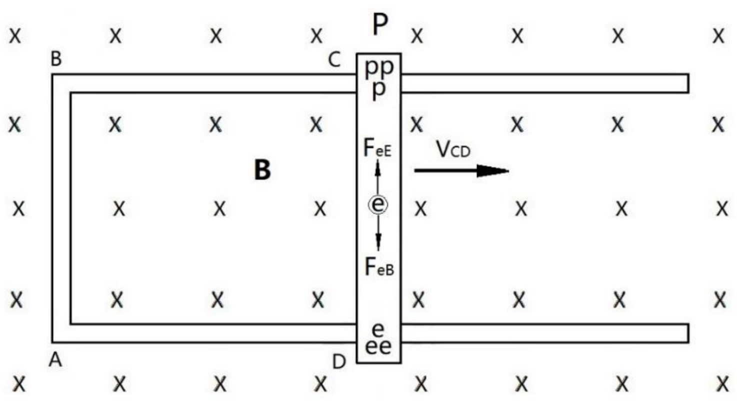

Figure 1.3 Force analysis of electrons in metal wire CD

Let the motional electromotive force between the two ends of wire CD be ε_{CDO}; the electric field intensity is:

$$E_{CD} = \varepsilon_{CDO}/L_{CD}$$

The electric field force on a free electron e is:

$$F_{eE} = e\ E_{CD}$$

$$F_{eE} = e\ \varepsilon_{CDO}/L_{CD} \qquad (1\text{-}5)$$

The electrons e within wire CD move together at V_{CD} relative to magnetic field B. According to Lorentz’s magnetic field force theorem, the Lorentz magnetic field force on the free electrons e is:

$$F_{eB} = e\ V_{CD}\ B \qquad (1\text{-}6)$$

The electric field force F_{eE} and the Lorentz magnetic field force F_{eB} are equal in magnitude and opposite in direction. From formulas (1-5) and (1-6), the following can be obtained:

$$e\ \varepsilon_{CDO}/L_{CD} = e\ V_{CD}\ B$$

Then, the motional electromotive force of metal wire CD is:

$$\varepsilon_{CDO} = B\ L_{CD}\ V_{CD} \qquad (1\text{-}7)$$

Formulas (1-7) and (1-4) are the same. Formula (1-7) is derived based on Lorentz’s magnetic field force theorem, and formula (1-4) is derived based on Faraday's law of electromagnetic induction. Lorentz’s magnetic field force theorem is the microscopic physical essence of the electromotive force, while Faraday's law of electromagnetic induction is the macroscopic physical manifestation of the electromotive force.

2. Further study on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem

The above equations (1-7) and (1-4) are the same, and wire CD moves at a constant speed V_{CD} relative to magnetic field B. If the speed V_{CD} is variable, then the following question arises: are the motional electromotive forces of wire CD derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem still the same?

Before derivation, the electron motion resistance force theorem is first introduced. Under the action of the electric field force F_{eE}, the electrons moving in the metal wire will be affected by the electron motion resistance force F_{eR}. As shown in Figure 2.1, the circuit has a constant voltage power supply, the electrons move at a uniform speed V_e in the wire, and the electric field force F_{eE} and motion resistance force F_{eR} are balanced, that is, they are equal in magnitude and opposite in direction.

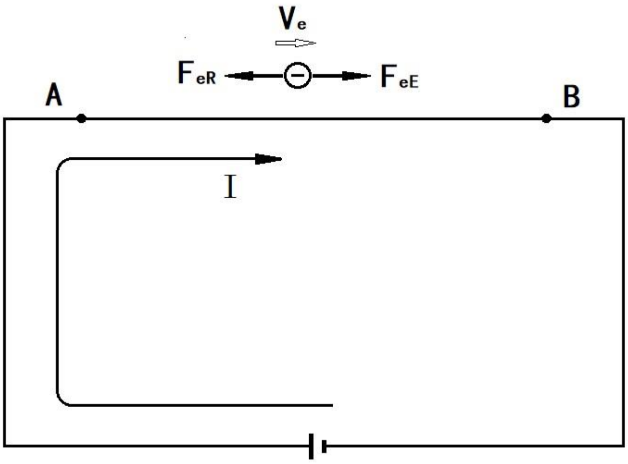

Figure 2.1 Force analysis of an electron flowing through a wire

Let the voltage between the two ends of wire AB be U_{AB}; then, the electric field force on an electron is:

$F_{eE} = e\ U_{AB}/L_{AB}$

$F_{eE} = e\ E_{AB}$ (2-1)

Let the length of metal wire AB be L_{AB}, the cross-sectional area be S, and the resistivity be ρ; the resistance R_{AB} of wire AB is:

$R_{AB} = L_{AB}\ \rho/S$

According to Ohm's law, the current passing through wire AB is:

$I_{AB} = U_{AB}/R_{AB}$

$I_{AB} = U_{AB}\ S/(L_{AB}\ \rho)$ (2-2)

Let the number of free electrons per unit volume of the above metal wire be n_e and the speed of electrons in the wire be V_e; the amount of electricity flowing through the cross-section of the metal wire within 1 second, that is, the current passing through wire AB, is:

$I_{AB} = e\ n_e\ S\ V_e$ (2-3)

From formulas (2-2) and (2-3), the following can be obtained:

$U_{AB}\ S/(L_{AB}\ \rho) = e\ n_e\ S\ V_e$

$U_{AB}/L_{AB} = V_e(e\ n_e\ \rho)$

Then, the electric field intensity of wire AB is:

$E_{AB} = (e\ n_e\ \rho)V_e$

The electrons move at a constant speed V_e in the wire, and the electric field force F_{eE} and motion resistance force F_{eR} are balanced, that is, F_{eE} and F_{eR} are equal in magnitude and opposite in direction. Then, the electron motion resistance force is:

$F_{eR} = F_{eE}$

$= e\ E_{AB}$

$= (e^2\ n_e\ \rho)V_e$

Let $k_{ev} = e^2 n_e \rho$ be the coefficient of resistance of electron motion; then, the electron motion resistance force is:

$$F_{eR} = - k_{ev} V_e \qquad (2\text{-}4)$$

The above equation indicates that electrons move in the metal wire under the action of the motion resistance force F_{eR}, and the magnitude of the electron motion resistance force is proportional to the speed of the electrons, whereas the direction is opposite. Equation (2-4) is called the electron motion resistance force theorem.

The coefficient of resistance of electron motion k_{ev} is related to the wire materials. For copper wires, the number of free electrons per unit volume n_e is 8.5×10^{28} m^{-3}, the charge amount of an electron e is 1.6×10^{-19} C, and the resistivity ρ is 1.75×10^{-8} Ωm. Then, the coefficient of resistance of electron motion of the copper wire is $k_{ev} = e^2 n_e \rho = 3.81 \times 10^{-17}$ $\Omega C^2 m^{-2}$.

The electron motion resistance force theorem reveals that an electron moving in the metal wire will be affected by the motion resistance force, and the electron motion resistance force is proportional to the speed of the electron. The electrons moving in the metal wire will be affected by the motion resistance force, which heats up the metal wire. In Figure 2.1, the driving force of electrons is the electric field force F_{eE}, and the electric field force F_{eE} and motion resistance force F_{eR} are balanced, that is, they are equal in magnitude and opposite in direction. In addition to the electric field force, the driving force of electrons can also be the Lorentz magnetic field force, electrochemical driving force, etc.

Below, the electromotive force of metal wire CD moving at variable speed will be separately derived based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem.

Without loss of generality, wire CD moves back and forth sinusoidally with O as the central zero point along the x-axis, and X_M is the maximum distance of the left and right movement of wire CD, as shown in Figure 2.2.

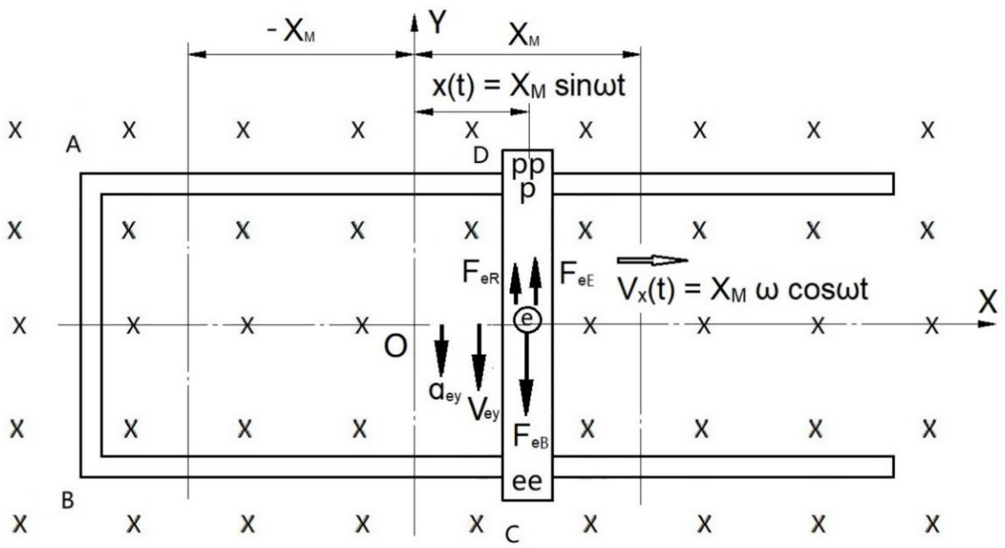

Figure 2.2 Electromotive force of wire CD moving back and forth sinusoidally

At time t, the position of the wire CD on the x-axis is:

$$x(t) = X_M \sin\omega t \qquad (2\text{-}5)$$

where $\omega=2\pi f$ is the angular frequency and f is the frequency in Hz. Then, the speed of wire CD in the x direction at time t is:

$$V_x(t) = X_M\ \omega \cos\omega t \qquad (2\text{-}6)$$

Below, first, the electromotive force of wire CD will be derived according to Faraday's law of electromagnetic induction. At time t, wire CD is at position x(t), and its speed is $V_x(t)$. Then, the magnetic flux increase in wire frame ABCD within dt time is:

$$d\Phi = V_x(t)\ L_{CD}\ B\ dt$$
$$= L_{CD}\ B\ dt\ X_M\ \omega \cos\omega t$$

Then, the change rate of the magnetic flux is:

$$d\Phi/dt = L_{CD}\ B\ X_M\ \omega \cos\omega t \qquad (2\text{-}7)$$

According to Faraday's law of electromagnetic induction, at time t and position x(t), the electromotive force of wire CD is obtained from equation (2-7) as follows:

$$\varepsilon_{CDO}(t) = L_{CD}\ B\ X_M\ \omega \cos\omega t \qquad (2\text{-}8)$$

Equation (2-8) is derived from Faraday's laws of electromagnetic induction.

Below, the electromotive force of wire CD will be derived based on Lorentz's magnetic field force theorem.

Metal wire CD moves back and forth sinusoidally, and the electrons in wire CD are no longer stationary such the electrons within wire CD move along the y-axis at a speed of $V_{ey}(t)$ and an acceleration of $\alpha_{ey}(t)$, as shown in Figure 2.2. Below, we derive the electromotive force of metal wire CD based on the force analysis of electrons within wire CD.

At time t, the metal wire CD is at position x(t). Let the motional electromotive force of the metal wire CD be ε_{CDO}; then, the electric field force on an electron is:

$$F_{eE}(t) = e\ \varepsilon_{CDO}(t)/L_{CD} \qquad (2\text{-}9)$$

The Lorentz magnetic field force on an electron is:

$$F_{eB}(t) = e\ B\ X_M\ \omega \cos\omega t \qquad (2\text{-}10)$$

Driven by the Lorentz magnetic field force, let the speed of electrons along the y-axis within wire CD be $V_{ey}(t)$; then, the electron motion resistance force is:

$$F_{eR}(t) = k_{ev}\ V_{ey}(t) \qquad (2\text{-}11)$$

Driven by the Lorentz magnetic field force, let the acceleration of electrons along the y-axis within wire CD be $\alpha_{ey}(t)$ and the mass of an electron be m_e; then, the electron acceleration driving force is:

$$F_{e\alpha}(t) = m_e\ \alpha_{ey}(t) \qquad (2\text{-}12)$$

According to the force analysis of the electron in Figure 2.2, the following can be obtained:

$$F_{e\alpha}(t) = F_{eB}(t) - F_{eE}(t) - F_{eR}(t)$$

$$m_e \alpha_{ey}(t) = e B X_M \omega \cos\omega t - e \varepsilon_{CDO}(t)/L_{CD} - k_{ev} V_e(t)$$

At time t and position x(t), the electromotive force of wire CD is:

$$\varepsilon_{CDO}(t) = L_{CD} B X_M \omega \cos\omega t - (k_{ev} L_{CD}/e) V_{ey}(t) - (m_e L_{CD}/e) \alpha_{ey}(t) \qquad (2\text{-}13)$$

Equations (2-5) and (2-13) are derived based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, respectively.

There is only one term of $L_{CD}BX_M\omega\cos\omega t$ on the right-hand side of equation (2-5). The first term on the right-hand side of equation (2-13) is the same as the $L_{CD}BX_M\omega\cos\omega t$ term in equation (2-5), whereas equation (2-13) has two additional terms, $(k_{ev} L_{CD}/e) V_{ey}(t)$ and $(m_e L_{CD}/e) \alpha_{ey}(t)$, which are related to the speed $V_{ey}(t)$ and acceleration $\alpha_{ey}(t)$ of electrons. Therefore, when the speed of wire CD changes relative to the magnetic field, the electromotive forces of wire CD derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem are different. For Lorentz's magnetic field force theorem, its theoretical results and experimental observations regarding high-energy electron cyclotrons are consistent within a fairly high degree of accuracy, even when the electron speed is close to the speed of light, reaching 0.99C, after considering the relativistic effect, its theoretical results and experimental observations are still consistent. Lorentz's magnetic force theorem is a universal fundamental electromagnetic theorem. Therefore, Faraday's law of electromagnetic induction is an engineering approximation formula.

In equations (2-5) and (2-13), the term $L_{CD}BX_M\omega\cos\omega t$ can be directly calculated from mathematical formula. However, in equation (2-13), the two additional terms $(k_{ev} L_{CD}/e)V_{ey}(t)$ and $(m_e L_{CD}/e)\alpha_{ey}(t)$ are difficult to directly calculate from mathematical formulas, and it is necessary to perform a force analysis of each electron with the Lorentz's magnetic field force and electric field force to obtain the position, speed $V_{ey}(t)$ and acceleration $\alpha_{ey}(t)$ of each free electron within wire CD. For the determined wire material and structure, with the help of computer numerical processing, the calculation results that meet the engineering application need can be obtained.

In equations (2-5) and (2-13), the radiation of electric and magnetic fields is not considered; for general transformers and motors, their operating frequencies are below several hundred hertz, and the radiated energy of electric and magnetic fields can be negligible. When the angular frequency ω is high enough, a considerable part of the energy in the circuit becomes the radiated energy of the electric and magnetic fields. Therefore, Faraday's law of electromagnetic induction and formula (2-13) derived from Lorentz's magnetic field force theorem are both engineering approximation formulas for calculating the electromotive force.

3. An extension of Lorentz's magnetic field force theorem

Lorentz's magnetic field force theorem reveals that an electric charge moving in a uniform magnetic field is affected by the Lorentz magnetic field force, which is the microscopic physical essence of the electromotive force generated in metal wires and coils. For charge q moving at speed **v** in a magnetic field with magnetic induction intensity **B**, charge q is affected by the Lorentz magnetic field force:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \qquad (3\text{-}1)$$

The magnetic force $\mathbf{F}_B$, charge velocity **v**, and magnetic induction intensity **B** obey the right-hand rule.

As shown in Figure 3.1, a ball with a charge of q and a mass of m is suspended by a quartz filament from a beam.

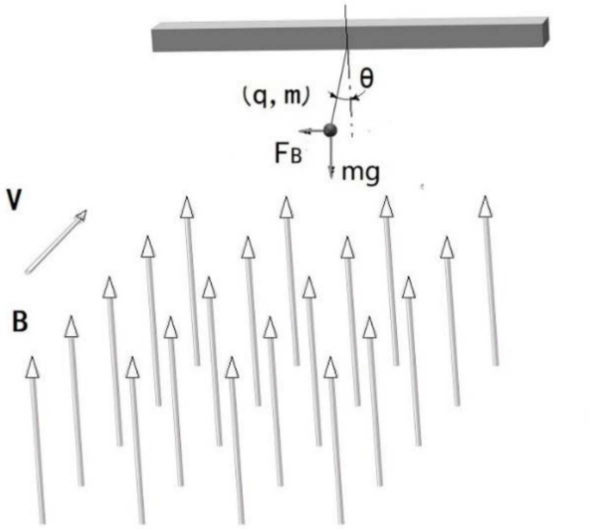

Figure 3.1 Lorentz magnetic field force of a charged ball moving in a uniform magnetic field

Let a uniform magnetic field with magnetic induction intensity B move at V relative to the charged ball; the Lorentz magnetic field force acting on the charged ball is:

$$F_B = q\, v\, B$$

The charged ball is simultaneously affected by gravity mg and the tensile force of the quartz filament, and the Lorentz magnetic field force is obtained from the force analysis of the charged ball in Figure 3.1:

$$F_B = mg\, tg\theta$$

A charge at rest in a changing magnetic field wave is also affected by the Lorentz magnetic field force, which is the microscopic physical essence of the induced electromotive force in metal wires and coils. For a sinusoidal magnetic field wave with angular frequency ω and maximum magnetic induction intensity B_{max}, based on preliminary research, it can be concluded that the Lorentz magnetic field force on the charge is:

$$F_B(t) = k_B\, q\, \omega\, B_{max} \sin(\omega t) \quad (3\text{-}2)$$

In the above equation, $F_B(t)$ is also a sinusoidal function, and k_B is the proportional coefficient of the magnetic field force, which requires further theoretical and experimental studies. When $\sin(\omega t) = 1$, the sinusoidal alternating Lorentz magnetic field force $F_B(t)$ in equation (3-2) is the maximum:

$$F_{Bmax} = k_B\, q\, \omega\, B_{max} \quad (3\text{-}3)$$

As shown in Figure 3.2, a ball with charge q and mass m is suspended by a quartz filament from a beam.

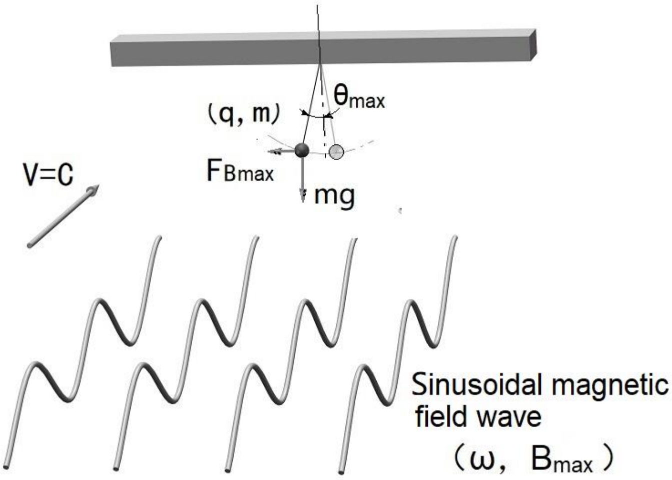

Figure 3.2 Lorentz magnetic field force of a charged stationary ball
in a sinusoidal magnetic field wave

The sinusoidal magnetic field wave has an angular frequency of $\omega=2\pi f$, and the maximum magnetic induction intensity B_{max} passes through the charged ball at the speed of light. The charged ball oscillates left and right at a frequency of f under the action of the alternating Lorentz magnetic field force. Let the maximum swing angle be θ_{max} and the corresponding maximum Lorentz magnetic field force be F_{Bmax}. The charged ball is simultaneously affected by gravity mg and the tensile force of the quartz filament. When the swing angle is θ_{max}, the maximum Lorentz magnetic field force is obtained from the force analysis of the charged ball in Figure 3.2:

$$F_{Bmax} = mg\, tg\theta_{max} \quad (3\text{-}4)$$

From equations (3-3) and (3-4), the following can be concluded:

$$k_B\, q\, \omega\, B_{max} = mg\, tg\theta_{max}$$

Then, the proportional coefficient of the magnetic field force is:

$$k_B = mg\, tg\theta_{max} /(q\, \omega\, B_{max}) \quad (3\text{-}5)$$

Equation (3-5) provides an experimental method for calibrating the proportional coefficient of the magnetic field force.

4. Conclusion

Faraday's law of electromagnetic induction reveals that the induction electromotive force generated in a metal coil is proportional to the change rate of the magnetic flux passing through the coil. Lorentz's magnetic field force theorem reveals that an electric charge moving in a magnetic field is affected by the Lorentz magnetic field force. Lorentz's magnetic field force theorem is the microscopic physical essence of the induction electromotive force, and Faraday's law of electromagnetic induction is the macroscopic physical manifestation of the induction electromotive force.

An induction electromotive force will be generated between the two ends of a metal wire moving in a magnetic field. In this study, the calculation formulas of the electromotive force of metal wires were separately derived based on Faraday’s law of electromagnetic induction and Lorentz’s magnetic field force theorem. When a metal wire moves at a uniform speed in a magnetic field, the results derived from both are the same. When a metal wire moves back and forth sinusoidally in a magnetic field, the electromotive forces of the wire derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem are different. Lorentz's magnetic force theorem is a universal fundamental electromagnetic theorem, so Faraday's law of electromagnetic induction is an engineering approximation formula.

An electric charge moving in a uniform magnetic field is affected by the Lorentz magnetic field force, which is the microscopic physical essence of the motional electromotive force generated in metal wires and loops. An electric charge at rest in a changing magnetic field wave is also affected by the Lorentz magnetic field force, which is the microscopic physical essence of the induced electromotive force in metal wires and loops. The electromotive force in metal wires and loops is essentially the result of the counter-potential movement of electric charges under the action of the Lorentz magnetic field force. Faraday's law of electromagnetic induction and the formula for calculating the electromotive force derived from Lorentz's magnetic field force theorem are both engineering approximation formulas.

Faraday's law of electromagnetic induction is one of the 4 equations of Maxwell's equations. This paper reveals that Faraday's law of electromagnetic induction is an engineering approximation formula, which is a great challenge for Maxwell's equations and the fundamental electromagnetic theorems. This will have a profound impact on scientific discovery and technological progress.

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Availability of Data and Materials:

All data generated or analysed during this study are included in this published article and its supplementary information files.

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