Theoretical value of the Gravitational constant

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Abstract

In this paper it will be presented the theoretical value of the Gravitational constant. From the dimensionless unification of the fundamental interactions we will find the formulas for the Gravitational constant. From these equations it will be calculated the theoretical value of the Gravitational constant. This value is very close to the CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances.

Keywords

Cosmological constant , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant

1. Introduction

Gravity is a natural phenomenon by which all things with mass or energy,including planets,stars, galaxies,and even light,are brought toward one another. On Earth,gravity gives weight to physical objects,and the Moon's gravity causes the ocean tides. Gravity is most accurately described by the general theory of relativity,which describes gravity not as a force,but as a consequence of masses moving along geodesic lines in a curved spacetime caused by the uneven distribution of mass. However,for most applications,gravity is well approximated by Newton's law of universal gravitation,which describes gravity as a force causing any two bodies to be attracted toward each other,with magnitude proportional to the product of their masses and inversely proportional to the square of the distance between them. Gravity is the weakest of the four fundamental interactions of physics. As a consequence,it has no significant influence at the level of subatomic particles. In contrast,it is the dominant interaction at the macroscopic scale,and is the cause of the formation, shape and trajectory of astronomical bodies.

In the late 17th century,Isaac Newton's description of the long-distance force of gravity implied that not all forces in nature result from things coming into contact. Newton's work in his Mathematical Principles of Natural Philosophy dealt with this in a further example of unification,in this case unifying Galileo's work on terrestrial gravity,Kepler's laws of planetary motion and the phenomenon of tides by explaining these apparent actions at a distance under one single law: the law of universal gravitation. In 1814,building on these results,Laplace famously suggested that a sufficiently powerful intellect could,if it knew the position and velocity of every particle at a given time,along with the laws of nature,calculate the position of any particle at any other time. In 1900,David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis to all of physics. In this problem he thus asked for what today would be called a theory of everything. After 1915,when Albert Einstein published the theory of gravity (general relativity),the search for a unified field theory combining gravity with electromagnetism began with a renewed interest. In Einstein's day,the strong and the weak forces had not yet been discovered,yet he found the potential existence of two other distinct forces,gravity and electromagnetism,far more alluring. coupling),is a number that determines the strength of the force exerted in an interaction. In attributing a relative strength to the four fundamental forces,it has proved useful to quote the strength in terms of a coupling constant. The coupling constant for each force is a dimensionless constant. The gravitational constant is an empirical physical constant that participates in the calculation of gravitational force between two bodies. It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. In Newton's law, it is the proportionality constant connecting the gravitational force between two bodies with the product of their masses and the inverse square of their distance. In the Einstein field equations, it quantifies the relation between the geometry of spacetime and the energy–momentum tensor. The modern notation of Newton's law involving G was introduced in the 1890s by C.V. Boys. The first implicit measurement with an accuracy within about 1% is attributed to Henry Cavendish in a 1798 experiment.

The physicist Sir Isaac Newton in 1687 published his book "Philosophiae Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper,Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However,these must be adjusted by introducing the gravity constant G. The gravitational constant G occupies an anomalous position among the other constants of physics. The mass M of any celestial object cannot be determined independently of the gravitational attraction that it exerts. Thus,the combination G·M,not the separate value of M,is the only meaningful property of a star,planet,or galaxy. According to general relativity and the principle of equivalence,G does not depend on material properties but is in a sense a geometric factor.

The concept of a different cosmology G first appears in the work of Edward Arthur Milne a few years before Dirac formulated LNH. Milne was inspired not by a large number of coincidences but by a contradiction of Einstein's general theory of relativity. For Milne, the space was not a structured object but merely a frame of reference in which relations such as this could accommodate Einstein's conclusions:

$$G = \frac{c^3}{\mathrm{M}_U} T_U$$

According to this relationship, G increases with time. Dirac hypothesized that the constant of universal attraction G varies with time. Dirac's hypothesis went so far as to claim that such coincidences could be explained if the very physical constants changed with T_U, especially the gravitational constant G, which must decrease with time:

$$G \approx \frac{1}{t}$$

However, according to general relativity, G must also be constant over time. Although George Gamow noted that such a time variation is not necessarily due to Dirac's assumptions, no corresponding change of G has been found. According to general relativity, G is constant, otherwise the law of conservation of energy is violated. Dirac dealt with this difficulty by introducing into the Einstein field equations a gauge function β that describes the structure of spacetime in terms of a ratio of gravitational and electromagnetic units [1]. The gravitational constant is defined as:

$$G = \alpha_G \frac{\hbar c}{m_e^2}$$

The expressions for the gravitational constant G in terms of Planck units are:

$$G = \frac{c^3 l_{pl}^2}{\hbar} = \frac{\hbar c}{m_{pl}^2} = \frac{l_{pl} c^2}{m_{pl}} = \frac{c^5 t_{pl}^2}{\hbar}$$

A surprisingly close relationship between gravity and the electrostatic interaction. The gravitational constant G and the Coulomb constant ke are expressed in terms of Planck units as:

$$G = \frac{K_e q_e^2}{a m_{pl}^2}$$

Also another beautiful expression that proves the close relationship between gravity and electrostatic interaction is:

$$G = \frac{\alpha c^4 l_{pl}^2}{K_e q_e^2}$$

The 2018 CODATA recommended value of gravitational constant is $G=6.67430\times10^{-11}$ $m^3/kg\cdot s^2$ with standard uncertainty 0.00015×10^{-11} $m^3/kg\cdot s^2$ and relative standard uncertainty 2.2×10^{-5}. The gravitational constant G is quite difficult to measure because gravity is much weaker than other fundamental forces, and an experimental apparatus cannot be separated from the gravitational influence of other bodies. The first direct measurement of gravitational attraction between two bodies in the laboratory was performed in 1798, seventy-one years after Newton's death, by

Henry Cavendish. He determined a value for G implicitly, using a torsion balance invented by the geologist Rev. John Michell. His result corresponds to the value of $G=6.74(4)\times10^{-11}$ $m^3/kg\cdot s^2$. It is surprisingly accurate, about 1% above the modern value. The accuracy of the measured value of G has increased only modestly since the original Cavendish experiment. In the January 2007 issue of Science, Fixler et al. described a measurement of the gravitational constant by a new technique, atom interferometry, reporting a value of $G=6.693(34)\times10^{-11}$ $m^3/kg\cdot s^2$, 0.28% (2800 ppm) higher than the 2006 CODATA value. An improved cold atom measurement by Rosi et al. was published in 2014 of $G=6.67191(99)\times10^{-11}$ $m^3/kg\cdot s^2$. Although much closer to the accepted value (suggesting that the Fixler et al. measurement was erroneous), this result was 325 ppm below the recommended 2014 CODATA value, with non-overlapping standard uncertainty intervals. As of 2018, efforts to re-evaluate the conflicting results of measurements are underway, coordinated by NIST, notably a repetition of the experiments reported by Quinn et al.

In August 2018,a Chinese research group announced new measurements based on torsion balances, $6.674184(78)\times10^{-11}$ $m^3/kg\cdot s^2$ and $6.674484(78)\times10^{-11}$ $m^3/kg\cdot s^2$ based on two different methods [2]. These are claimed as the most accurate measurements ever made, with standard uncertainties cited as low as 12 ppm. The difference of 2.7σ between the two results suggests there could be sources of error unaccounted for. The study is an example of excellent craftsmanship in precision measurements. However, the true value of G remains unclear. Various determinations of G that have been made over the past 40 years have a wide spread of values. Although some of the individual relative uncertainties are of the order of 10 parts per million, the difference between the smallest and largest values is about 500 parts per million .

2. Unification of the fundamental interactions

In [3] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. New interpretation and very accurate values of the fine-structure constant has been discovered in terms of the Archimedes constant and the golden radio. We propose in [4] , [5] and [6] the exact formula for the fine-structure constant α with the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}=137.035999164... \quad (1)$$

Also we propose in [6] , [7] and [8] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2=137.035999078... \quad (2)$$

We propose in [9] the exact mathematical expression for the proton to electron mass ratio:

$$\mu^{32}=\varphi^{-42}\cdot F5^{160}\cdot L5^{47}\cdot L19^{40/19} \Rightarrow \mu=1836.15267343... \quad (3)$$

$$7\cdot\mu^{3}=165^{3}\cdot\ln^{11}10 \Rightarrow \mu=1836.15267392... \quad (4)$$

$$\mu=6\cdot\pi^{5}+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}=1836.15267343... \quad (5)$$

Also in [9] was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42) \quad (6)$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12 \quad (7)$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231 \quad (8)$$

$$\mu-807\cdot\alpha=1205\cdot\pi-518\cdot\varphi-411\cdot e \quad (9)$$

In [10] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot10^{2}\cdot\cos(\mu\cdot\alpha^{-1})+13^{2}=0 \quad (10)$$

The exponential form of this equation is:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \tag{11}$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i \tag{12}$$

It was presented in [11] the mathematical formulas that connects the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$\alpha_{G(p)}=\mu^2\cdot\alpha_G \tag{13}$$

$$\alpha=\mu\cdot N_1\cdot\alpha_G \tag{14}$$

$$\alpha\cdot\mu=N_1\cdot\alpha_{G(p)} \tag{15}$$

$$\alpha^2=N_1^2\cdot\alpha_G\cdot\alpha_{G(p)} \tag{16}$$

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1 \tag{17}$$

$$\mu^2=4\cdot e^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \tag{18}$$

$$\mu\cdot N_1=4\cdot e^2\cdot\alpha^3\cdot N_A^2 \tag{19}$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1=1 \tag{20}$$

$$\mu^3=4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{21}$$

$$\mu^2=4\cdot e^2\cdot\alpha_G\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1^2 \tag{22}$$

$$\mu=4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{23}$$

In [12] we presented the recommended value for the strong coupling constant:

$$\alpha_s=\frac{Euler'\,number}{Gerford's\ constant}=\frac{e}{e^{\pi}}=e^{1-\pi}=0,11748.. \tag{24}$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. In the papers [13] , [14] , [15] and [16] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant α_s and the weak coupling constant α_w. We reached the conclusion of the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot\alpha_s=10^7\cdot\alpha_w \tag{25}$$

$$\alpha_s^2=i^{2i}\cdot10^7\cdot\alpha_w \tag{26}$$

Resulting the unity formulas that connects the strong coupling constant α_s and the fine-structure constant α:

$$\alpha_s\cdot\cos\alpha^{-1}=i^{2i} \tag{27}$$

$$\cos\alpha^{-1}=\frac{\alpha_s^{-1}}{e^{\pi}} \tag{28}$$

The figure 1 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius e^{π}.

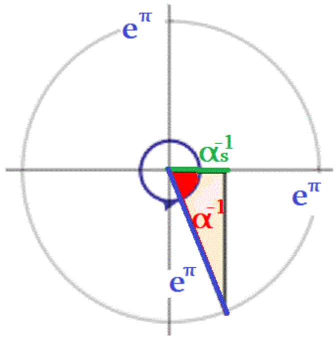

Figure 1. *The angle in α^{-1} radians.*

We reached the conclusion of the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^{\pi} \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \tag{29}$$

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i} \tag{30}$$

The figure 2 below shows the geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.

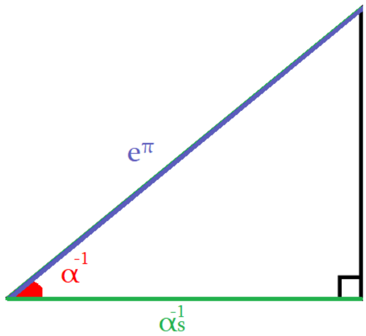

Figure 2. *Geometric representation of the dimensionless unification of the strong nuclear and the electromagnetic interactions.*

The electroweak theory,in physics,is the theory that describes both the electromagnetic force and the weak force. We reached the conclusion of the dimensionless unification of the weak nuclear and the electromagnetic forces:

$$10^{7} \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \cdot i^{2i} \tag{31}$$

The figure 3 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius $10^{7} \cdot e^{\pi-1}$.

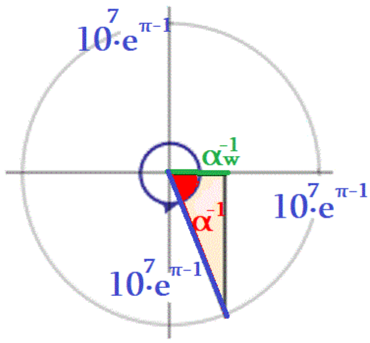

Figure 3. *The angle in α^{-1} radians.*

The figure 4 below shows the geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions.

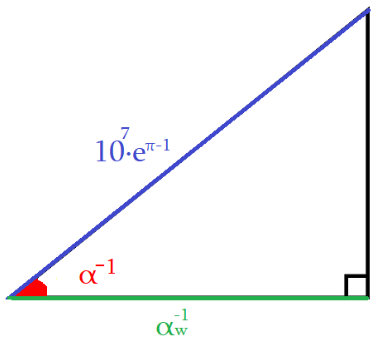

Figure 4. *Geometric representation of the dimensionless unification of the weak nuclear and the electromagnetic interactions*

Resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w and the fine-structure constant α:

$$10^7 \cdot \alpha_w \cdot \cos\alpha^{-1} = \alpha_s \tag{32}$$

$$\cos \alpha^{-1} = \frac{\alpha_s \alpha_w^{-1}}{10^7} \tag{33}$$

The figure 5 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^7.

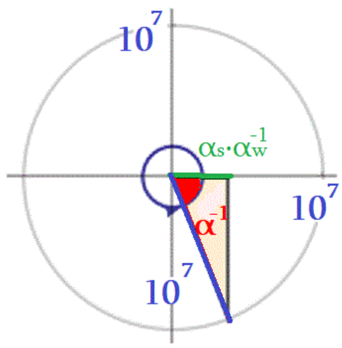

Figure 5. *The angle in α^{-1} radians.*

The figure 6 below shows the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.

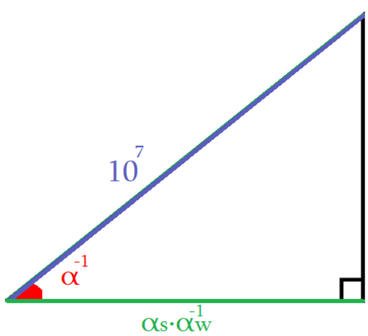

Figure 6. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic interactions.*

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear and the electromagnetic forces:

$$10^7 \cdot \alpha_W \cdot (e^{i/\alpha}+e^{-i/\alpha})=2 \cdot \alpha_S \quad (34)$$

Resulting the unity formula that connects the fine-structure constant α,the gravitational coupling constant α_G and the Avogadro's number N_A:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=1 \quad (35)$$

$$\alpha^{-2} \cdot \cos^2\alpha^{-1}=4 \cdot \alpha_G \cdot N_A^2 \quad (36)$$

The figure 7 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.

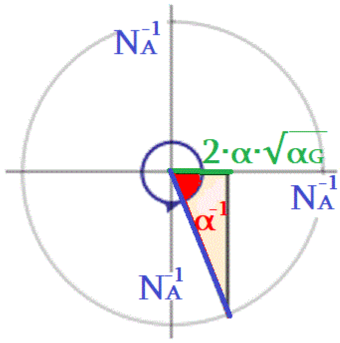

Figure 7. *The angle in α^{-1} radians.*

The figures 8 below show the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.

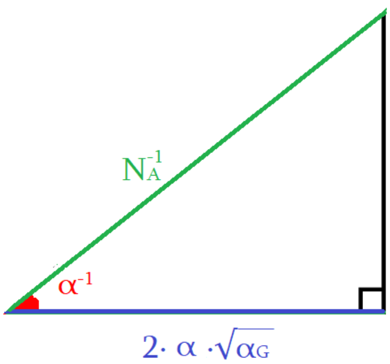

Figure 8. *Geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

We reached the conclusion of the dimensionless unification of the gravitational and the electromagnetic forces:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=1 \quad (37)$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=(e^{i/\alpha}+e^{-i/\alpha})^2 \quad (38)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4 \cdot \alpha_S^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2=i^{4i} \quad (39)$$

$$\alpha^2 \cdot (e^{i/\alpha}+e^{-i/\alpha}) \cdot \alpha_S^4 \cdot \alpha_G \cdot N_A^2=i^{8i} \quad (40)$$

The figure 9 below shows the geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions.

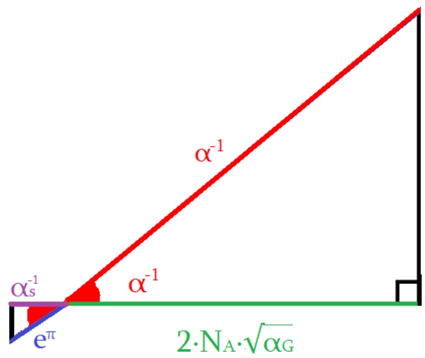

Figure 9. *Geometric representation of the dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions*

We reached the conclusion of the dimensionless unification of the weak nuclear,the gravitational and electromagnetic forces:

$$4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2=i^{4i}\cdot e^2 \quad (41)$$

$$10^{14}\cdot \alpha^2\cdot (e^{i/\alpha}+e^{-i/\alpha})^2\cdot \alpha_w^2\cdot \alpha_G\cdot N_A^2=i^{8i} \quad (42)$$

The figure 10 below shows the geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions.

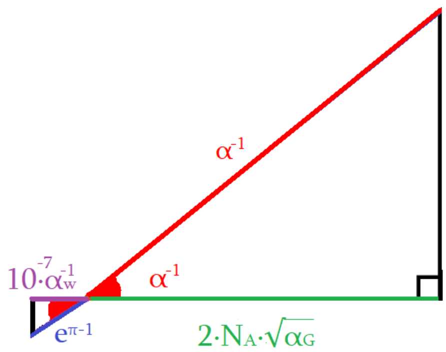

Figure 10. *Geometric representation of the dimensionless unification of the weak nuclear,the gravitational and the electromagnetic interactions*

Resulting the unity formula that connect the strong coupling constant α_s,the weak coupling constant α_w,the fine-structure constant α and the gravitational coupling constant $\alpha_{G(p)}$ for the proton:

$$4\cdot 10^{14}\cdot N_A^2\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_{G(p)}=\mu^2\cdot \alpha_s^2 \quad (43)$$

We reached the conclusion of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2=4\cdot 10^{14}\cdot \alpha_w^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2 \quad (44)$$

$$8\cdot 10^{7}\cdot N_A^2\cdot \alpha_w\cdot \alpha^2\cdot \alpha_G=\alpha_s\cdot (e^{i/\alpha}+e^{-i/\alpha}) \quad (45)$$

The figure 11 below shows the geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions.

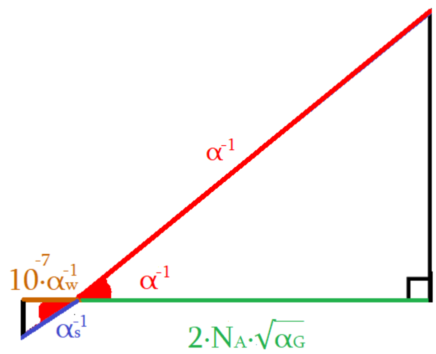

Figure 11. *Geometric representation of the dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions*

From these expressions resulting the unity formulas that connects the strong coupling constant α_s,the weak coupling constant α_w,the proton to electron mass ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro's number N_A,the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_{G(p)}$:

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2 \tag{46}$$

$$\mu^2\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_{G(p)}\cdot N_A^2 \tag{47}$$

$$\mu\cdot N_1\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^3\cdot N_A^2 \tag{48}$$

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1 \tag{49}$$

$$\mu^3\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1 \tag{50}$$

$$\mu\cdot\alpha_s=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha_G\cdot G(p)^2\cdot N_A^2\cdot N_1^2 \tag{51}$$

$$\mu\cdot\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1 \tag{52}$$

3. Unification of atomic physics and cosmology

In [17] and [18] resulting in the dimensionless unification of atomic physics and cosmology. The relevant constant in atomic physics is the fine-structure constant α,which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant α_g. It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant α_g is an equivalent way to express the biggest issue in theoretical physics. The gravitational fine structure constant α_g is defined as:

$$\alpha_g=\frac{l_{pl}^3}{r_e^3} \tag{53}$$

$$\alpha_g=\frac{\sqrt{\alpha_G^3}}{\alpha^3} \tag{54}$$

$$\alpha_g=\sqrt{\frac{\alpha_G^3}{\alpha^6}} \tag{55}$$

with numerical value:

$$\alpha_g=1.886837\times10^{-61}$$

Also equals:

$$a_g^2 \cdot a^6 = a_G^3$$

$$a_g^2 = a_G^3 \cdot a^{-6}$$

$$\alpha_g^2 = \left(\frac{\alpha_G}{\alpha^2}\right)^3 \quad (56)$$

The expression that connects the gravitational fine-structure constant a_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\varphi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (57)$$

Resulting the unity formula for the gravitational fine-structure constant a_g:

$$a_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3} \quad (58)$$

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3} \quad (59)$$

$$a_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-3} \quad (60)$$

$$a_g = (10^7 \cdot a_w \cdot a_G^{1/2} \cdot e^{-1} \cdot a_s^{-1} \cdot a^{-1})^3 \quad (61)$$

$$a_g^2 = (10^{14} \cdot a_w^2 \cdot a_G \cdot e^{-2} \cdot a_s^{-2} \cdot a^{-2})^3 \quad (62)$$

$$a_g = 10^{21} \cdot i^{6i} \cdot a_w^3 \cdot a_G^{3/2} \cdot a_s^{-6} \cdot a^{-3} \quad (63)$$

So the unity formulas for the gravitational fine-structure constant a_g are:

$$a_g^2 = 10^{42} \cdot i^{12i} \cdot a_w^6 \cdot a_G^3 \cdot a_s^{-12} \cdot a^{-6} \quad (64)$$

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly,but it is required in cosmological equations in order to reconcile theory with our observations of the universe. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (65)$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (66)$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6} \quad (67)$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6 \quad (68)$$

$$a_s^{12} \cdot a^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6 \quad (69)$$

For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 \mathrm{N_A}\right)^{-6} \frac{c^3}{G\hbar} \quad (70)$$

$$\Lambda = i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6} \frac{c^3}{G\hbar} \quad (71)$$

$$\Lambda = i^{12i} e^6 \left(2\, 10^7 \alpha_w a^3 N_A\right)^{-6} \frac{c^3}{G\hbar} \quad (72)$$

$$\Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 \alpha^2}\right)^3 \frac{c^3}{G\hbar} \tag{73}$$

$$\Lambda = 10^{42} i^{12}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \frac{c^3}{G\hbar} \tag{74}$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4}\right)^3 \tag{75}$$

4. Formulas of the Gravitational constant

Now we will find the formulas for the gravitational constant G using the unity formulas for the coupling constants that we calculated. From expression (37) the gravitational coupling constant α_G can be written in the form:

$$4 \cdot e^2 \cdot N_A^2 \cdot \alpha^2 \cdot \alpha_G = 1$$

$$\alpha_G = (2 \cdot e \cdot \alpha \cdot N_A)^{-2} \tag{76}$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \tag{77}$$

From equivalent expressions (39) the gravitational coupling constant α_G can be written in the forms:

$$4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$\alpha_G = (2 \cdot e^{\pi} \cdot \alpha_s \cdot \alpha \cdot N_A)^{-2} \tag{78}$$

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i}$$

$$\alpha_G = i^{4i} \cdot (2 \cdot \alpha_s \cdot \alpha \cdot N_A)^{-2} \tag{79}$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^{\pi}\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \tag{80}$$

$$G = i^{4i}(2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \tag{81}$$

From expressions (41) the gravitational coupling constant α_G can be written in the form:

$$4 \cdot 10^{14} \cdot e^{2\pi} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^2$$

$$\alpha_G = (2 \cdot e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A)^{-2} \tag{82}$$

$$4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i} \cdot e^2$$

$$\alpha_G = i^{4i} \cdot e^2 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A)^{-2} \tag{83}$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = \left(2e^{\pi-1}10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{84}$$

$$G = i^{4i}e^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{85}$$

From expression (46) the gravitational coupling constant α_G can be written in the form:

$$4\cdot 10^{14}\cdot N_A^2\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G=\alpha_s^2$$

$$\alpha_G=\alpha_s^2\cdot(2\cdot 10^7\cdot\alpha_w\cdot\alpha\cdot N_A)^{-2} \tag{86}$$

Therefore from this expression the formula for the gravitational constant is:

$$G = \alpha_s^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2} \tag{87}$$

5. Theoretical value of the Gravitational constant

The fine-structure constant is one of the most fundamental constants of physics. In particular, the fine-structure constant sets the strength of electromagnetic interaction between light (photons) and charged elementary particles such as electrons and muons. Many eminent physicists and philosophers of science have pondered why α itself has the value that it does,because the value shows up in so many important scenarios and aspects of physics. The fine-structure constant α is defined as:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c}$$

The 2018 CODATA recommended value of the fine-structure constant is α=0.0072973525693(11) with standard uncertainty $0.0000000011\times 10^{-3}$ and relative standard uncertainty 1.5×10^{-10}. Also the fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}}\frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

In physics,the gravitational coupling constant α_G is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by α_G. The choice of units of measurement,but only with the choice of particles. The gravitational coupling constant α_G is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale,atomic scale,and cosmological scale). The gravitational coupling constant can be used for comparison of length,range and force values. The gravitational coupling constant α_G is defined as:

$$\alpha_G = \frac{Gm_e^2}{\hbar c}$$

There is so far no known way to measure α_G directly. The value of the constant gravitational coupling α_G is only known in four significant digits. The approximate value of the constant gravitational coupling is $\alpha_G=1.7518099\times 10^{-45}$. Also the gravitational coupling constant is universal scaling factor:

$$\alpha_G = \frac{m_e^2}{m_{pl}^2} = \frac{\alpha_{G(p)}}{\mu^2} = \frac{\alpha}{\mu N_1} = \frac{\alpha^2}{N_1^2 \alpha_{G(p)}} = \left(\frac{2\pi l_{pl}}{\lambda_e}\right)^2 = \left(\alpha \frac{l_{pl}}{r_e}\right)^2 = \left(\frac{l_{pl}}{\alpha \alpha_0}\right)^2$$

The gravitational coupling constant $\alpha_{G(p)}$ for the proton is produced similar to the electron,but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton $\alpha_{G(p)}$ is defined as:

$$\alpha_{G(p)} = \frac{Gm_p^2}{\hbar c}$$

The approximate value of the constant gravitational coupling of the proton is $\alpha_{G(p)}=5.9061512\text{x}10^{-39}$. Also other expression for the gravitational coupling constant is:

$$\alpha_{G(p)} = \frac{m_p^2}{m_{pl}^2} = \mu^2 \alpha_G = \frac{\alpha\mu}{N_1} = \frac{\alpha^2}{N_1^2 \alpha_G}$$

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1904. But Weyl and Eddington suggested that the number was about 10^{40} and was related to cosmological quantities. The ratio N_1 of electric force to gravitational force between electron and proton is defined as:

$$N_1 = \frac{\alpha}{\mu \alpha_G} = \frac{\alpha\mu}{\alpha_{G(p)}} = \frac{\alpha}{\sqrt{\alpha_G \alpha_{G(p)}}} = \frac{k_e q_e^2}{Gm_e m_p} = \frac{\alpha \hbar c}{Gm_e m_p}$$

The approximate value of the ratio of electric force to gravitational force between electron and proton is $N_1=2.26866072\text{x}10^{39}$. The ratio N_1 of electric force to gravitational force between electron and proton can also be written in expression:

$$N_1 = \frac{5}{3} 2^{130} = 2,26854911 \times 10^{39}$$

According to current theories N_1 should be constant. The ratio N_2 of electric force to gravitational force between two electrons is defined as:

$$N_2 = \mu N_1 = \frac{\alpha}{\alpha_G} = \frac{N_1^2 \alpha_{G(p)}}{\alpha} = \frac{k_e q_e^2}{Gm_e^2} = \frac{\alpha \hbar c}{Gm_e^2}$$

The approximate value of N_2 is $N_2=4.16560745\text{x}10^{42}$. According to current theories N_2 should grow with the expansion of the universe.

Avogadro's number N_A is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. Avogadro's number N_A is the fundamental physical constant that links the macroscopic physical world of objects that we can see and feel with the submicroscopic,invisible world of atoms.The name honors the Italian mathematical physicist Amedeo Avogadro,who proposed that equal volumes of all gasses at the same temperature and pressure contain the same number of molecules. The most accurate definition of the Avogadro's number value involves the change in molecular quantities and,in particular,the change in the value of an elementary charge. The exact value of the Avogadro's number is $N_A=6,02214076\times10^{23}$. The value of the Avogadro's number N_A can also be written in expressions:

$$N_A=84446885^3=6.02214076\times10^{23}$$

$$N_A=2^{79}=6.04462909\times10^{23}$$

A Planck length l_{pl} is about 10^{-20} times the diameter of a proton,meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck l_{pl} has dimension [L]. The length Planck l_{pl} can be defined by three fundamental natural constants,the speed of light at vacuum c,the reduced Planck constant and the gravity constant G as:

$$l_{pl} = \sqrt{\frac{\hbar G}{c^3}} = \frac{\hbar}{m_{pl}c} = \frac{h}{2\pi m_{pl}c} = \frac{m_p r_p}{4m_{pl}}$$

The 2018 CODATA recommended value of the Planck length is $l_{pl}=1.616255\times10^{-35}$ m with standard uncertainty 0.000018×10^{-35} m and relative standard uncertainty 1.1×10^{-5}. The Bohr radius a_0 is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius a_0 is defined as:

$$a_0 = \frac{\hbar}{\alpha m_e c} = \frac{r_e}{\alpha^2} = \frac{\lambda_c}{2\pi\alpha}$$

The 2018 CODATA recommended value of the Bohr radius is $a_0=5.29177210903\times10^{-11}$ m with standard uncertainty $0.00000000080\times10^{-11}$ m and relative standard uncertainty 1.5×10^{-10}.

The Planck constant,or Planck's constant,is a fundamental physical constant of foundational importance in quantum mechanics. The constant gives the relationship between the energy of a photon and its frequency,and by the mass-energy equivalence,the relationship between mass and frequency. Specifically,a photon's energy is equal to its frequency multiplied by the Planck constant. The constant is generally denoted by h. The reduced Planck constant,equal to the constant divided by $2\cdot\pi$,is denoted by $\hbar$. For the reduced Planck constant $\hbar$ apply:

$$\hbar=a\cdot m_e\cdot a_0\cdot c$$

So from these expressions we have:

$$\hbar^2=a^2\cdot m_e^2\cdot a_0^2\cdot c^2$$

$$(\hbar\cdot G/c^3)=a^2\cdot m_e^2\cdot a_0^2\cdot(G/\hbar\cdot c)$$

$$(\hbar\cdot G/c^3)=a^2\cdot a_0^2\cdot(G\cdot m_e^2/\hbar\cdot c)$$

$$l_{pl}^2=a^2\cdot a_G\cdot a_0^2$$

So the new formula for the Planck length l_{pl} is:

$$l_{pl} = a\sqrt{a_G}\alpha_0 \tag{88}$$

A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Max Planck proposed natural units that indirectly discovered the lowest-level properties of free space, all born from equations that simplified the mathematics of physics equations. The fundamental unit of length in this unit system is the Planck length l_{pl}. The smallest components of spacetime will never be seen with the human eye as it is orders of magnitudes smaller than an atom. Thus, it will never be directly observed but it can be deduced by mathematics. We proposed to be a lattice structure, in which its unit cells have sides of length $2\cdot e\cdot l_{pl}$. Perhaps for the minimum distance l_{min} apply:

$$l_{min}=2\cdot e\cdot l_{pl} \tag{89}$$

From expressions apply:

$$\cos a^{-1}=e^{-1}$$

$$\cos a^{-1}\cdot l_{min}=2\cdot l_{pl}$$

$$\cos\alpha^{-1} = \frac{2l_{pl}}{l_{min}} \tag{90}$$

The figures 12 below show the geometric representation of the fundamental unit of length.

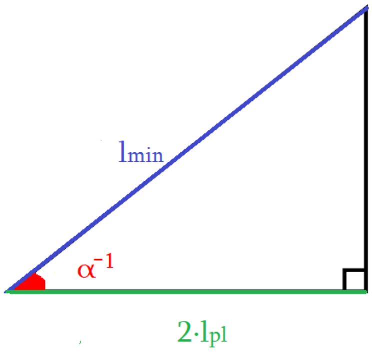

Figure 12. *Geometric representation of the fundamental unit of length.*

For the Bohr radius a_0 apply:

$$a_0 = N_A \cdot l_{min}$$

$$a_0 = 2 \cdot e \cdot N_A \cdot l_{pl} \quad (91)$$

The figures 13 below show the geometric representation of the relationship between the Bohr radius and the Planck length.

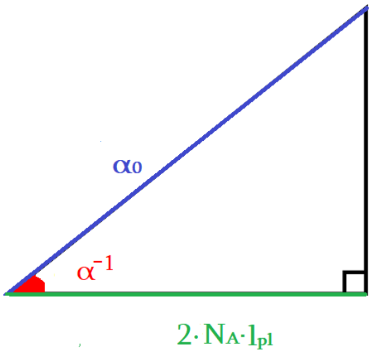

Figure 13. *Geometric representation of the relationship between the Bohr radius and the Planck length.*

We will use this expression and the new formula for the Planck length l_{pl} to resulting the unity formula that connects the fine-structure constant a and the gravitational coupling constant a_G:

$$a_0 = 2 \cdot e \cdot N_A \cdot l_{pl}$$

$$\alpha_0 = 2eN_A\alpha\sqrt{\alpha_G}\alpha_0$$

$$2eN_A\alpha\sqrt{\alpha_G} = 1$$

Therefore the unity formula that connects the fine-structure constant a,the gravitational coupling constant a_G and the Avogadro's number N_A is:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (92)$$

The unity formula is equally valid:

$$a^2 \cdot a_G = (2 \cdot e \cdot N_A)^{-2} \quad (93)$$

So the new formula for the Avogadro number N_A is:

$$N_A = \left(2e\alpha\sqrt{\alpha_G}\right)^{-1} \quad (94)$$

Also resulting the expressions:

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$4 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^{-2}$$

$$\cos^2\alpha^{-1} = 4 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2$$

$$\alpha^{-2} \cdot \cos^2\alpha^{-1} = 4 \cdot \alpha_G \cdot N_A^2 \quad (95)$$

This unity formula is equally valid:

$$\alpha^{-1} \cos\alpha^{-1} = 2N_A\sqrt{\alpha_G} \quad (96)$$

Resulting another elegant exponential form equations:

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot e^{-1}$$

$$4 \cdot e^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = 1$$

$$4 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = e^{-2}$$

$$16 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = (e^{i/\alpha} + e^{-i/\alpha})^2 \quad (97)$$

This unity formula is equally valid:

$$\alpha^{-1}\left(e^{\frac{i}{\alpha}} + e^{\frac{-i}{\alpha}}\right) = 4N_A\sqrt{\alpha_G} \quad (98)$$

Now we will find the theoretical value of the Gravitational constant G using the unity formulas for the coupling constants that we calculated. From expression (95) the gravitational coupling constant α_G can be written in the form:

$$\alpha^{-2} \cdot \cos^2\alpha^{-1} = 4 \cdot \alpha_G \cdot N_A^2$$

$$\alpha_G = (2 \cdot \alpha \cdot N_A)^{-2} \cdot \cos^2\alpha^{-1} \quad (99)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2\alpha N_A)^{-2} \cos^2\alpha^{-1} \frac{\hbar c}{m_e^2} \quad (100)$$

Using the 2018 CODATA recommended value of the the fundamental constants resulting the theoretical value of the Gravitational constant G:

$$G = 6.67448 \times 10^{-11}\ m^3/kg \cdot s^2 \quad (101)$$

The concept of power of two supports an idea of holographic concepts of the Universeor some of the fractal theories. Also it is used in wave mechanics,and it could be viewed in accordance with wave properties of the elementary particles in quantum physics. The figure 14 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius N_A^{-1}.

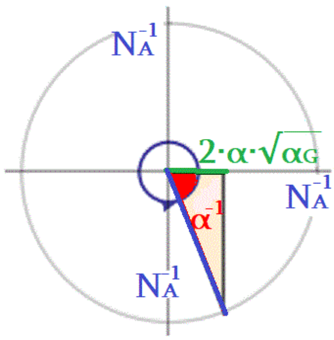

Figure 14. *The angle in α^{-1} radians.*

In his experiments of 1849–50,Michael Faraday was the first to search for a unification of gravity with electricity and magnetism. However,he found no connection. The figure 15 and 16 below shows the geometric representation of the dimensionless unification of the gravitational and the electromagnetic interactions.

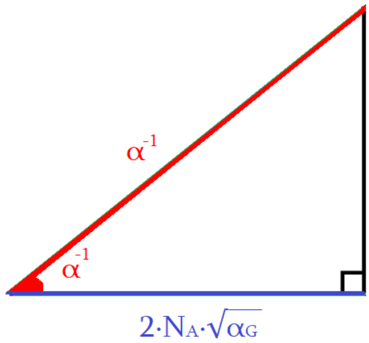

Figure 15. *First geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

In 1900,David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem,he challenged researchers to find an axiomatic basis for all of physics.

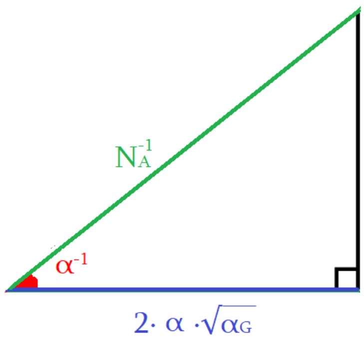

Figure 16. *Second geometric representation of the dimensionless unification of the gravitational and the electromagnetic* interactions

Gravity and electromagnetism are able to coexist as entries in a list of classical forces,but for many years it seemed that gravity could not be incorporated into the quantum framework,let alone unified with the other fundamental forces. For this reason,work on unification,for much of the twentieth century,focused on understanding the three forces described by quantum mechanics:electromagnetism and the weak and strong forces.

6. Conclusions

It presented the theoretical value of the Gravitational constant $G=6.67448\times10^{-11}$ $m^3/kg\cdot s^2$. This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring 6.674184×10^{-11} $m^3/kg\cdot s^2$ and 6.674484×10^{-11} $m^3/kg\cdot s^2$-of-swinging and angular acceleration methods, respectively. From the dimensionless unification of the fundamental interactions we find the formulas for the Gravitational constant:

$$G = (2e\alpha N_A)^{-2}\frac{\hbar c}{m_e^2}$$

$$G = (2\alpha N_A)^{-2}\cos^2\alpha^{-1}\frac{\hbar c}{m_e^2}$$

$$G = (2e^{\pi}\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2}$$

$$G = i^{4i}(2\alpha_s\alpha N_A)^{-2}\frac{\hbar c}{m_e^2}$$

$$G = \left(2e^{\pi-1}10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2}$$

$$G = i^{4i}e^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2}$$

$$G = \alpha_s^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2}$$

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