PROBLEM WITH THE DERIVATION OF NAVIER-STOKES EQUATIONS

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ABSTRACT. The English idiom "Where there's a will, there's a way" means that if someone really wants to do something, she or he will find a way to do it.
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My most recent progress is Ref. [1].

1. Mathematical Problem

The derivation begins with ansatz:

$$\rho(\vec{r},t)\,\frac{\partial\,\vec{u}(\vec{r},t)}{\partial\,t} = \vec{f}(\vec{r},t)\,. \tag{1}$$

Let us insert $\vec{r} = \vec{r}(t)$,

$$\rho(\vec{r}(t),t)\,\frac{d\,\vec{u}(\vec{r}(t),t)}{d\,t} = \vec{f}(\vec{r}(t),t)\,,$$

$$\rho(\vec{r}(t),t)\,\left(\frac{\partial\,\vec{u}(\vec{r},t)}{\partial\,t}|_{\vec{r}=\vec{r}(t)} + \frac{d\,\vec{r}(t)}{dt}\frac{\partial\,\vec{u}(\vec{r},t)}{\partial\,\vec{r}}|_{\vec{r}=\vec{r}(t)}\right) = \vec{f}(\vec{r}(t),t)\,.$$

Let us remove the unnecassary notation $\vec{r}(t) \to \vec{r}$, and we get the Navier-Stokes equations

$$\rho(\vec{r},t)\,\left(\frac{\partial\,\vec{u}(\vec{r},t)}{\partial\,t} + \vec{v}\,\nabla\vec{u}\right) = \vec{f}(\vec{r},t)\,. \tag{2}$$

The $\vec{v} \equiv \vec{u}$, if $\vec{u}$ is velocity at the point $\vec{r}$, at the moment t.

With all respect to Dr. Stokes, equation (2) contradicts equation (1).

2. Physical Problem

Let the velocity pattern of the fluid is $\vec{u}$. Let us run toward the fluid with velocity $\vec{w}$. Then according to us, the fluid approaches us with velocity $\vec{V} = \vec{u} - \vec{w}$, where $\vec{w}$ is a constant vector. All other inner

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parameters of the fluid (ρ and $\vec{f}$) remain the same. If Eq.(1) is the description of this fluid, then

$$\rho \frac{\partial \vec{V}}{\partial t} = \vec{f}. \tag{3}$$

As you see, the Eq.(3) is the same as Eq.(1), but with velocity $\vec{V}$ in the role of $\vec{u}$.

If Eq.(2) is the description of the fluid, then

$$\rho \left(\frac{\partial \vec{V}}{\partial t} + \vec{V}\, \nabla \vec{V} + \vec{w}\, \nabla \vec{V} \right) = \vec{f}. \tag{4}$$

As you see, the Eq.(4) is not the same as Eq.(2), with velocity $\vec{V}$ in the role of $\vec{u}$.

In other words, Eq.(2) is not Invariant under Galilean Coordinate Transformation. All non-relativistic Physical systems (low-velocity systems) satisfy the Galilean Coordinate Transformation.

References

[1] Dmitri Martila, Stefan Groote, "Evaluation of the Gauss Integral." Stats. 5(2): 538–545 (2022).
https://doi.org/10.3390/stats5020032
Dmitri Martila, Stefan Groote, "Analytic Error Function and Numeric Inverse Obtained by Geometric Means." Stats. 6(1): 431–437 (2023).
https://doi.org/10.3390/stats6010026