Cardinal numbers And Cantor's continuum hypothesis

---Tranclosed logic princiole and its inference (6)

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Abstract: In this paper, the cardinal number problem is discussed separately in the axiom systems for set theory *SZF*+ and *SZF*-. It can be proved that:ordinal numbers and cardinal numbers are unified. There is no uncountable cardinal number; in *Cantor*'s set theory, definitions, theorems and propositions based on uncountable cardinal numbers and ordinal numbers are all false. In fact, *Cantor*'s continuum hypothesis is the cardinal number of a set of natural numbers, and whether there are other cardinal numbers between cardinal numbers of set of natural numbers |N| and cardinal numbers of power set |P (N)|, different interpretations are given in different axiom systems.

1.Cardinal numbers And Cantor's continuum hypothesis of axiom system (*SZF*+) of non-standard set theory

In the previous article we analyzed (see *Contradiction and reconstruction of axiom of ZF system ----Tranclosed logic princiole and its inference* (5)) that the infinity axiom in the *ZF* system is wrong. Two complementary axiom systems for set theory *SZF*+ and *SZF*- are established with the new infinity axioms (δ+1 = δ and δ+1 ≠ δ), and here we will discuss the problem of the cardinal number of sets and the well-known Continuum hypothesis problem in these two systems, respectively.

1.1 Cardinal numbers (Inference of axiom system (*SZF*+) of non-standard set theory)

Definition 1.1.1 Let M be a set and α be an ordinal number, if there exists bijective relation $f: M \sim \alpha$, $\alpha \in N^*$, set M can construct one-to-one correspondence with ordinal number α , we call cardinal number of set M as α and denote as : $|M| = \alpha$.

Theorem1.1.1 Ordinal number of α is itself, that is $|\alpha| = \alpha$.

Proof: there exists a bijective relation $f(x) = x$, $f: \alpha \sim \alpha$, that is $|\alpha| = \alpha$.

Cardinal number order of ordinal number is the order of ordinal number itself. In non-standard infinity, same as finite natural numbers, the cardinal number of the infinite set is the ordinal number of the last element

$$\cdots|\varpi-1|<|\varpi|<|\varpi+1|<\cdots<|\varpi+n|<\cdots<|2\varpi|<\cdots<|n\varpi|<\cdots<|\varpi^2|<|\varpi^3|$$

$$<\cdots<|\varpi^n|<\cdots<|n^{\varpi}|<\cdots<|\varpi^{\varpi}|<\cdots<|\varpi^{\varpi^{\varpi}}|<\cdots<|\varpi^{\varpi^{\cdot^{\cdot^{\cdot^{\varpi}}}}}|<\cdots.$$

Example1.1.1 Cardinal numbers of non-standard infinite set

$$A=\left\{0\cdot 1\cdot 4\cdot 9\cdots\cdots n^2\cdots\cdots(\varpi-1)^2\right\},$$

$$\varpi=\left\{0\cdot 1\cdot 2\cdot 3\cdots\cdots n\cdots\cdots(\varpi-1)\right\},$$

$$f(x)=x^2,\quad x\in A\leftrightarrow f(x)\in\varpi,$$

$$|A|=\varpi,$$

Theorem1.1.2 For a non-standard infinite set, it cannot form bijective relation with its own subset.

Prove: given infinite sets A, B and A is the proper set of B, $A\subset B$

Let $|B|=f(\varpi),|A|=f(\varpi)-\alpha,\alpha\in N^*$,

If sets A and B can construct bijective relation,

there exists $|B|=|A|$, $f(\varpi)-\alpha=f(\varpi)$,

According to operation rule of transfinite natural number: $f(\varpi)-\alpha<f(\varpi)$,

Thus, non-standard infinity itself cannot form bijective relation with its own subset.

Example1.1.2 Non-standard infinite set

Set of even numbers

$$S_1=\{\ 0\cdot 2\cdot 4\cdots\cdots 2(\varpi-1)\ \},$$

$$S_2=\{\ 0\cdot 2\cdot 4\cdots\cdots 2(\varpi-1)\cdot 2\varpi\cdot 2(\varpi+1)\ \}.$$

$$S_1\subset S_2,\quad |S_1|=\varpi,\quad |S_2|=\varpi+2$$

Under non-standard infinity, infinite set S_1 is the subset of S_2, S_2 have two elements $2\varpi, 2(\varpi+1)$ more than S_1, S_1 and S_2 cannot form bijective relation.

Example1.1.3 Under different counting method, the cardinal number of set of transfinite natural number is different

(1) Set of natural numbers $A_1=\left\{0\cdot 1\cdot 2\cdot 3\cdots\cdots n\right\}$, grows in manner of successor plus 1, its infinite set is :

$$\mu\lim_{n\to\infty}A_1=\left\{0\cdot 1\cdot 2\cdot 3\cdots\cdots\varpi\right\},\text{ its cardinal number is }|\mu\lim_{n\to\infty}A_1|=\varpi+1;$$

(2)Set of natural numbers $A_2=\left\{0\cdot 1\cdot 2\cdot 3\cdot 4\cdots\cdot 2n\right\}$, grows by two times and its infinite set is:

$$\mu\lim_{n\to\infty}A_2=\left\{0\cdot 1\cdot 2\cdot 3\cdot 4\cdots\cdot 2\varpi\right\},\text{ its cardinal number is : }|\mu\lim_{n\to\infty}A_2|=2\varpi+1;$$

(3) Set of natural numbers $A_3 = \{0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots n^2\}$, grows in manner of n^2, its finite set is :

$$\mu \lim_{n\to\infty} A_3 = \{0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots \varpi^2\}, \text{ its cardinal number is } |\mu \lim_{n\to\infty} A_3| = \varpi^2 + 1;$$

Similarly, if $A_4 = \{0 \cdot 1 \cdot 2 \cdots\cdots 2^n\}$, then

$$\mu \lim_{n\to\infty} A_4 = \{0 \cdot 1 \cdot 2 \cdots\cdots 2^{\varpi}\}, \text{ its cardinal number is :} |\mu \lim_{n\to\infty} A_4| = 2^{\varpi} + 1;$$

If $A_5 = \{0 \cdot 1 \cdot 2 \cdots\cdots \overline{a_0 a_1 \cdots a_{n-1}}\}$, $\overline{a_0 a_1 \cdots a_{n-1}}$ is regarded as n-digit number in the decimal system, $\overline{a_0 a_1 \cdots a_{\omega-1}}$, then

$$\mu \lim_{n\to\infty} A_5 = \{0 \cdot 1 \cdot 2 \cdots\cdots \overline{a_0 a_1 \cdots a_{\omega-1}}\}, \text{ its cardinal number is } |\mu \lim_{n\to\infty} A_3| = 10^{\varpi} + 1;$$

If $A_6 = \{0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots n^n\}$, then

$$\mu \lim_{n\to\infty} A_6 = \{0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots \varpi^{\varpi}\}, \text{ its cardinal number is } |\mu \lim_{n\to\infty} A_6| = \varpi^{\varpi} + 1;$$

If $A_7 = \{0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots f_i(n)\}$, grows in manner of $f_i(n)$, then,

$$\mu \lim_{n\to\infty} A_7 = \{0 \cdot 1 \cdot 2 \cdot 3 \cdots\cdots f_i(\varpi)\}, \text{ its cardinal number is } |\mu \lim_{n\to\infty} A_7| = f_i(\varpi) + 1.$$

Under different calculation methods, cardinal numbers of set of transfinite natural numbers are different.

Theorem1.1.3 Cardinal numbers of set of transfinite vary with encoding system and it can be any cardinal number but not a definite number.

Definition 1.1.2 Hyperreal numbers

Natural numbers or decimal numbers containing infinite digits are called hyperreal numbers, denoted with $\overline{a_\alpha \cdots a_2 a_1 a_0 \bullet b_1 b_2 b_3 \cdots b_\beta}$.

Assume that $R^* = \{x \mid x = \overline{a_\alpha \cdots a_2 a_1 a_0 \bullet b_1 b_2 b_3 \cdots b_\beta}\}$ is a set of hyperreal numbers, (a_i and b_i are complete arrangements of $0,1,2,\cdots,9$; α, β are ordinal numbers (or transfinite natural numbers), x is a sequence of 2 numeric segments, separated by a decimal point, thereinto, $a_\alpha \cdots a_2 a_1 a_0$ are called natural number bits , while natural number bits have finite digits or infinite digits; $b_1 b_2 b_3 \cdots b_\beta$ are called decimal digits, while decimal digits have finite digits or infinite digits.

Its decimal expansion is:

$$x = a_\alpha 10^\alpha + \cdots + a_\varpi 10^\varpi + \cdots + a_n 10^n \cdots + a_1 10^1 + a_0 10^0$$

$$+b_1 10^{-1}+b_2 10^{-2}\cdots+b_n 10^{-n}+\cdots+b_{\varpi}10^{-\varpi}+\cdots+b_{\beta}10^{-\beta}$$

Like transfinite natural numbers, cardinal numbers of set of hyperreal numbers vary with encoding system.

Theorem1.1.4 The cardinal number of set of hyperreal numbers which vary with encoding system may be any a cardinal number but not a definite number.

If

$$R^* = \{x \mid x = \overline{a_\alpha \cdots a_2 a_1 a_0 \bullet b_1 b_2 b_3 \cdots b_\beta}\}$$

Then

Its cardinal number is $|R^*| = 10^{\alpha+\beta+1}$

Based on above analysis, we can conclude that:

In non-standard form, under different operation modes, the cardinal number of infinite set can be different, and the sizes of two real infinite sets can only be compared in the same operation.

1.2 Continuum hypothesis（Inference of axiom system (*SZF*+) of non-standard set theory ）

Definition 6.1 Existence definition of power set

Denote the power set $\{0,1,2,3,\cdots,n-1\}$, of set of all natural numbers from 0 to $n-1$ with $\wp(n)$, namely,

$\wp(n)=\{x \mid x\subseteq n\}$, $n=\{0,1,2,3,\cdots,n-1\}$, $\wp(n)=\wp(\{0,1,2,3,\cdots,n-1\})$, power set of all natural numbers is denoted as: $\wp(N)=\{x \mid x\subseteq N\}$.

The above existence definition of the power set is the "power set axiom" in the axiom set theory ZF system, i.e.:

Power set axiom: $(\forall y)(\exists A)(\forall x)(x\in A \leftrightarrow x\subseteq y)$, denote as $A=\wp(y)$;

Theorem 1.2.1 Cardinal numbers of finite set's power set

Let $n=\{0\cdot 1\cdot 2\cdot 3\cdots\cdots n-1\}$, $|\wp(n)|$ denotes number of subset in $\wp(n)$,then $|\wp(n)|=2^n$

Prove:

$n=\{0\cdot 1\cdot 2\cdot 3\cdots\cdots n-1\}$, there are n elements in it, $\wp(n)$ is the set formed by all the different combinations of these n elements.

Thus, $|\wp(n)|$ is the sum of all the different combinations of n elements, That is $|\wp(n)| = C_n^0 + C_n^1 + C_n^2 + C_n^3 + \cdots + C_n^n = 2^n$,

So $|\wp(n)| = 2^n$ is true.

Theorem1.2.2 Cardinal number infinite set's power set ϖ

For any infinite set, $\varpi = \{0 \cdot 1 \cdot 2 \cdot 3 \cdot \cdots \cdot \varpi - 1\}$, then $|\wp(\varpi)| = 2^{\varpi}$.

Prove: according to above theorem $|\wp(n)| = 2^n$ and according to transfinite natural number induction, "for any proposition, if it is true to finite numbers, it is true to infinite numbers."

So $|\wp(\varpi)| = 2^{\varpi}$, the theorem is true obviously.

Theorem1.2.3 If the cardinal number of set of transfinite natural number is definite, then the cardinal number of its power set is a definite number, that is :

For any infinite set $\alpha \in N^*$, $\alpha = \{0 \cdot 1 \cdot 2 \cdot 3 \cdot \cdots \cdot \alpha - 1\}$, then $|\wp(\alpha)| = 2^{\alpha}$.

Prove: since $\alpha = \{0 \cdot 1 \cdot 2 \cdot 3 \cdot \cdots \cdot \alpha - 1\}$, $|\wp(\alpha)| = 2^{\alpha}$;

That is: the cardinal number of its power set is a definite number.

If $\alpha = \varpi$, then $|\wp(\varpi)| = 2^{\varpi}$, if $\alpha = 2^{\varpi}$, then $|\wp(2^{\varpi})| = 2^{2^{\varpi}}$, and so on.

$|\wp(\alpha)|$ varies with α.

Note 1.2.1 Cantor's continuum hypothesis

【Cantor proves that 2^{ϖ}, $2^{2^{\varpi}}$, $\cdots 2^{2^{\cdot^{\cdot^{2^{\varpi}}}}}$, $\cdots$ are uncountable cardinal numbers, which is not consistent with our intuition.

It is mistake that Cantor denotes number of elements in $\wp(N)$ as $|\wp(N)| = 2^{\aleph_0}$ and believes that they are uncountable.

In fact, Cantor's notation $|\wp(N)| = 2^{\aleph_0}$ is equal to $|\wp(\varpi)| = 2^{\varpi}$; cardinal numbers of set of natural numbers' power set are countable.

Simultaneously, Cantor believes that $|\underbrace{\wp\,\wp \cdots \wp}_{n}(\varpi)| = \left.2^{\cdot^{\cdot^{2^{\varpi}}}}\right\} n$ is uncountable cardinal number of higher layer, it is wrong either.

Cantor's proof

$0, 1, 2, \cdots, \omega, \cdots \omega 2, \omega 3, \cdots, \omega^2, \omega^3, \cdots, \omega^{\omega}, \omega^{\omega^{\omega}}, \cdots, \omega^{\omega^{\cdot^{\cdot^{\omega}}}}, \cdots$ are all countable cardinal numbers.

ω_1, ω_2, $\cdots \omega_{\omega}$, $\cdots \omega_{\alpha}$, $\cdots$ are all uncountable cardinal numbers.

Denote countable cardinal number as ω_0 and get a series of pedigrees from countable cardinal numbers to uncountable cardinal numbers

$$\omega_0,\quad \omega_1,\quad \omega_2,\quad \cdots\omega_\omega,\quad \cdots\omega_\alpha,\quad \cdots.$$

In some documents, it also is denoted as:

$$\aleph_0,\quad \aleph_1,\quad \aleph_2,\quad \cdots\aleph_\omega,\quad \cdots\aleph_\alpha,\quad \cdots.$$

Cantor proves that $2^{\aleph_0}$ is uncountable cardinal number and makes a conjecture about continuum hypothesis that there is no other cardinal number between $2^{\aleph_0}$ and $\aleph_0$, or $2^{\aleph_0}=\aleph_1$ and generalized continuum hypothesis

Cantor's continuum hypothesis

$$\neg\exists S\ (\ \aleph_0<|S|<2^{\aleph_0}\ );$$

Generalized continuum hypothesis:

$$\neg\exists S\ (\ \aleph_\alpha<|S|<2^{\aleph_\alpha}\ ).$$

Since $\aleph_\alpha$ can be regarded as cardinal number of an infinite set, $2^{\aleph_\alpha}$ can be regarded as cardinal number of infinite set's power set of, Cantor's continuum hypothesis can be regarded as that:

Is there other cardinal number between cardinal number $\aleph_\alpha$ of an infinite set and cardinal number

$2^{\aleph_\alpha}$ of infinite set's power set? 】

In standard axiom system and non-standard axiom system, there are uncountable ordinal numbers and cardinal numbers, Cantor's continuum hypothesis can be regarded as that:

Is there other cardinal number between cardinal number α of an infinite set and cardinal number 2^α of the infinite set's power set?

We will answer this question in the standard axiom system and the non-standard axiom system respectively.

Theorem1.2.4 Negation of Cantor Continuum Hypothesis (CH)

In non-standard infinite axiom system: $\exists S\ (\ \varpi<|S|<2^{\varpi}\ )$, $\exists S\ (\ \alpha<|S|<2^{\alpha}\ )$.

Prove:

$$\varpi=\{0\cdot1\cdot2\cdot3\cdots\cdot\varpi-1\}$$

According to transfinite natural number induction, cardinal number of power set $\wp(\varpi),\ \wp\ \wp(\varpi),\ \cdots,\underbrace{\wp\ \wp\cdots\wp}_{n}(\varpi),\cdots$ is :

$$|\wp(\varpi)|=2^{\varpi},|\wp\wp(\varpi)|=2^{2^{\varpi}},\cdots,|\underbrace{\wp\wp\cdots\wp}_{n}(\varpi)|=2^{\cdot^{\cdot^{2^{\varpi}}}}\Big\}n,\cdots$$

$$\varpi<2^{\varpi}<\quad 2^{2^{\varpi}}<\quad\cdots<2^{2^{\cdot^{\cdot^{2^{\varpi}}}}}<\quad\cdots$$

For any cardinal number α

$$\alpha<2^{\alpha}<\quad 2^{2^{\alpha}}<\quad\cdots<2^{2^{\cdot^{\cdot^{2^{\alpha}}}}}<\quad\cdots$$

There are $\varpi,\varpi+1,\varpi+2,\cdots$ these cardinal numbers between ϖ and 2^{ϖ}.

There are $\alpha,\alpha+1,\alpha+2,\cdots$ these cardinal numbers between α and 2^{α}

$$\varpi<\varpi+1<\varpi+2<\cdots<2^{\varpi}$$

$$\alpha<\alpha+1<\alpha+2<\cdots<2^{\alpha}$$

Let $S=\varpi+1$, $S=\alpha+1$, that is:

$$\exists S\ (\varpi<|S|<2^{\varpi}),\ \exists S\ (\alpha<|S|<2^{\alpha}).$$

Therefore, in non-standard infinite axiom system: both Cantor's continuum hypothesis (CH) and generalized continuum hypothesis (GCH) are false.

●2.Cardinal numbers And Cantor's continuum hypothesis of axiom system (*SZF*-) of standard set theory

●2.1(Inference of axiom system (*SZF*-) of standard set theory)

In standard infinity, order of cardinal numbers of ordinal number is the order of ordinal numbers themselves, there is only an infinite cardinal number and cardinal numbers and ordinal numbers are unified.

$$\cdots|\infty-1|=|\infty|=|\infty+1|=\cdots=|\infty+n|=\cdots=|2\infty|=\cdots=|n\infty|=\cdots=|\infty^2|=|\infty^3|$$

$$\cdots=|\infty^n|=\cdots=|n^{\infty}|=\cdots=|\infty^{\infty}|=\cdots=|\infty^{\infty^{\infty}}|=\cdots=\infty.$$

Example2.1.1 Cardinal numbers of standard infinite set

$$A=\{0\cdot 1\cdot 4\cdot 9\cdot\cdots\cdot n^2\cdot\cdots\cdot\infty\},$$

$$N=\{0\cdot 1\cdot 2\cdot 3\cdot\cdots\cdot n\cdot\cdots\cdot\infty\}=\infty,$$

$$f(x)=x^2,\quad x\in N\leftrightarrow f(x)\in A,$$

$$|A|=\infty,$$

Theorem2.1.1 Standard infinite set itself can form bijective relation with its own subset.

For example: $f(x)=x^2$

$$A\subset N\,,x\in N\leftrightarrow f(x)\in A$$

Therefore , Standard infinite set itself can form bijective relation with its own subset.

Example2.1.2 Standard infinite set

Set of even numbers

$$S_1=\{\ 0\cdot2\cdot4\cdots\cdots2(\infty-1)\ \},$$

$$S_2=\{\ 0\cdot2\cdot4\cdots\cdots2(\infty-1)\cdot2\infty\cdot2(\infty+1)\ \}.$$

$$S_1=S_2\,,|\,S_1\,|=\infty\,,|\,S_2\,|=\infty$$

Example2.1.3 Under different calculation mode, cardinal numbers of set of standard natural numbers' infinite set of are same.

(1)Set of natural numbers $A_1=\{0\cdot1\cdot2\cdot3\cdots\cdots n\}$, grows in manner of successor plus 1, its infinite set is:

$\lim\limits_{n\to\infty}A_1=\{0\cdot1\cdot2\cdot3\cdots\cdots\infty\}$, its cardinal number is: $|\lim\limits_{n\to\infty}A_1|=\infty$;

(2)Set of natural numbers $A_2=\{0\cdot1\cdot2\cdot3\cdot4\cdots\cdot2n\}$, grows by two times, its infinite set is:

$\lim\limits_{n\to\infty}A_2=\{0\cdot1\cdot2\cdot3\cdot4\cdots\cdot\infty\}$, its cardinal number is $|\lim\limits_{n\to\infty}A_2|=\infty$;

(3) Set of natural number $A_3=\{0\cdot1\cdot2\cdot3\cdots\cdots n^2\}$, grows in manner of n^2, its infinite set is:

$\lim\limits_{n\to\infty}A_3=\{0\cdot1\cdot2\cdot3\cdots\cdots\infty\}$, its cardinal number is $|\lim\limits_{n\to\infty}A_3|=\infty$;

Similarly, if $A_4=\{0\cdot1\cdot2\cdots\cdots2^n\}$, then

$\lim\limits_{n\to\infty}A_4=\{0\cdot1\cdot2\cdots\cdots\infty\}$, its cardinal number is: $|\lim\limits_{n\to\infty}A_4|=\infty$;

If $A_5=\{0\cdot1\cdot2\cdots\cdots\overline{a_0a_1\cdots a_{n-1}}\}$, $\overline{a_0a_1\cdots a_{n-1}}$ is regarded as the n-digit number in the decimal system , $\overline{a_0a_1\cdots a_{\omega-1}}$, then

$\lim\limits_{n\to\infty}A_5=\{0\cdot1\cdot2\cdots\cdots\infty\}$, its cardinal number is $|\lim\limits_{n\to\infty}A_5|=\infty$;

If $A_6=\{0\cdot1\cdot2\cdot3\cdots\cdots n^n\}$, then

$\lim\limits_{n\to\infty}A_6=\{0\cdot1\cdot2\cdot3\cdots\cdots\infty\}$, its cardinal number is :$|\lim\limits_{n\to\infty}A_6|=\infty$;

If $A_7=\{0\cdot1\cdot2\cdot3\cdots\cdots f_i(n)\}$, grows in manner of $f_i(n)$, then

$$\lim_{n\to\infty} A_7 = \{0\cdot 1\cdot 2\cdot 3\cdot\cdots\cdot\infty\}$$, its cardinal number is $|\lim_{n\to\infty} A_7| = \infty$.

Under different calculation mode, the infinite cardinal number of set of standard natural number is the same.

Theorem 2.1.2 By any encoding system, infinite cardinal number of set of standard natural numbers is ∞.

Theorem2.1.3 By any encoding system, infinite cardinal number of set of standard real numbers is ∞.

If

$$R = \{x \mid x = \overline{a_n \cdots a_2 a_1 a_0 \bullet b_1 b_2 b_3 \cdots b_\infty}\}$$

Then its cardinal number is :

$$|R| = 10^{n+\infty} = \infty.$$

●2.2 (*SZF*-) Inference of axiom system of (*SZF*-) standard set theory

Theorem2.2.1 Cardinal numbers of infinite set's power set

For any infinite set, $N = \{0\cdot 1\cdot 2\cdot 3\cdot\cdots\cdot\infty\}$, then $|\wp(N)| = 2^\infty = \infty$.

Prove: according to above theorems and transfinite natural number induction, the theorem is true obviously.

Theorem2.2.2 $|\wp(N)| = |\wp\,\wp(N)| = \cdots = |\underbrace{\wp\,\wp\cdots\wp}_{n}(N)| = \cdots = \infty$

In standard set theory *SZF*- axiom system, Cantor's continuum hypothesis (CH) is true.

Theorem 2.2.3 Affirmation of Cantor's continuum hypothesis (CH)

In standard infinite axiom system: $\neg\exists S\ (\infty < |S| < 2^\infty)$.

Prove:

$$N = \{0\cdot 1\cdot 2\cdot 3\cdot\cdots\cdot\infty\}$$

According to transfinite natural number induction, the cardinal number of power set $\wp(N), \wp\,\wp(N), \cdots, \underbrace{\wp\,\wp\cdots\wp}_{n}(N), \cdots$ is:

$$|\wp(N)| = 2^\infty, |\wp\,\wp(N)| = 2^{2^\infty}, \cdots, |\underbrace{\wp\,\wp\cdots\wp}_{n}(N)| = 2^{\cdot^{\cdot^{2^\infty}}}\Big\} n, \cdots$$

$$\infty = 2^\infty = 2^{2^\infty} = \cdots = 2^{2^{\cdot^{\cdot^{2^\infty}}}} = \cdots$$

In standard set theory *SZF*- axiom system, $2^\infty = \infty$, there is no other cardinal number between ∞ and 2^∞, namely,

$$\neg\exists S\ (\infty < |S| < 2^\infty),$$

Therefore, in standard infinite axiom system: “both Cantor’s continuum hypothesis (CH) and generalized continuum hypothesis (GCH) are true.”

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