An Elementary Solution of the Four-color Conjecture

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Abstract

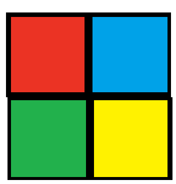
The four-color conjecture (Guthrie's problem after F. Guthrie) is divided into 5 simple groups, every region is shared a common boundary with 1, 2, 3, 4, and unlimited regions sharing a common boundary (other than a single point) do not share the same color to make the problem much simpler and clearer, The four-color theorem is solved perfectly by using these simpler models.

Key words: four-color conjecture, Guthrie’s problem

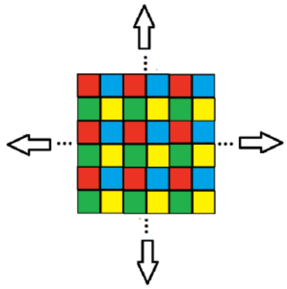
The four-color conjecture (Guthrie's problem after F. Guthrie) states that any map in a plane can be colored using four-colors in such a way that regions sharing a common boundary (other than a single point) do not share the same color.

At the now time, the solutions of mathematical problems are too mathematical and make problems more complex, if we divide a complex problem into a few of the simpler problems and solve them one by one, the solutions will be much simpler and clearer.

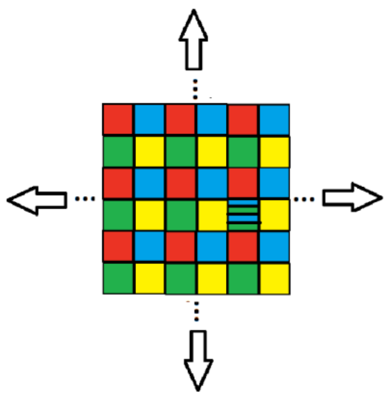
First, let us divide the four-color theorem into a simplest and special situation: every region is shared a common boundary with 4 regions and we can draw four squares with four-color (red, blue, green and yellow) and merge them as the following:



Then make unlimited copies of it, them put unlimited copies to left, right, up and down sides of this square, we can get unlimited regions sharing a common boundary (other than a single point) do not share the same color as the following:

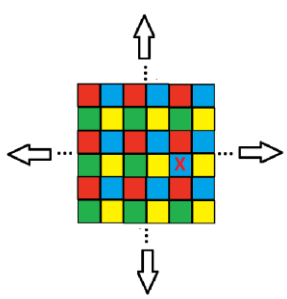


Second, to let every region is shared a common boundary with unlimited regions as following (due to the square size limit, the green square is only divided into 4 regions, however, it can be divided to unlimited regions, the divided regions must be color with the original color “green” and the color “blue” at its diagonal position alternately，the divided regions sharing a common boundary (other than a single point) do not share the same color as following:

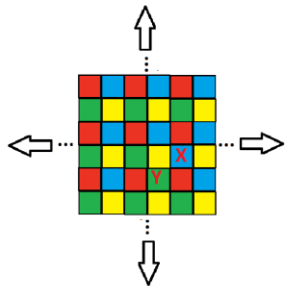


Same thing can be done to other three sides, and each square can be divided into 2, 3, 4…n.

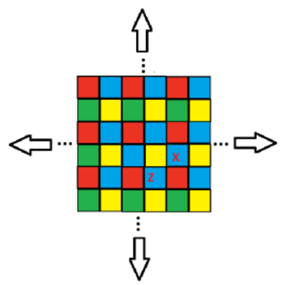
Third, to let every region is shared a common boundary with only 3 regions, we just merge the 2 sides of the region into one and select one color of the 2 original colors (see the labeled square with X), such as:



Fourth, to let every region is shared a common boundary with 2 regions, we just merge the other 2 sides of the region into one and select one color (green) of the 2 original colors (see the labeled square with Y), such as:



Final, to let every region is shared a common boundary with only 1 region, we just merge the all four sides of the region into one and select one color of the 2 original colors (see the labeled square with X and Z both same color), such as:



At all above 5 models, any region can share with any number regions sharing a common boundary (other than a single point) do not share the same color, the four-color conjecture (Guthrie's problem after F. Guthrie) is solved perfectly.