Winning at War: Comparing different strategies in a card game

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(Dated: April 22, 2023)

The card game "war" is a simple game usually assumed to not include any element of strategy, only luck. I challenge this notion by noticing that the order of placing cards back into the deck can be used as a strategy. I simulate the game with different strategies, and find that the strategies can significantly increase the chances of winning, but usually increase the time it takes to complete the game. This is however dependent on your opponent using specific strategies. The best advice on strategy seems to be tricking your opponent into following an ordered strategy, while you use a random strategy, a strategy some might object to.

INTRODUCTION

The card game 'War' is a simple card game that can be played by two or more players. Usually only two, as it is one of the few games that is fun to play with only two. It is known for taking long to complete, and is by some called "starve-to-death", due to its long duration.

It has long been considered a game of pure chance, as the players have no say in what card they can play. However, in this paper we will show that there are strategies that can be used to significantly increase the chances of winning, and also for reducing the time it takes to complete. These strategies involve the order the cards are placed back into the deck, and several such orderings will be considered in this paper.

I will first give an account of the rules of the game, and which assumptions that go into the simulation I built for it. Then I will give an account of the strategies that will be considered. Then I will present the results of the simulations, and finally I will discuss the results and give some conclusions.

RULES

The game is played with a standard 52-card deck of French-suited playing cards. While more than 2 players are possible, this work will only consider the case of two players. At the beginning of the game, the deck is shuffled and divided evenly between the two players, giving each a stack of 26 cards. Each player places their stack face down in front of them. The game is played in rounds, and in each round both players turn up a card from their stack. The player with the higher card wins. Only the value of the card matters, the suit is disregarded. The cards played in the round are placed at the bottom of the winners deck. This is repeated until one player has won all the cards.

In the case of a tie, "war" is declared. Both players turn up three cards face down, and then one card face up. The player with the higher card wins the war, and the cards are placed at the bottom of the winners deck. If the cards are tied again, another war is declared. This is repeated until one player wins the war. Should one player run out of cards during a war, the last card they have is used in the final battle. The other player still places out 3 cards face down, so the number of cards from each player may differ. If a war is declared with the last card, the player loses and the game is over.

STRATEGIES

There is not much choice each player has during the game. It is therefore natural to think the game is purely a game of chance and no strategy. There is however one point where the player has to make a decision; the order the cards are placed back into the deck. No account of the rules I know reference how this should be done, and it therefore a potential source of strategy. In this section* I will give an account of the strategies that will be considered in this paper.

What most players of this game already do is probably not think about it at all, and just place the cards back into the deck in a random order. Note that this is not truly random, as in the case of war, the face-down card are usually kept in the same order. How this is done however is not consistent, and it is therefore not straight forward to reproduce the same order in a simulation. The first strategy I will consider is the truly random shuffle before putting the pot back into the deck. This is quite straight forward and not much more to be said about this.

The first non-random strategies involve stacking. Simply concatenate the cards played by each player before placing the put into your deck. A choice has to be made of which to place first. The two strategies are called Stacking, Me First (SMF), and Stacking, Me Last (SML). These are joined by their reverse, RSMF and RSML. This is the process of reversing the pot before placing it into your deck.

The next few strategies involve interleaving. This is the process of taking the two decks placed by each player, and alternating which one to pick up a card from. This is illustrated in figure 1 for two decks of different size. In this example, the top deck is the one that is picked up from first. This is the losing deck, as can be seen in the last number in each deck. Therefore, the illustration in figure 1 showcases the strategy Interleave, Me Last (IML). An alternative is Interleave, Me First (IMF). Both of these are accompanied by their reverse, RIML and RIMF. This is the process of reversing the pot before placing it into your deck.

Note that in most cases many of these strategies will be equivalent. In a simple battle, you only have two choices. Put your card first or last into the deck. In this case, SMF, RSML, IMF and RIML are all equivalent, while SML, RSMF, IML and RIMF are all the opposite. However, after a war, they are all distinct. Certain strategies are very simmilar. RIMF and IML only differ in the case of a player not having enough cards to lay 3 face-down during war. Some of these strategies might be frowned upon by the other player, and it is therefore possible that some of them are not used. This is not considered in this paper.

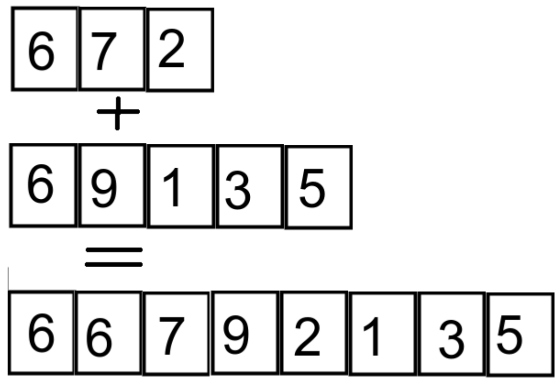

FIG. 1: Illustration of interleaving.

SIMULATION

The simulation of the game is implemented in Python. A game is simulated between to players handed half of a shuffled deck, until one player wins. Each player is given a strategy, which is consistently followed throughout the game. Some strategies are prone to produce very long-lasting games. Therefore, I set a limit of 5.000 rounds, after which the game is declared a draw. This is done to avoid the simulation taking too long. The simulation is run 10.000 times for each strategy combination. Only games that finish in less rounds than the limit are counted. The number of rounds needed to finish, and the winner is recorded. The results are averaged over all the games. The results are presented in the next section.

RESULTS

The win-rate for player 1 with each strategy combination is shown in figure 2. Most combinations do not have a significant difference in win-rate. Noticable exceptions are RSMF and SML, which have a win-rate of 0.6 against IMF, RIML, SMF and RSML. The very best is SML which has a win-rate of 0.67 against SMF. Also note that the matrix is anti-symmetric, meaning that the win-rate for player 2 is the same as for player 1, but with the strategies reversed.

Interestingly, player 1 using the random strategy is significantly worse than 50/50 when used against any of the other strategies except itself. Using random strategy against an ordered strategy makes you lose about 55% of the time. But player 2 using the random strategy is significantly better. Why random should be better for player 2 against both IMF and IML, with switching roles give the opposite result I can not explain at the moment. Further investigation is needed.

As mentioned, I set a limit of 5000 rounds, and some combinations of strategies lead to games lasting longer than this. Figure 3 shows the chance of a game lasting longer than 5000 rounds. This is as high as 80% for SML against SMF. Opposite strategies seem to produce a high chance of this, especially the stacking strategies.

In figure 4, the average length of each game is shown. Here there is quite a large variation. Any combination involving the random strategy takes around 280 rounds. Several combinations are all close to 180 rounds, which seems to be the minimum. The worst combinations are RSMF against SMF and RSML against RSMF and SML, both taking on average 1100 rounds. However, there is a very high standard deviation, barely lower than the mean itself. This is the case for all combinations. This means that it is highly variable how long a game will last, some lasting as low as 9 rounds. The longest games lasted between 1500 and 2500 rounds for the combinations that do not lead to games lasting longer than the round-limit. For the combinations that do, the average length is much higher.

FURHTER WORK

This work has only considered the case of two player. It remains to be seen how the strategies compare when more players are involved. Further, I have only considered static strategies. I think it is likely that some adaptive strategy could out-perform any of the ones tried here, for instance one dependent on the values of the card, your opponents last move or your position in the game. It would be interesting to see if this is the case.

CONCLUSION

It is not straight forward to recommend one strategy to use. It seems that any strategy that gives a higher than random chance of winning also has a high chance of lasting very long, and is dependent on your opponent using an ordered strategy, which is unlikely. The only way seems to be to trick them into using an ordered strategy, and use random yourself, while positioning yourself to be player 2. Then the game probably will not last very long, and you will have a better-than-random chance of winning.

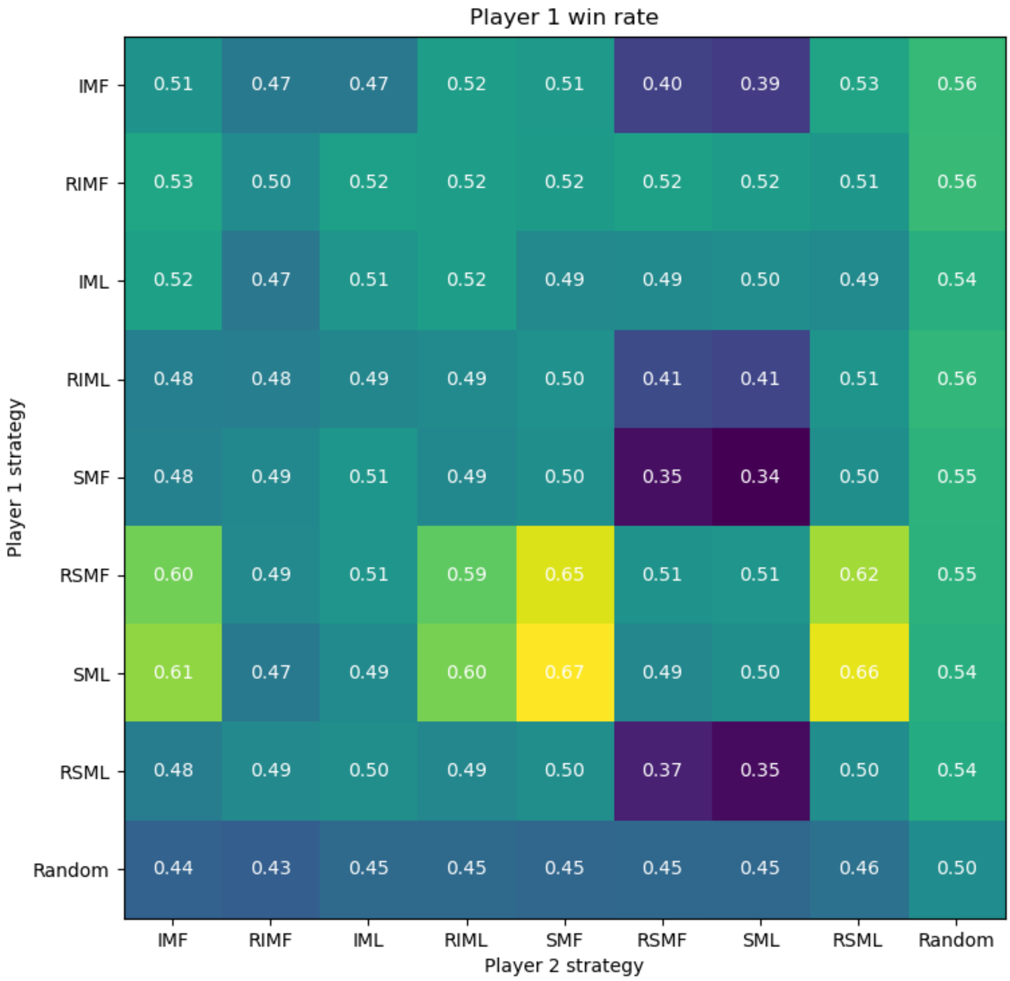

FIG. 2: Chance of winning for each strategy combination.

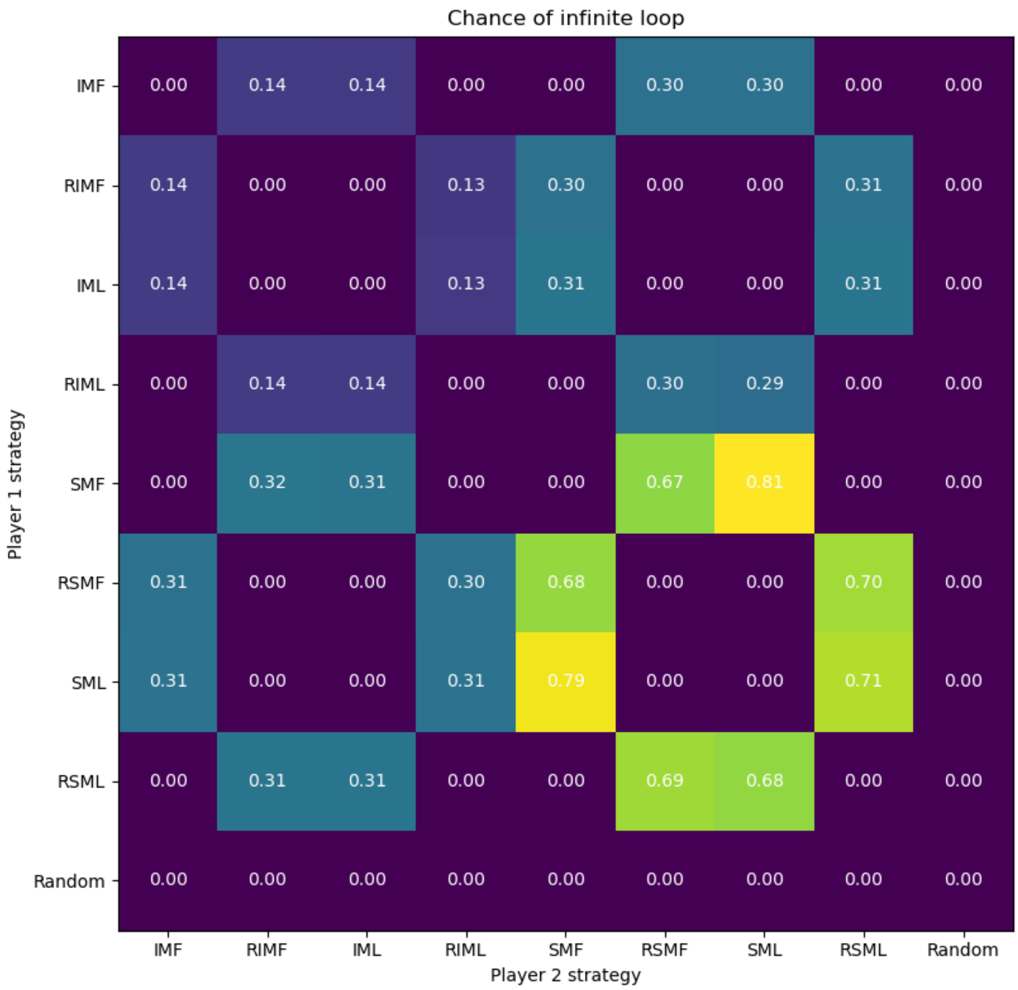

FIG. 3: Chance of a game lasting longer than the round-limit.

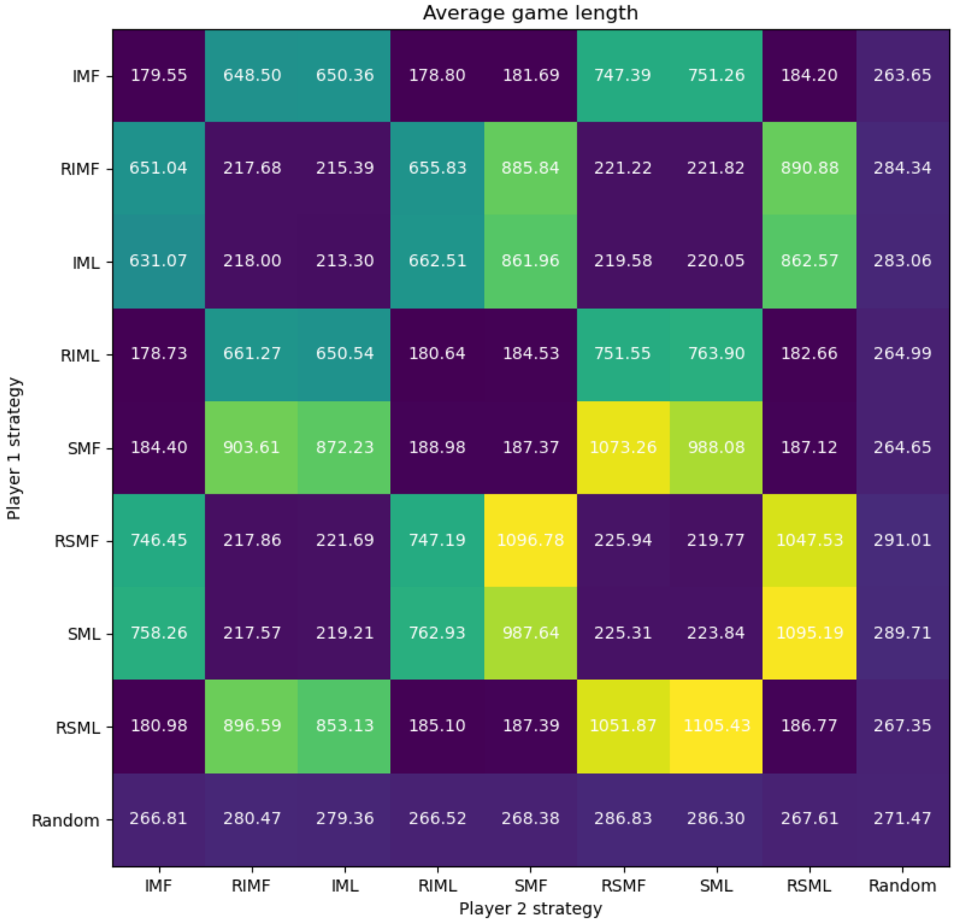

FIG. 4: Average number of rounds needed to finish a game.