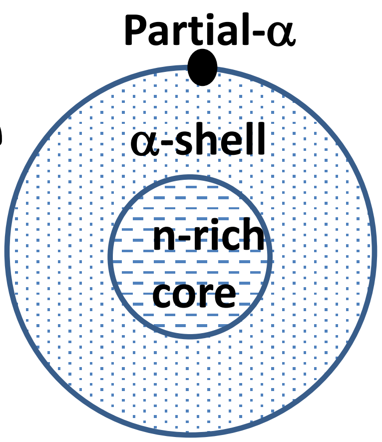
Alpha-Crystal: A Simple Model for Nuclear Structure



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American Physical Society Virtual Meeting, April 24, 2023

Summary

- A simple real-space picture of nuclear structure is proposed:
  - Large nuclei are quasi-spherical close-packed crystals of αs.
  - But central αs are converted to neutron-rich phase by beta decay in large electrostatic potential.
  - The n-rich core may be made of pure-n or p3n nuclei.
  - A single partial-α may also be present on the outer surface.
  - A continuum approximation to this α-crystal is particularly simple.
- This model accounts for general trends in stable nuclei and in nuclear decay.
- Reasons are discussed why this was *not* proposed in 1930s.
- This model may provide greater insight than standard shell and droplet models for both instruction and calculation.

Atoms, Molecules, & Crystals

- I approach nuclear structure with a background in solid-state physics.
- Everyone knows that atoms bind to form molecules, and the ground state of an array of identical molecules is a crystal.
- An example of a molecular crystal is solid hydrogen, consisting of a close-packed crystal of tightly-bound H_2 molecules.
  - Weak Van der Waals binding between molecules.
  - Surface emission consists of individual H_2 molecules, *not* H-atoms or clusters.
- Another atom or molecule may adhere to the outer surface of the crystal.

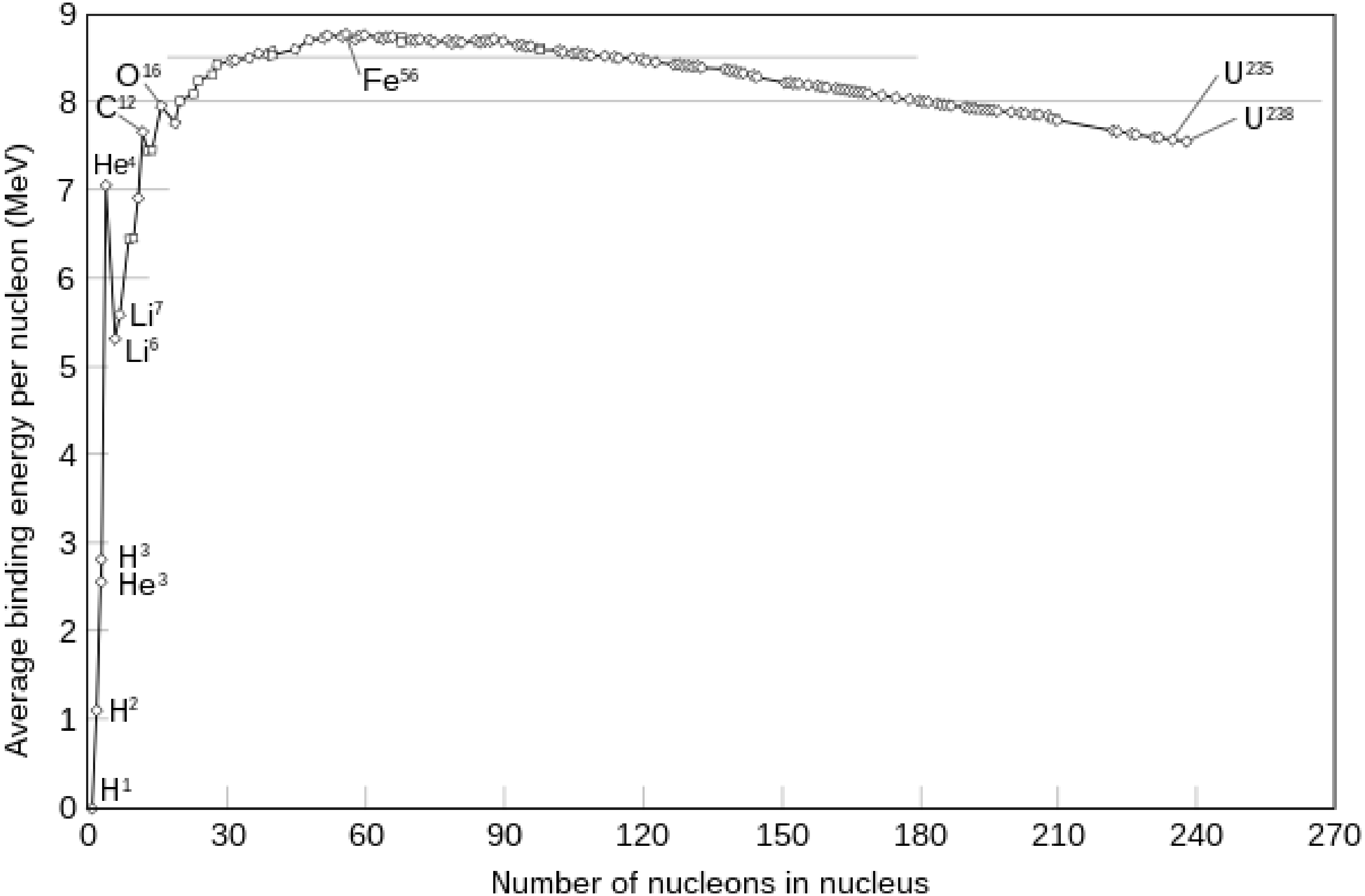
Electrons and Nucleons

- In 1930s, protons and neutrons were believed to be elementary particles/waves similar to electrons.
  - So nuclei analogous to atoms, with delocalized overlapping orbitals.
  - Led to quantum shell models for ground and excited nuclear states, and droplet models for mechanical properties of large nuclei.
- But since 1970s, quarks known to be elementary particles/waves analogous to electrons, and nucleons are more analogous to atoms ~ 1 fm which bind to one another.
  - That would also make small nuclei (deuterons, tritons, αs) more analogous to molecules, and large nuclei analogous to molecular clusters or crystals.
  - Novel suggestion – nucleus is “molecular crystal” of αs (He-4 nuclei).

Protons, Neutrons, and Beta Decay

- Proton (938.3 MeV) and neutron (939.6 MeV) are very similar.
- 1.3 MeV excess energy of n – decays to p by beta decay in ~ 15 minutes, via weak interaction.
- But n is quite stable inside nucleus, where electric potential raises energy of p relative to n.
  - In some cases, p in nucleus can decay to n via beta decay.
- α is the most stable small nucleus, with 28 MeV binding energy (7 MeV per nucleon).
  - This high stability is analogous to a molecule with saturated bonds.
- Other small stable “molecules”:
  - Deuteron (pn) – 1.1 MeV/nucleon
  - He-3 (ppn) – 2.5 MeV/nucleon

Curve of Binding Energy

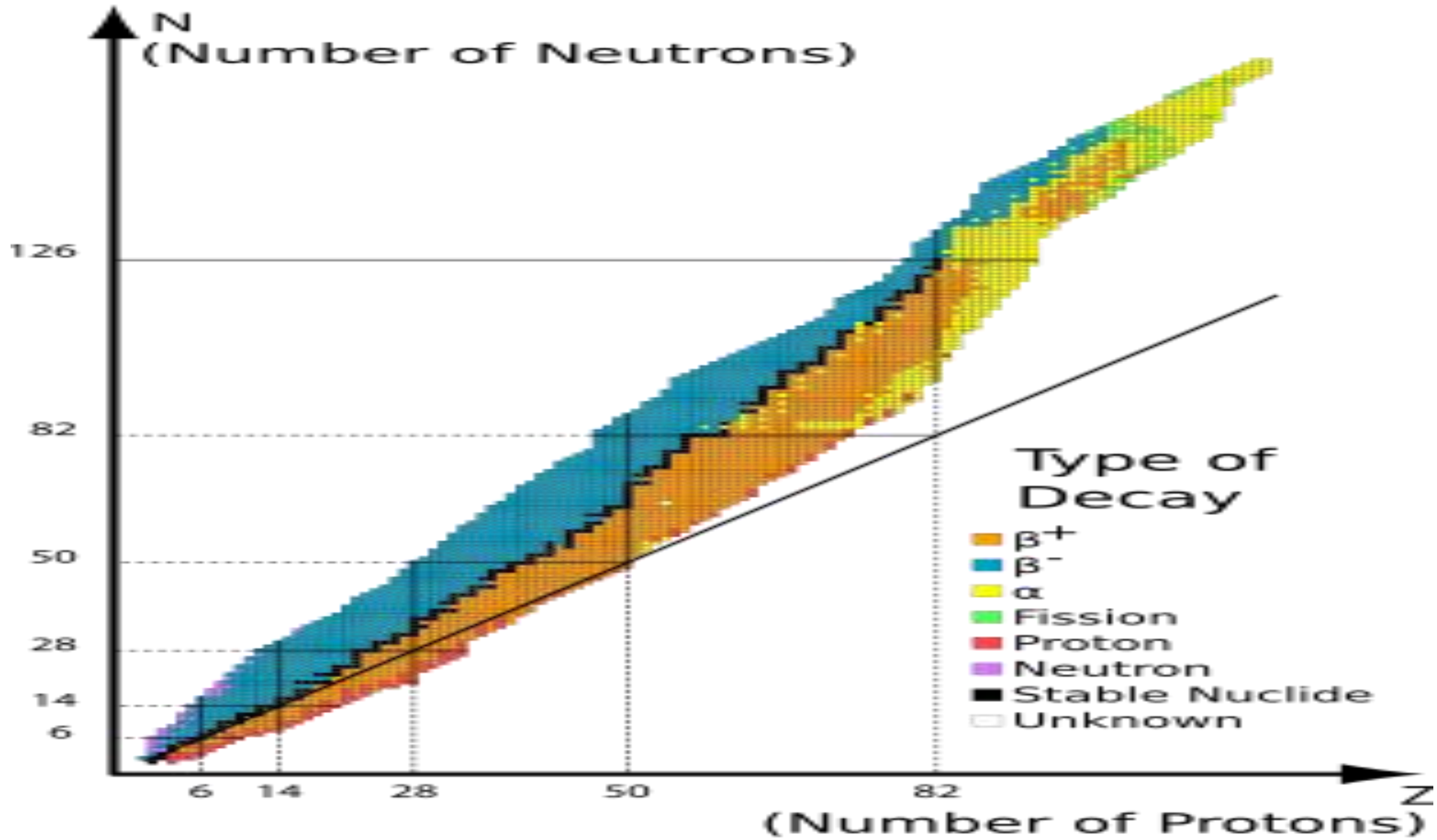


From https://en.wikipedia.org/wiki/Nuclear_binding_energy

General Explanation of Binding Energy

- Nuclei have short-range attraction due to strong force, but long-range repulsion of protons due to electric force.
- Fe-56 has largest binding energy, ~8.8 MeV per nucleon.
- Larger nuclei have lower net binding energy, but remain virtually stable through Bi-210.
  - Attractive strong forces greater than repulsive electric forces.
- Heavy nuclei could reduce energy by splitting in half (fission), but spontaneous fission very unlikely, at least up to Uranium.
- Primary nuclear decay mechanism for heavy nuclei is α–emission, *not* emission of p or n.
- No simple quantitative picture of binding energy available in conventional models.

Curve of Excess Neutrons



From https://en.wikipedia.org/wiki/Stable_nuclide

Why Excess Neutrons?

- Small stable nuclei (up to S-32) have equal numbers of p and n, but heavy nuclei have extra neutrons.
  - Ratio of neutrons to protons rises from 0.5 for He-3 to 1.5 for U-238.
- Narrow range of stable nuclei – excess neutrons or excess protons decay by β-decay.
- But α-decay is primary decay mechanism of unstable heavy nuclei, such as uranium.
  - Suggests that αs are primary component on surface of large nuclei.
  - The excess neutrons may be hidden in the interior of the nucleus.
- General understanding that larger electric potential raises stability of neutrons over protons, but no simple quantitative model of neutron excess is available in conventional models.

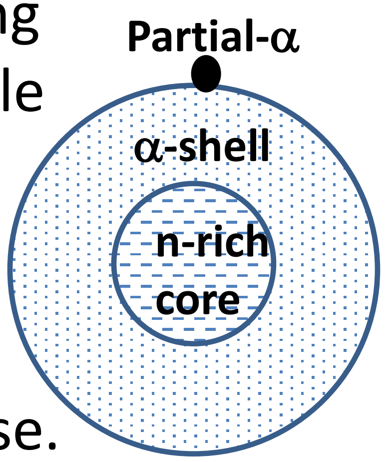
Nucleonic Liquid or Lattice?

- Most nuclear models have treated the nucleus as a liquid droplet, with nucleons moving around in it.
  - A static lattice model is quite different. [1],[2]
- The preference for a liquid model stemmed from the early belief that nucleons were elementary particles like electrons.
  - According to wave-particle duality, electrons are point particles moving within a distributed delocalized wave.
  - But we now know that nucleons have internal quark wave components, but are externally localized objects ~ 1 fm.
  - In a dynamic liquid model, all nucleons have the same average environment.
  - But in a static lattice model, central nucleons see a much higher electric potential than those near the surface.

[1] Hafstad and Teller, "The Alpha Particle Model of the Nucleus", Phys. Rev. 54, 681 (1938).
[2] Pauling, "The Close-Packed Spheron Model of Atomic Nuclei", Proc. NAS 54, 989 (1965).

Spherical α-crystal with Neutron-Rich Core

- New simple picture of spherical nucleus comprising close-packed αs with neutron-rich core and a single partial-α on outer surface.
- Electric potential highest in core, where neutron-rich phase is more stable than α.
- In α-shell, α is more stable than neutron-rich phase.
- Partial-α consists of tightly bound 1, 2, or 3 nucleons present as n, np, pnn, or ppn, depending on surface electric potential.
- Crystal binding is short-range contact attraction, with long-range electric repulsion.
- Long-range repulsion makes very large nuclei unstable toward α-emission from surface or fission into smaller nuclei with reduced repulsion.



Even Simpler Spherical Continuum Approx.

- Outer radius r_2 given by r_2 = 1.2 fm*$A^{1/3}$ where A = Z + N = "atomic mass".
  - Density 0.14/fm^3 close-packed nucleons.
- For simplicity, first consider pure-neutron core with inner radius r_1 inside outer α-shell
  - Charge density $\rho = 0.5$ e/nucleon = 0.07 e/fm^3 for $r_1 < r < r_2$, 0 for $r < r_1$.
  - Calculate electric potential $\phi(r)$, determine r_1 by $\phi(r_1) = \phi_0 \approx 10$ MV.
  - Model inter-particle attraction as positive volume binding energy and negative surface energy.
  - Ignore partial α on surface.
- Simple calculation of attractive and repulsive energies leads to Curve of Excess Neutrons and Curve of Binding Energy.

Electric Fields and Potentials

- Consider first small nucleus with only α-sphere without neutron-rich core (r_1=0)
  - Uniform-density sphere – standard problem in electricity and gravity
- $E(r) = Ze/(4\pi\varepsilon_0 r^2)$, $r > r_2$

  $= Ze/(4\pi\varepsilon_0 r_2^2)*(r/r_2)$, $r < r_2$
- $\phi(r) = -\int E(r)\,dr$

  $= Ze/(4\pi\varepsilon_0 r)$, $r > r_2$

  $= Ze/(4\pi\varepsilon_0 r_2)[1 + 0.5r^2/r_2^2]$, $r < r_2$
- $\phi(0) = 1.5Ze/(4\pi\varepsilon_0 r_2)$
- While E is max at surface, ϕ is max at center.
- Example [Note that $e/(4\pi\varepsilon_0)$ = 1.44 MV-fm]
  - For Z= 16 and A= 32 (S-32), r_2 = 3.8 fm and $\phi(0)$ = 9.1 MV.

Large Nucleus with n-rich Core

- Solve by linear superposition of two α-spheres with r_2 and r_1.
  - Define $Q_2 = 4\pi\rho r_2^3/3$, $Q_1 = 4\pi\rho r_1^3/3$.
  - For pure-n-core, $Z = (Q_2 - Q_1)/e$, $N = (Q_2 + Q_1)/e$, $A = 2Q_2/e$
- $\phi(0) = 1.5Q_2/(4\pi\varepsilon_0 r_2) - 1.5Q_1/(4\pi\varepsilon_0 r_1)$

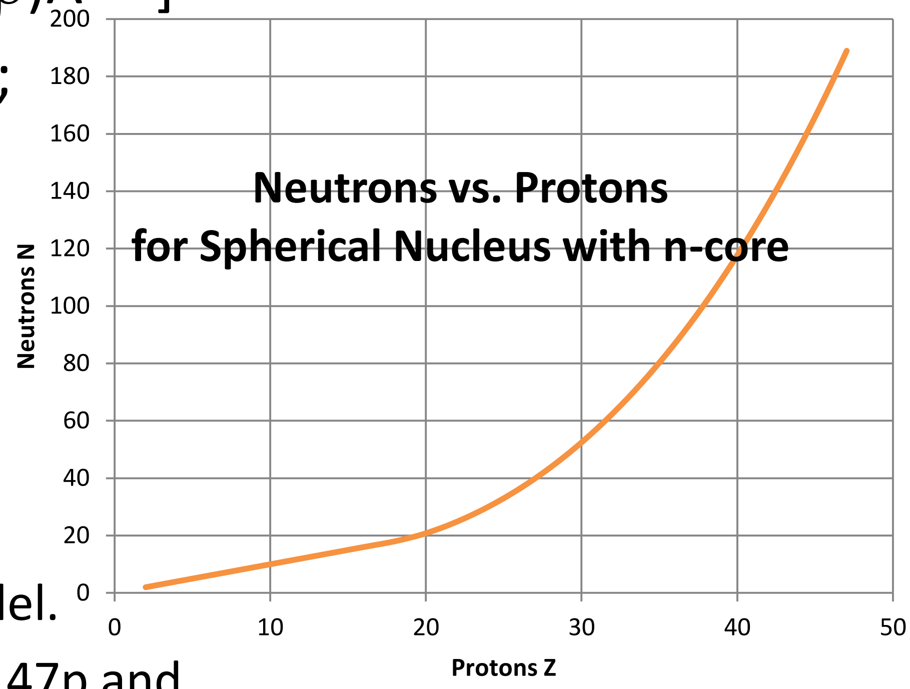
  $= (\rho/2\varepsilon_0)(r_2^2 - r_1^2)$
- $\phi(0) = \phi(r_1) = \phi_1$ is the constant potential inside the n-core.
- Solve for r_1 as a function of A and ϕ_1.
  - $r_1 = [(1.2\, A^{1/3})^2 - 2\varepsilon_0\phi_1/\rho]^{1/2}$
- So n-rich core nucleates $r_1>0$ for $A = 0.58(2\varepsilon_0\phi_1/\rho)^{1.5} = 36$, using $\phi_1 = 10$ MV and $\rho/e = 0.07\ \mathrm{fm}^{-3}$.
  - In fact, both S-32 (with N=Z) and S-36 (with N=Z+4) are stable.
  - Larger nuclei have extra neutrons.

Excess Neutrons from Spherical n-core

- $Q_1/Q_2 = (r_1/r_2)^3 = [1 - 0.7(2\varepsilon_0\phi_1/\rho)A^{-2/3}]^{3/2}$

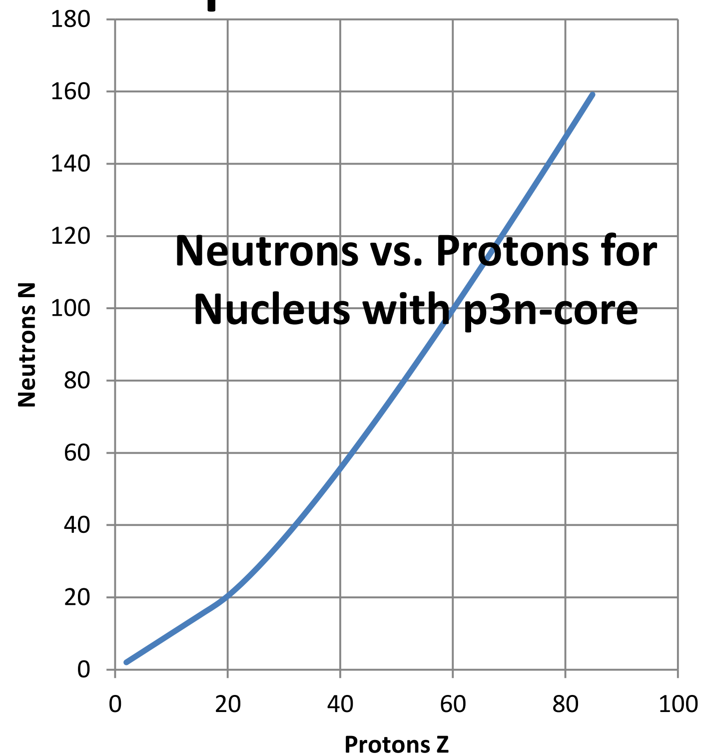
  $= [1 – 11A^{-2/3}]^{3/2}$, for $A > 36$;

  $= 0$ for $A < 36$.
- $Z = (A/2)(1 - Q_1/Q_2)$;
- $N = (A/2)(1 + Q_1/Q_2)$
- Plot of N vs. Z similar to plot of stable nuclei
  - But N rises too fast in simple model.
  - For example, predicts A= 232 has 47p and 185n, but Th-232 actually has 90p and 142n.
- What is needed is core where N increases half as fast.



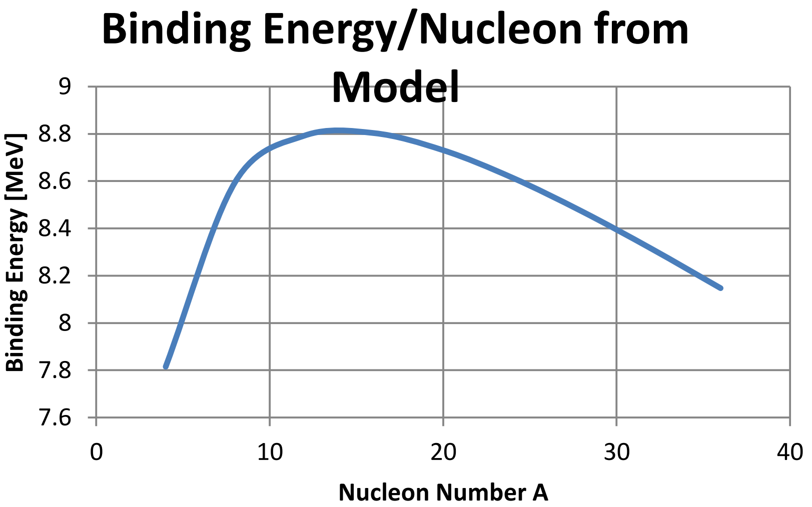
Excess Neutrons from Spherical p3n-core?

- Alternative n-rich phase – p3n = H-4 nucleus.
  - H-4 nucleus unstable, but may be stable inside nucleus with large ϕ.
  - May be more stable than 4n in nucleus.
- Excess N due to p3n core should rise half as fast as 4n core.
  - $Z = (A/2)(1 - Q_1/2Q_2)$;
  - $N = (A/2)(1 + Q_1/2Q_2)$
- Plot of N vs. Z much closer to plot of stable nuclei
  - Predicts A= 232 has 81p and 151n, closer to Th-232 with 90p and 142n.
  - May be acceptable agreement for this crude continuum approx.



Contributions to Total Binding Energy

- Consider first small nucleus (up to A=36) without n-rich core.
  - Binding of α: U_α = 7 MeV/nucleon.
  - Cohesive energy U_c = 8 MeV/nucl.
  - Surface energy $U_s = -10A^{-1/3}$ MeV/nucl.
  - Electric energy $U_e = -\phi^*(e/2)$ /nucl. = $-0.3Ae^2/4\pi\varepsilon_0 r_2 = -0.35A^{2/3}$ MeV
  - $U_{tot} = U_\alpha + U_c + U_s + U_e = 7 + 8 - 10A^{-1/3} - 0.35A^{2/3}$ [MeV/nucleon]
  - Curve rises for small A due to reduced relative surface energy, falls for large A due to electric repulsion, both on scale ~ 1 MeV.
  - Enough adjustable parameters to obtain reasonable agreement with measured binding energy curve, but not critical test.



Binding Energy with n-rich core

- Depends in detail on which core model (pure n or ¾-n) used.
  - Results of excess neutron calculation suggests p3n may be more appropriate.
- For A>36, binding energy reduced by ~ 5 MeV per nucleon in core, but electrostatic repulsion also reduced.
  - Overall effect to slow decline in binding energy for large A.
- Additional adjustable parameters should enable reasonable fit to experiment, but other data are needed to more clearly identify whether these parameters are correct.

Evidence for α-crystal with n-rich Core

- Nuclear α-decay suggests that αs are on outside of nucleus, in agreement with model.
  - The lack of decay by emission of single neutrons suggests that excess neutrons are *not* on the surface, again in agreement.
- If a nucleus splits via fission, part of the n-rich core is likely to be exposed, but would no longer be subject to stabilizing electric potential.
  - Fission products do indeed emit extra neutrons, which tend to produce chain reactions.
- A fractured spherical nucleus will not itself be spherical, and may reflect a flat fracture surface.
  - Relaxation to spherical nucleus may require sequence of neutron emission, beta decay, and gamma emission associated with rearrangement.

Educational Value of Spherical Nuclear Model

- Modern physics is often presented primarily as abstract equations without consistent physical pictures.
- But simple visual models with analytical solutions have great value for undergraduate courses and textbooks, particularly if they are a good first approximation.
- This spherical crystal model of the nucleus looks similar to what students have already seen in chemistry classes, and enables electrostatic calculations similar to those that they have seen in introductory physics classes.
- If this model is correct, it may provide a good introduction to nuclear physics, enabling insight into important issues of nuclear stability and energetics.

Conclusions

- A simple model of nuclear structure is presented, which could have been proposed in the 1930s, but was evidently overlooked.
- Within this model, a small nucleus ($A<36$) consists of a close-packed quasi-spherical crystal of αs, in analogy to a molecular crystal.
- In larger nuclei, a central core of αs converts to a neutron-rich phase, thus accounting for excess neutrons.
- Both pure-n and p3n phases are considered, but the p3n phase (H-4 nuclei) fits the data better.
- This model enables both a simple picture and analytic formulas that may be particularly useful for introducing nuclear physics to undergraduate students.

References

1. L. Hafstad and E. Teller, “The Alpha Particle Model of the Nucleus”, Phys. Rev. 54, 681 (1938).
2. L. Pauling, “The Close-Packed Spheron Model of Atomic Nuclei”, Proc. NAS 54, 989 (1965).
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5. Wikipedia, “Nuclear Structure”. https://en.wikipedia.org/wiki/Nuclear_structure
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