A DARK MATTER THEORY BY GRAVITATION GOT IN M31 AND MILKY WAY-V3

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1. ABSTRACT

In this paper is developed carefully an original theory of dark matter, whose main hypothesis is that DM is generated by the own gravitational field. In this work is introduced the best version of the theory physical and mathematically, previously published since 2014.

The hypothesis of DM by gravitation has two main consequences: the first one is that the law of DM generation has to be the same, in the halo region, for all the galaxies and the second one is that the haloes are unlimited so the total DM goes up without limit. Both properties are crucial for the success of this theory. Apparently the hypothesis of unlimited haloes would seem impossible but further cosmological studies may demonstrate that dark energy may counter balance the growing of DM.

This paper is similar to one previous paper published in 2019, although there is an important newness, which is the new rotation curve of Milky Way published by Sofue,Y. in 2020. Results got with this new data in Milky Way back strongly the rightness of ***DM by gravitation*** theory.

This work begins studying the Rotation curves data come from Sofue,Y.2015 for M31 galaxy and from Sofue 2020 for Milky Way. Thanks these remarkable rotation curves, both regression curves at halo region are fitted with power regression functions whose exponents are the same for both galaxies.

By the fitted function is possible to calculate a dark matter density function depending on radius which is transformed into a DM density depending on gravitational field. This change is the core of the theory because at such moment it is possible to study the formula of dark matter density by the Buckingham theorem in order to change the statistical calculus by physical formulas which depend on the Universal constants G, h and c. From now on the statistical theory becomes a perfect physical theory that despite it is based on the Newtonian framework allows to get new formulas such as DM density and total DM mass.

In chapters 10 ,11 and 12 are calculated masses for M31, Milky Way and M33 with excellent results. But it is in the chapter 13 where are calculated the most brilliant results got in this work. The total mass of the Local group of galaxies at 770 kpc is 4.8E+12 Msun which match perfectly with the dynamical mass at 770 kpc currently accepted by the Local Group 5E+12 Msun. Also it is calculated the mass of MW+M31+M33 at 3 Mpc being 9,5E+12 Msun, which match with the results got by Azadeh Fattahi and Julio F. Navarro in 2020 about total mass of Local Group.
In the chapter 14 it is shown a method quite simple to extend the theory to cluster of galaxies.

The chapter 15 is dedicated to show how DM may be counter balanced by dark energy with negative mass at cluster scale. Namely is calculated that DE+DM is zero at 2.2 Mpc in the Local Group, also it is studied this phenomenon into the Virgo and Coma clusters.

2. INTRODUCTION

Since 2013 up to 2019 I have published several papers studying DM in galactic halos, especially in M31 and Milky Way although also I have published some papers studying DM in Coma cluster [3] Abarca,M.2019.

As reader knows M31 is the twin galaxy of Milky Way in the Local Group of galaxies. According [5] Sofue, Y. 2015. Its baryonic masses are $M_{BARYONIC\text{-}M31} = 1{,}61\cdot10^{11}\ M_{SUN}$ and $M_{BARYONIC\text{-}MILKY\ WAY} = 1{,}4\cdot10^{11}\ M_{SUN}$. Where Msun represents the mass of Sun. Msun = 1.99E+30 kg that it will be used frequently throughout the paper.

The DM by gravitation theory was introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* . It considers that DM is generated by the own gravitational field. In order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. i.e. radius bigger than 30 kpc for MW and 40 kpc for M31, as it will be shown in chapter 6.

This hypothesis has two main consequences: the first one is that the law of dark matter generation, in the halo region, has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit.

In the chapter 9, is demonstrated mathematically that total mass by Direct mass formula goes up proportionally to the root square of distance, so this property may explain how the ratio of dark matter versus baryonic matter at cluster scale is bigger than such ratio at galactic scale.

By other side the growing of the total mass is so slow that Dark energy phenomenon may counter balance the DM when it is considered radius measured in mega parsecs. Precisely, this fact may explain the size of galactic clusters. This issue will be studied in chapter 14.

The first consequence before mentioned, dark matter generated by a Universal law, has been studied by all my papers, especially inside M31 and Milky Way thanks the remarkable data of rotation curves published in papers [5] Sofue, Y.2015 and [6] Sofue, Y.2020.

In fact I could develop rigorously the theory because the rotation curve of M31 at halo region decreased with a power regression fitted curve whose exponent is -1/4. However with data published for Milky Way at the same paper (2015) it was not possible to fit rigorously the rotation curve with such exponent.

Fortunately, in a new paper [6] Sofue,Y.2020, the author gives a new rotation curve data for Milky Way at halo region whose fitted curve has an exponent -1/4. Such result is good news for DM by gravitation theory, because the theory states a universal law of DM generation in the halo region of galaxies or clusters.

In this paper it is firstly developed all the theory carefully with M31 rotation curve data up to chapter 10 and the chapter 11 is dedicated to apply the theory to Milky Way with magnificent results.

In the chapter 12 is estimated the total DM for M33, this estimation is made by Direct mass formula because is more simple than Bernoulli mass formula, and both give results very close.

The chapter 13 is dedicated to estimate the total mass of Local Group. The total mass calculated for Milky Way, M31 and M33 is $4{,}8*10^{12}$ Msun. As the dynamical mass for Local group at 770 kpc is estimated to be $5*10^{12}$ Msun, it is clear that both results are very close and even the agreement will be bigger when it is considered the mass of de Large Cloud of Magellan galaxy, which is approximately 4 times smaller than M33.

The importance of these findings is high because there is not any other theory able to explain such amount of mass. Therefore, the theory not only may explain the rotation curves in halo region for M31 and MW but also it is the only one that may explain the total mass of the Local Group of galaxies.

In fact in the paper [3] Abarca,M.2019, it was studied the Coma cluster with the theory and it was shown how works perfectly inside this cluster, which is one of the most massifs between known clusters.

As I have mentioned before, this theory has been developed assuming the hypothesis that DM is a quantum gravitational effect. However, it is possible to remain into the Newtonian framework to develop the theory. In my opinion there are two factors to manage the DM conundrum with a quite simple theory.
The first one, that it is developed into the halo region, where baryonic matter is negligible. The second one, that the mechanics movements of celestial bodies are very slow regarding velocity of light, which is supposed to be the speed of gravitational bosons. It is known that community of physics is researching a quantum gravitation theory since many years ago, but does not exist yet.

To use a more simple theory instead the general theory is a typical procedure in physics.
For example the Kirchhoff ´s laws are the consequence of Maxwell theory for direct current and remain valid for alternating current, introducing complex impedances, on condition that signals must have low frequency. However these laws do not work for electromagnetic microwaves because of its high frequency.

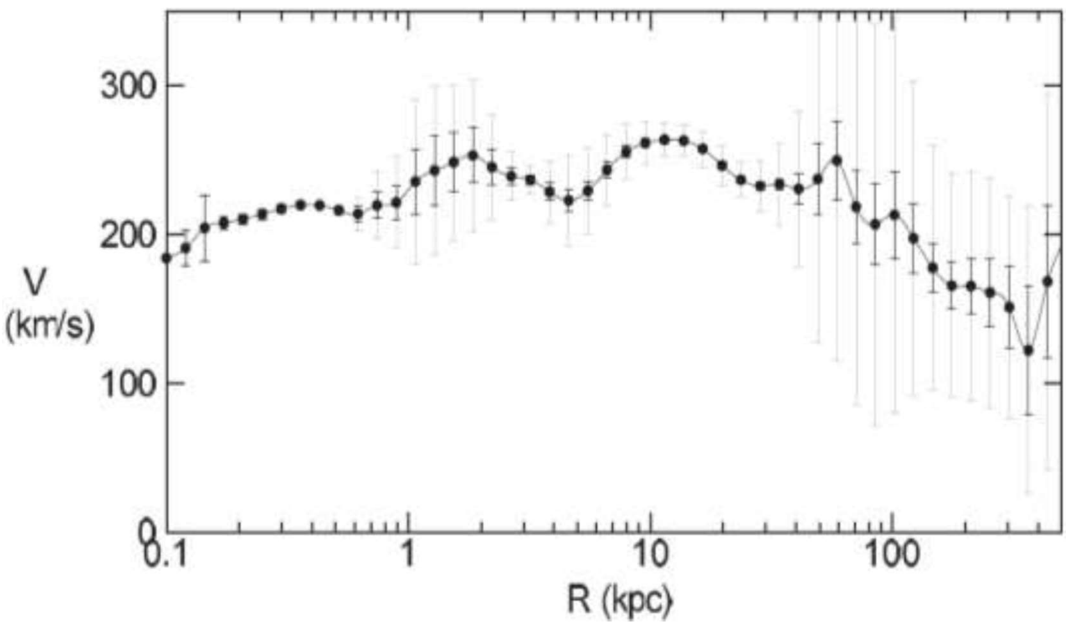
Thanks the possibility to study the gravitational effect of DM pure, in halo regions of M31 and MW, it have been possible to develop a theory mathematically simple. When baryonic mass is mixed with dark matter as it happens inside the galactic disc the mathematical treatment is by far more complex.

Taking into account that the only ones giant galaxies quite close to be able to study with accuracy the rotation curve at halo region are Milky Way and M31, the coincidence of the same exponent to the fitted function for the rotation curves for both galaxies is crucial in order to state that dark matter is generated according an Universal law.

3. OBSERVATIONAL DATA FOR M31 GALAXY FROM SOFUE. 2015 DATA

Graphic come from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. Although Sofue rotation curve ranges from 0,1 kpc up to 352 kpc the range of dominion considered for this work is only the halo region where ratio baryonic matter is negligible. In chapter 6, will be shown that this happens for radius bigger than 40 kpc, despite the fact that disc radius for M31 is accepted to be 35 kpc.

kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8



The measure at 352 kpc has been rejected because has a velocity too high, so does not match with the other measures.

3.1 POWER REGRESSION TO ROTATION CURVE

The measures of rotation curve have a very good fitted curve by power regression.

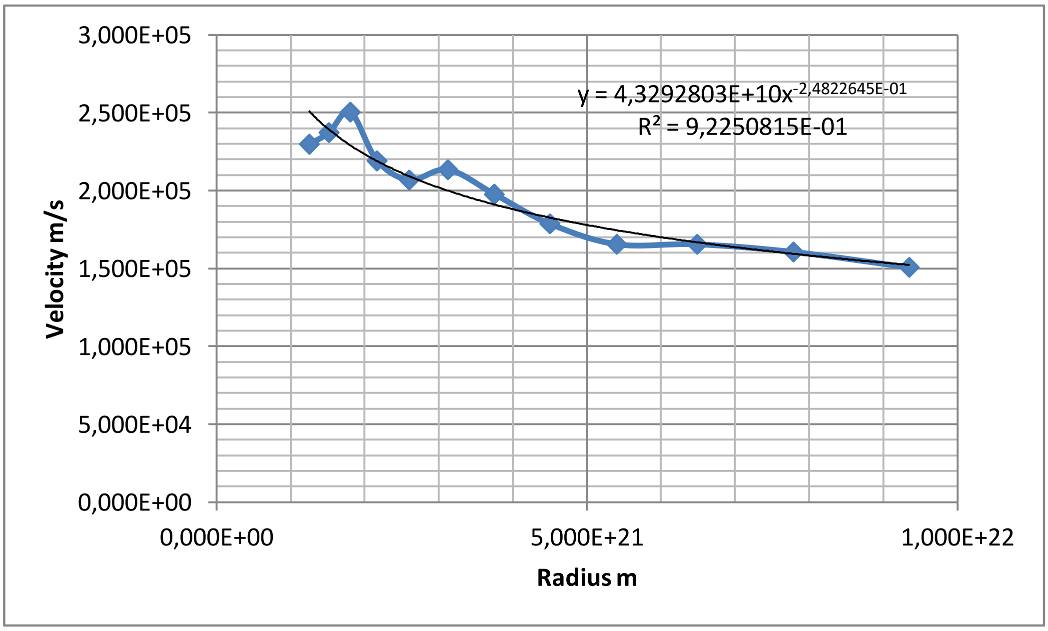
Power regression for M31 rot. curve	
V=a*r^b	
a	$4{,}32928 * 10^{10}$
b	-0.24822645
Correlation coeff.	0,96

In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

Data fitted are in grey columns below.
In fifth column is shown results of fitted velocity and sixth column shows relative difference between measures and fitted results.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Below is shown a graphic with measures data and power regression function.



Correlation coefficient equal to 0,96 which is a superb result especially when dominion measures is up to 303 kpc. There is not any other galaxy to measure a rotation curve so magnificent. According theory of DM generated by field, galaxy haloes are unlimited although up to a half of distance i.e. 375 kpc toward Milky Way direction is dominated by M31 field whereas the other half distance is dominated by Milky Way.

Furthermore, it has been calculated regression curve with another data placed at 363 kpc, but power regression is -0.28 and correlation coefficient is 0.954. This result shows that such data is not trustworthy because according dimensional analysis power has to be -0.25. As 363 kpc is placed in the border of M31 halo it is possible that such data might be influenced by a different field. Therefore it is better to study data only up to 303 kpc.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a \cdot r^b$. As $M_{DYNAMIC}(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$. Hereafter this formula will be called Direct Mass $M_{DIRECT}(<r) = \frac{a^2 \cdot r^{2b+1}}{G}$ because it has been got rightly from rotation curve.

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $dm = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G}$ and $dV = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot$ results $D_{DM}(r) = L \cdot r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m^3

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E BY VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, Virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a \cdot r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E_{VIRIAL} = a^2 \cdot r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A \cdot E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by the author in previous papers. [1] Abarca,M. [2] Abarca,M. [8] Abarca,M. y [10] Abarca,M.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

According parameters a & b got in previous chapter, A& B parameters are:

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3{,}6559956 \cdot 10^{-6}$
B	1,6682469

As conclusion, in this chapter has been demonstrated that a power law for velocity $v = a \cdot r^b$ is mathematically equivalent to a power law for DM density depending on E. $D_{DM} = A \cdot E^B$

5.3 THE IMPORTANCE OF $D_{DM} = A . E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ and $E = a^2 \cdot r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A \cdot E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be similar for different galaxies on condition that they were similar. In further chapters will be got that power B is exactly the same for M31 and Milky Way although A will be a bit different.

However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters. As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, for galaxy scale or bigger systems.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy. [5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi \cdot r \cdot \Sigma(r) dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$. To convert superficial baryonic density to volume density it is right to get the formula Volume density baryonic $= \dfrac{\Sigma(r)}{2r}$ so the ratio of both volume density is

$Ratio = \dfrac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$ where $D_{DM}(r) = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ a & b were got in epigraph 3.1. In table below are tabulated volume DM density, superficial baryonic density and ratio of both volume density.

Radius Kpc	Radius m	Ratio baryonic versus DM	$\Sigma(r)$ kg/m^2	Direct DM kg/m^3
36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23
38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.
This is the reason why in this work dominion for radius begins at 40 kpc.

7. A DIFFERENTIAL EQUATION FOR A GRAVITATIONAL FIELD

7.1 INTRODUCTION

This formula $D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \dfrac{G \cdot M(<r)}{r^2} = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM}

formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A \cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\frac{M'(r) \cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r^3}$ As $\varphi_{DM}(r) = A \cdot E^B(r)$ it is right to get $E'(r) = 4\pi \cdot G \cdot A \cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K \cdot E^B(r) - 2\frac{E(r)}{r}$ being $K = 4\pi \cdot G \cdot A$ Calling y to E, the differential equation is written in this simple way $y' = K \cdot y^B - \frac{2 \cdot y}{r}$ Bernoulli family equations $y' = K \cdot y^B - \frac{2 \cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$. This is $\frac{u'}{1-B} + \frac{2u}{r} = K$

The homogenous equation is $\frac{u'}{1-B} + \frac{2u}{r} = 0$ whose general solution is $u = C \cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u=z*r then it is got that $z = \frac{K \cdot (1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C \cdot r^{2B-2} + z \cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B} \right)^{\frac{1}{1-B}}$ $with\ B \neq 1\ and B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B-2$ $\beta = \frac{1}{1-B}$ and parameter $D = \left(\frac{K(1-B)}{3-2B} \right)$ then $E(r) = \left(Cr^{\alpha} + Dr \right)^{\beta}$

Calculus of parameter C through initial conditions Ro and Eo

Suppose $R_0 \; and \; E_0$ are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $\varphi_{DM}(r) = A \cdot E^B(r)$. Therefore this solution for field works only in the halo region and $R_0 \; and \; E_0$ could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E BY BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ Being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1, 6682469

In this case relative difference between B = 1, 6682469 and 10/7 is 16, 7 %. A 17% of error in cosmology could be acceptable. However by the end of the chapter it will be found a better solution.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G, h and c =$2.99792458 \cdot 10^8$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which can not be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi G A (B-1)}{2B-3} \right]^{\frac{2b-1}{2}}$ being $B \neq 1 \ and \ B \neq 3/2$.

As A is a positive quantity then $2b+1 > 0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ therefore $B \in (-\infty, 1) \cup (3/2, \infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2, \infty)$.

The main consequence this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is wrong because B=10/7 = 1.428. Therefore formula $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by two pi monomials it is more suitable formula.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3{,}6559956 \cdot 10^{-6}$
B	1,6682469

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6.

This way, power of E in formula $D_{DM} = A \cdot E^B$ will be 5/3 , which is the best approximation to B= 1.6682469.

Finally $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs less than 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a & b parameters and A&B parameters. Now considering B= 5/3

as $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$. It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Regression for M31 dominion 40-303 kpc	
$V=a*r^b$	
a	$4,32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Due to Buckingham theorem it is needed that b= -1/4 . Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a &b that fit perfectly to experimental measures of rotation curve in M31 halo.

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula $V= a*r^{-0.25}$ on condition that $\sum_e (v-v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

Where r_e represents each radius measure and v_e represents its velocity associated.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \cdot 10^{10}$$

New parameters a &b and A&B	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a new	$4,727513*10^{10}$
A new	$3.488152*10^{-6}$

9.2 FORMULAS OF DIRECT D.M. FOR DENSITY FOR MASS AND FOR VIRIAL FIELD

With these new parameters recalculated it is going to get the direct formulas got at the beginning of paper.

Function of Density DM depending on radius.

$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}}$ being $L = \dfrac{a^2 \cdot (2b+1)}{4\pi G} = \dfrac{a^2}{8 \cdot \pi \cdot G}$ =1,3326*10 [30]

Function of E depending on radius $E_{VIRIAL} = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}}$ being $a^2 = 2{,}235 \cdot 10^{21}$

Mass enclosed by a sphere of radius r, known as dynamical mass because it is calculated with velocity.

$M_{DYN}(<r) = \dfrac{v^2 \cdot R}{G}$ When velocity is replaced by its fitted function it is got $M_{DIRECT}(<r) = \dfrac{a^2 \cdot r^{2b+1}}{G} = \dfrac{a^2 \cdot \sqrt{r}}{G}$

9.3 BERNOULLI SOLUTION FOR E AND DENSITY IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to get formulas far simple because some parameters are simple fractions.

$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ being $\alpha = 2B - 2 = \dfrac{4}{3}$ being $\beta = \dfrac{1}{1-B} = \dfrac{-3}{2}$ then, the initial condition C

$C = \dfrac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$ becomes $C = \dfrac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ and $D = \left(\dfrac{4 \cdot \pi \cdot G \cdot A(1-B)}{3-2B}\right) = 8 \cdot \pi \cdot G \cdot A = a^{\frac{-4}{3}}$ so $D = a^{\frac{-4}{3}} = 5{,}85 \cdot 10^{-15}$

Therefore $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being C the initial condition of differential equation solution for E and $D = a^{\frac{-4}{3}}$ is a parameter closely related to the global rotation curve at halo region, being parameter **a** $= 4{,}7275 \cdot 10^{10}$

BERNUILLI SOLUTION FOR DENSITY

$$D_{BERNI} = A \cdot E^{\frac{5}{3}} = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}}$$

9.4 DARK MATTER AT A SPHERICAL CORONA BY BERNOULLI SOLUTION IN HALO REGION

Formula below express the dark matter contained inside a spherical corona defined by R_1 and R_2 belonging at halo.

$$M_{BERNI} = \int_{R1}^{R2} 4\pi \cdot r^2 \cdot D_{BERNI}\, dr = \int_{R1}^{R2} 4\pi \cdot r^2 A E^{B}\, dr = 4\pi A \int_{R1}^{R2} r^2 \left[C \cdot r^{4/3} + D \cdot r\right]^{\frac{-5}{2}} \cdot dr$$

The indefinite integral $I = 4\pi A \cdot \displaystyle\int \frac{r^2}{\left(C \cdot r^{4/3} + D \cdot r\right)^{2.5}} = \frac{8\pi A \sqrt{r}}{D \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$

As $\dfrac{8\pi A}{D} = \dfrac{1}{G}$. Calling $M(<r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ and **by** the Barrow`s rule, it is got

$\overset{R2}{\underset{R1}{M}}_{BERNI}$ = M(R2)-M(R1) that provided the DM contained inside the spherical corona defined by R2 and R1.

9.5 NEWTON`S THEOREM WITH BERNOULLI MASS FORMULA

The name for this theorem has been chosen because the relation between field E and total mass M(<r) is the same that in Newton's theory.

From Bernoulli field $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}} = \dfrac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D \right)}$

Bernoulli mass formula $M_{BERNI}(<r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D \right)^{\frac{3}{2}}}$ so $G \cdot M_{TOTAL}(<r) = \dfrac{\sqrt{r}}{\left(C \cdot \sqrt[3]{r} + D \right)^{\frac{3}{2}}}$ and

$$\frac{G \cdot M(<r)}{r^2} = \frac{\sqrt{r}}{r^2 \left(C \cdot \sqrt[3]{r} + D \right)^{\frac{3}{2}}} = \frac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D \right)^{3/2}} = E(r)$$

Therefore $E(r) = \dfrac{G \cdot M(<r)}{r^2}$ this is the intensity of field in Newton´s theory. This identity shows how the ***DM by gravitation theory,*** adding an extra of mass depending on radius, being the halo region unlimited, is able to explain the DM measures in galaxies and cluster in the Newtonian framework.

9.6 CALCULUS OF PARAMETER C

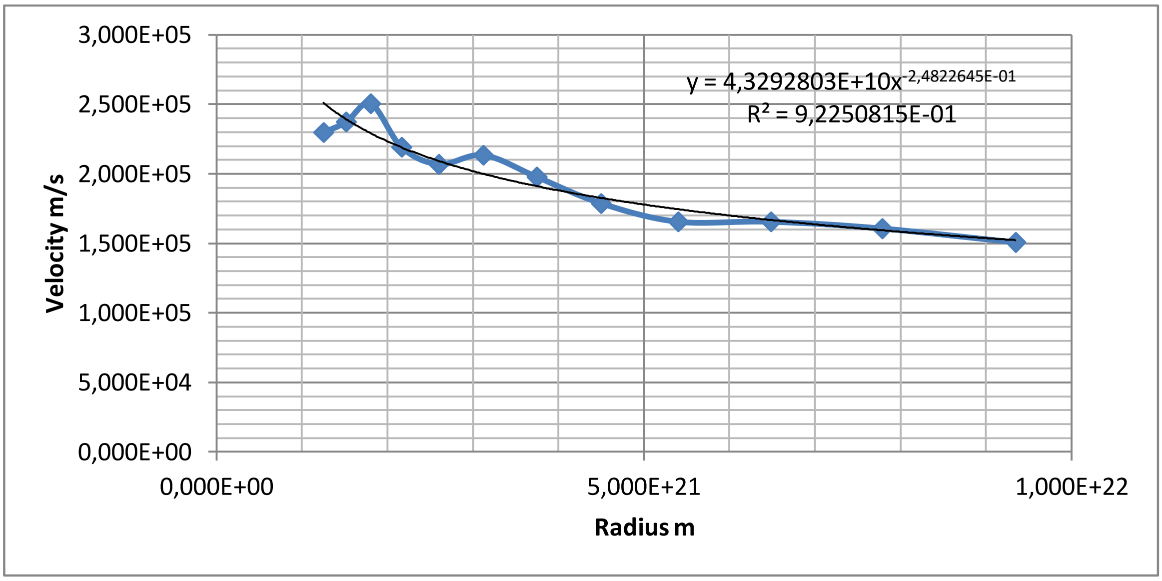
$$C_O = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

In this formula, E_O is the gravitational field at R_O radius which is V^2_0 / R_O i.e. the centripetal acceleration.

As experimental measures have errors, it will be calculated Co for every data in order to calculate Bernoulli dark matter mass for all of them.

	Radius	Radius	Velocity	Field Eo	Eo^(-2/3)	Eo^(-2/3)-D*R	Parameters
	kpc	m	m/s	m/s^2			Co
1	40,5	1,250E+21	2,299E+05	4,23E-11	8239256,95	9,27E+05	6,8830E-23
2	49,1	1,515E+21	2,374E+05	3,72E-11	8973615,749	1,11E+05	6,3753E-24
3	58,4	1,802E+21	2,505E+05	3,48E-11	9377625,482	-1,16E+06	-5,3082E-23
4	70,1	2,163E+21	2,192E+05	2,22E-11	12654654,13	1,20E+03	4,2804E-26
5	84,2	2,598E+21	2,069E+05	1,65E-11	15443489,73	2,45E+05	6,8683E-24
6	101,1	3,120E+21	2,135E+05	1,46E-11	16732951,68	-1,52E+06	-3,3317E-23
7	121,4	3,746E+21	1,978E+05	1,04E-11	20928758,45	-9,85E+05	-1,6934E-23
8	145,7	4,496E+21	1,788E+05	7,11E-12	27043372,43	7,42E+05	9,9988E-24
9	175	5,400E+21	1,656E+05	5,08E-12	33846627,1	2,26E+06	2,3822E-23
10	210,1	6,483E+21	1,656E+05	4,23E-12	38232870,38	3,08E+05	2,5446E-24
11	252,3	7,785E+21	1,607E+05	3,32E-12	44959131,06	-5,83E+05	-3,7771E-24
12	302,9	9,347E+21	1,508E+05	2,43E-12	55281485,72	6,02E+05	3,0572E-24

Values in yellow are negatives because these points are above the fitted curve . See graph below. In addition, the more close to curve the point is, the smaller, in absolute value, the parameter C is. The cyan value is the smaller.



BERNOULLI DM MASS INSIDE SPHERICAL CORONA FOR DIFFERENT PARAMETERS Co

Function of dark matter is $M(r) = \dfrac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$ being $D = a^{\frac{-4}{3}} = 5{,}85 \cdot 10^{-15}$ and parameters C calculated in previous paragraph.

The row 1 gives the value M(R1) in Msun units. See table below, the grey row.

The row 2 and the followings one give the dark matter inside the spherical corona with radius R_N and R_1, where n ranges from 2 to 14. For example, $M_{DM}{}_{R1}^{R2}$ = M(R2)-M(R1) that provided the DM contained inside the spherical corona defined by R2 and R1.

The columns 1 to 12 give the 12 dark matter functions inside spherical corona Rn –R1 ,where n ranges from 2 to 12, associated at different values of parameters C showed in previous table.

A good option to minimize error is to considerate the average function mass to try to adopt the most suitable parameter C for M31.

The last column, gives de Average dark matter function considering the 12 Bernoulli functions.

		1	2	3	4	5	6
	kpc	C at 40kpc	C at 49 kpc	C at 58 kpc	C at 70 kpc	C at 84kpc	C at 101kpc
1	40,5	4,9743E+11	5,8461E+11	6,9418E+11	5,9486E+11	5,8382E+11	6,542E+11
	Dark matter inside spherical corona between Rn and R1 with the formulas M(Rn)-M(R1) Dark matter inside spherical corona between Rn and R1 with the formulas M(Rn)-M(R1)						
2	49,1	4,4135E+10	5,8251E+10	7,8343E+10	6,0022E+10	5,8116E+10	7,0714E+10

3	58,4	8,6985E+10	1,1573E+11	1,5717E+11	1,1936E+11	1,1545E+11	1,4137E+11
4	70,1	1,3538E+11	1,8174E+11	2,4957E+11	1,8763E+11	1,8129E+11	2,2357E+11
5	84,2	1,8753E+11	2,5417E+11	3,5325E+11	2,627E+11	2,5352E+11	3,1506E+11
6	101,1	2,4344E+11	3,333E+11	4,6928E+11	3,449E+11	3,3242E+11	4,1656E+11
7	121,4	3,0327E+11	4,1969E+11	5,9926E+11	4,3487E+11	4,1854E+11	5,2918E+11
8	145,7	3,6717E+11	5,1392E+11	7,45E+11	5,3324E+11	5,1246E+11	6,5417E+11
9	175	4,3574E+11	6,173E+11	9,0967E+11	6,4144E+11	6,1548E+11	7,9385E+11
10	210,1	5,0878E+11	7,3E+11	1,095E+12	7,5974E+11	7,2775E+11	9,4917E+11
11	252,3	5,8674E+11	8,5327E+11	1,3047E+12	8,8953E+11	8,5053E+11	1,1227E+12
12	302,9	6,6967E+11	9,8779E+11	1,5422E+12	1,0316E+12	9,8449E+11	1,3164E+12
13	770	1,178E+12	1,8919E+12	3,3875E+12	1,9981E+12	1,884E+12	2,7375E+12
14	1000	1,3479E+12	2,226E+12	4,1909E+12	2,36E+12	2,216E+12	3,3166E+12

	7	8	9	10	11	12	
kpc	C at 121 kpc	C at 145 kpc	C at 175 kpc	C at 210 kpc	C at 252kpc	C at 302 kpc	Average DM mass Msun
40,5	6,2388E+11	5,7887E+11	5,5783E+11	5,9077E+11	6,0119E+11	5,8994E+11	
	Dark matter inside spherical corona between Rn and R1 with formulas M(Rn)-M(R1)						
49,1	6,5156E+10	5,7271E+10	5,3736E+10	5,9313E+10	6,1126E+10	5,917E+10	6,0446E+10
58,4	1,2991E+11	1,1372E+11	1,065E+11	1,1791E+11	1,2162E+11	1,1761E+11	1,2028E+11
70,1	2,0481E+11	1,7848E+11	1,6679E+11	1,8527E+11	1,9131E+11	1,8479E+11	1,8922E+11
84,2	2,8768E+11	2,4946E+11	2,3257E+11	2,5928E+11	2,6805E+11	2,5859E+11	2,6515E+11
101,1	3,7898E+11	3,269E+11	3,0403E+11	3,4025E+11	3,5218E+11	3,3931E+11	3,4846E+11
121,4	4,7959E+11	4,1134E+11	3,8156E+11	4,2878E+11	4,444E+11	4,2755E+11	4,3984E+11
145,7	5,9039E+11	5,0332E+11	4,6558E+11	5,2548E+11	5,4539E+11	5,2392E+11	5,4E+11
175	7,1321E+11	6,0407E+11	5,5712E+11	6,3175E+11	6,5666E+11	6,2979E+11	6,5051E+11
210,1	8,4859E+11	7,1372E+11	6,5619E+11	7,4779E+11	7,7853E+11	7,4537E+11	7,7172E+11
252,3	9,9841E+11	8,3347E+11	7,6371E+11	8,7494E+11	9,1248E+11	8,7199E+11	9,0521E+11
302,9	1,164E+12	9,6392E+11	8,801E+11	1,014E+12	1,0594E+12	1,0104E+12	1,052E+12
770	2,331E+12	1,8348E+12	1,6391E+12	1,9551E+12	2,0665E+12	1,9465E+12	2,0708E+12
1000	2,7857E+12	2,1543E+12	1,9104E+12	2,3056E+12	2,4468E+12	2,2947E+12	2,4629E+12

Comparing the Average DM mass placed in the last column with the others columns, it can be checked that ,relatives differences between values of mass at column C at 252 kpc and Average mass are under 1% . See green columns.

So the most suitable parameter C is at 252 kpc C= -3,777E-24. In my opinion, this is the way to get the best parameter C for M31 galaxy. Because there is no way to know which experimental data is more reliable, so taking the average dark matter mass function it will possible to reduce errors.

STUDYING CASE WHEN C = 0

Now It will be investigated the conditions to get C= 0. In such case, formula $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ leads to

$E_0^{\frac{-2}{3}} = D \cdot R_0 = a^{\frac{-4}{3}} \cdot R_0$ because $D = a^{\frac{-4}{3}}$ when B=5/3 as it was shown at epigraph 9.3

and as $E_{VIRIAL} = a^2 \cdot r^{2b-1}$ then $E_0^{\frac{-2}{3}} = a^{\frac{-4}{3}} \cdot R_0^{\frac{2-4b}{3}} = D * R_0 = a^{\frac{-4}{3}} \cdot R_0$ and by equation of power of R_0

$\frac{2-4b}{3} = 1$ it is got b= -1/4

At the beginning of chapter was shown that B= 5/3 leads rightly to b = -1/4.

Therefore C will be zero, when a point of fitted curve V = a*r $^{-0.25}$ of rotation curve in halos region is considered as the initial condition for the Bernoulli solution.

In the epigraph 9.7 it will be shown that for C= 0 the Bernoulli solution for field becomes direct formula for field, and the same happens with Bernoulli DM density and mass formulas.

9.7 GETTING DIRECT FORMULAS BY BERNOULLI FIELD WHEN PARAMETER C = 0

Thanks demonstration made above, it is clear why points measures close to fitted curve (see graph pag 15) gives values for C very close to zero. The more close point measure to fitted curve is, the more close to zero C is. See graph and tables in epigraph 9.6

FOR FIELD E

When in formula $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}}$ C= 0 then it is got $E = a^2 \cdot r^{\frac{-3}{2}}$ being $a^2 = 2.235 \cdot 10^{21}$ which is precisely direct formula for E_{VIRIAL}.

FOR D.M. DENSITY

As $D_{DM} = A*E^B$ Using field got by Bernoulli solution it is right to get

$D_{DM}(r) = A\left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-5}{2}}$ Being **A=** 3,488*10^{-6} **D=**5,85*10^{-15} if **C** = 0 then formula becomes

$D_{DM}(r) = A \cdot D^{\frac{-5}{2}} \cdot r^{\frac{-5}{2}} = L \cdot r^{\frac{-5}{2}}$ being $L = \frac{a^2}{8 \cdot \pi \cdot G}$ = 1.3326*10^{30} which is direct DM density formula.

FOR DIRECT MASS FORMULA

If C=0 then $M_{BERNI}(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ becomes $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $\frac{a^2}{G} = 3.349 \cdot 10^{31}$.

10. MASSES IN M31

In this chapter, It will be calculated some different types of masses related to M31.

10.1 DYNAMICAL MASS VERSUS DIRECT MASS

As it is known, dynamical mass represents the total mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius, so it is right to consider dynamical mass as the total mass, baryonic and DM mass, enclosed at radius R. Ranging radius in the interval of radius measured.

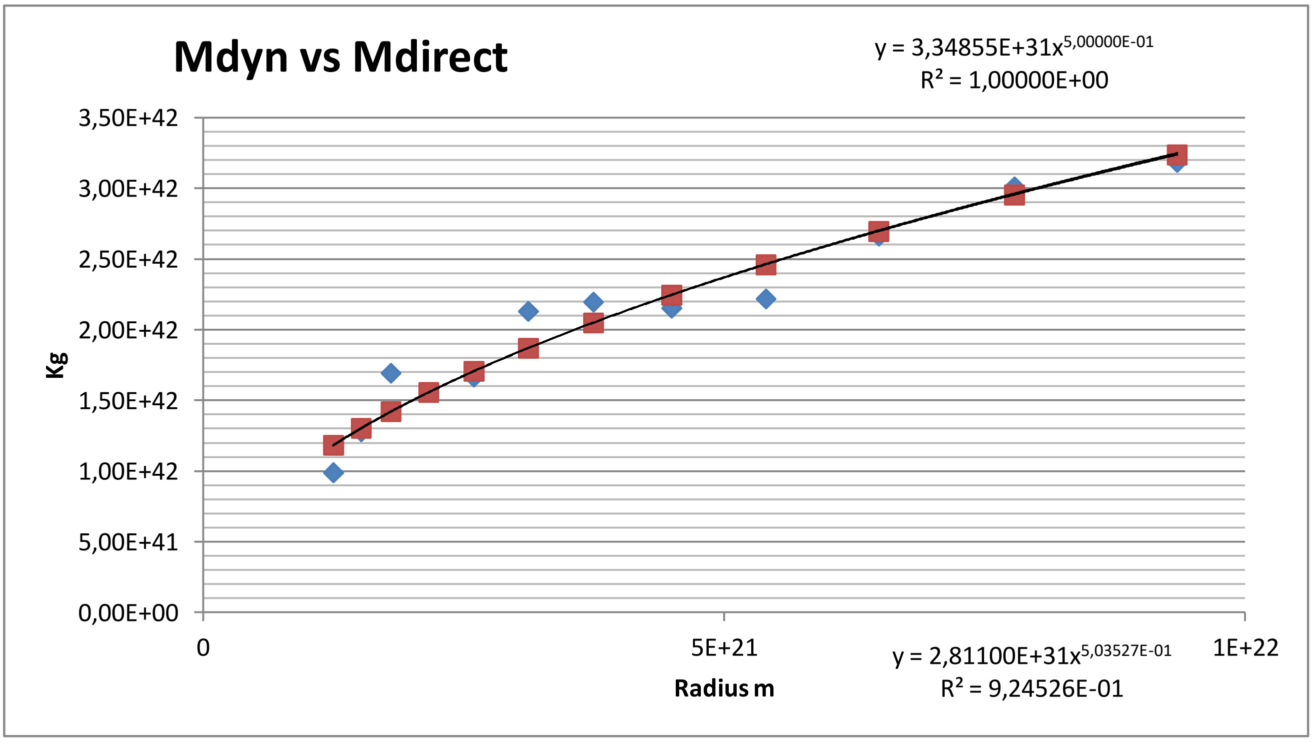
The formula of dynamical mass is $M_{DYN}(<r) = \frac{V^2 \cdot r}{G}$. and $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3{,}35.10^{31}$

			Dyn Mass	Direct mass	Rel diff
kpc	m	m/s	Msun	Msun	%
40,5	1,250E+21	2,299E+05	4,974E+11	5,95E+11	1,639E+01
49,1	1,515E+21	2,374E+05	6,429E+11	6,55E+11	1,849E+00
58,4	1,802E+21	2,505E+05	8,514E+11	7,14E+11	-1,919E+01
70,1	2,163E+21	2,192E+05	7,825E+11	7,83E+11	1,419E-02
84,2	2,598E+21	2,069E+05	8,373E+11	8,58E+11	2,373E+00
101,1	3,120E+21	2,135E+05	1,071E+12	9,40E+11	-1,392E+01
121,4	3,746E+21	1,978E+05	1,103E+12	1,03E+12	-7,143E+00
145,7	4,496E+21	1,788E+05	1,082E+12	1,13E+12	4,087E+00
175	5,400E+21	1,656E+05	1,115E+12	1,24E+12	9,833E+00
210,1	6,483E+21	1,656E+05	1,339E+12	1,35E+12	1,204E+00
252,3	7,785E+21	1,607E+05	1,514E+12	1,48E+12	-1,951E+00
302,9	9,347E+21	1,508E+05	1,600E+12	1,63E+12	1,629E+00

In the fifth column is tabulated the direct masses in order to be compared with dynamical masses.

Below in the graph are plotted both functions, blue points are dynamical masses and brown point are direct masses.

The first and third points have the maximum difference regarding fitted curve whereas relative differences decreased as radius increased. Namely, relative differences are below 10% for radius bigger than 120 kpc and are below for radius bigger than 210 kpc. So direct mass is a very good approximation for total mass (baryonic and DM) enclosed at radius R. Ranging radius in the interval of radius measured.



10.2 BERNOULLI MASS VERSUS DIRECT MASS

Below are both function formulas.

$M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3{,}35.10^{31}$, as was pointed in previous paragraph, will be used to approximate total mass at radius R.

According the theory, carefully developed in previous chapters, Bernoulli mass formula is the primitive to calculate the dark matter in the halo region by the Barrow`s rule. However, now it will be shown that also is a very good approximation to direct mass and consequently a good approximation to total mass.

$$M(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

M31 PARAMETERS C & D
$C_{M31} = -3{,}777 * 10^{-24}$
$D_{M31} = a^{\frac{-4}{3}} = 5{,}85 * 10^{-15}$

Below are tabulated both function and its relative difference. It is remarkable that even at 3 Mpc its difference is only 4,35 %, despite the fact that its dominion has been extended 30 times. So hereafter, Bernoulli mass and Direct mass will be considered as equivalent throughout unlimited dominion of haloes.

		Direct mass	Bernoulli mass	Rel diff
kpc	m	Msun	Msun	%
40,5	1,250E+21	5,949E+11	6,011E+11	1,04E+00
60	1,851E+21	7,240E+11	7,327E+11	1,19E+00
80	2,469E+21	8,361E+11	8,471E+11	1,31E+00
100	3,086E+21	9,347E+11	9,481E+11	1,41E+00
200	6,171E+21	1,322E+12	1,346E+12	1,77E+00
385	1,188E+22	1,834E+12	1,875E+12	2,20E+00
500	1,543E+22	2,090E+12	2,142E+12	2,40E+00
770	2,376E+22	2,594E+12	2,668E+12	2,77E+00
1000	3,086E+22	2,956E+12	3,048E+12	3,02E+00
1500	4,629E+22	3,620E+12	3,750E+12	3,46E+00
2000	6,171E+22	4,180E+12	4,346E+12	3,80E+00
3000	9,257E+22	5,120E+12	5,353E+12	4,35E+00

Bernoulli mass have been got with the DM by gravitation theory so its dominion is the unlimited haloes. However dominion of dynamical mass and direct mass range where have been got the measures.

In this paragraph , also in epigraph 9.7 , has been demonstrated that direct mass is very close to Bernoulli mass so, DM by gravitation theory back the rightness of direct mass on unlimited halo dominion. It is obvious that direct mass is easier to calculate than Bernoulli mass because it has only one parameter. Afterwards it will be used in chapters 12, 13,14 and 15.

11. DARK MATTER BY GRAVITATION THEORY IN MILKY WAY

In [17] Abarca M.2019. was introduced the extended halo of M31 whose radius was defined as the distance between M31 and Milky Way. Similarly the extended halo for Milky Way is the same distance. The reason to consider this extension is backed by the concept of DM nature according with theory of gravitational field as generator of DM.

This concept will be discussed newly in chapter 13, where is calculated the total mass of Local Group of galaxies.

11. 1 ROTATION CURVE OF MILKY WAY BY SOFUE 2020 DATA

This table of rotation curve of Milky Way comes from [2] Sofue.

kpc	km/s	radius m	vel m/s
30,448	229,6	9,40E+20	229600
33,493	222,5	1,03E+21	222500
36,842	215	1,14E+21	215000
40,527	207,1	1,25E+21	207100
44,579	200,3	1,38E+21	200300
49,037	194,7	1,51E+21	194700
53,941	189,8	1,66E+21	189800
59,335	186,2	1,83E+21	186200
65,268	184,7	2,01E+21	184700
71,795	183,9	2,22E+21	183900
78,975	181,4	2,44E+21	181400
86,872	175,5	2,68E+21	175500
95,56	167,7	2,95E+21	167700

This new set of Sofue data is very important for the theory of DM by gravitation theory because gives a rotation curve at halo region with a power for radius very close to -1/4 which is the same for M31. This fact backs strongly the hypothesis of this theory.

In the previous paper [5] Sofue, Y.2015, the author gave an extended dominion up to 300 kpc, However data with radius bigger than 100 kpc have too high velocity and fitted power function did not fit properly with exponent -1/4.

The logical explanation about these data is to consider that such celestial objects are not in dynamical equilibrium. Perhaps they came from the outskirts of MW and were captivated by MW gravitational field afterwards.

Anyway, the important data are those closer, because it is right to think that celestial objects with lower radius belong to MW from times of MW formation so these objects may have a better dynamic equilibrium.

11.2 FITTED FUNCTION VELOCITY VERSUS RADIUS AND PARAMETER a AT HALO REGION

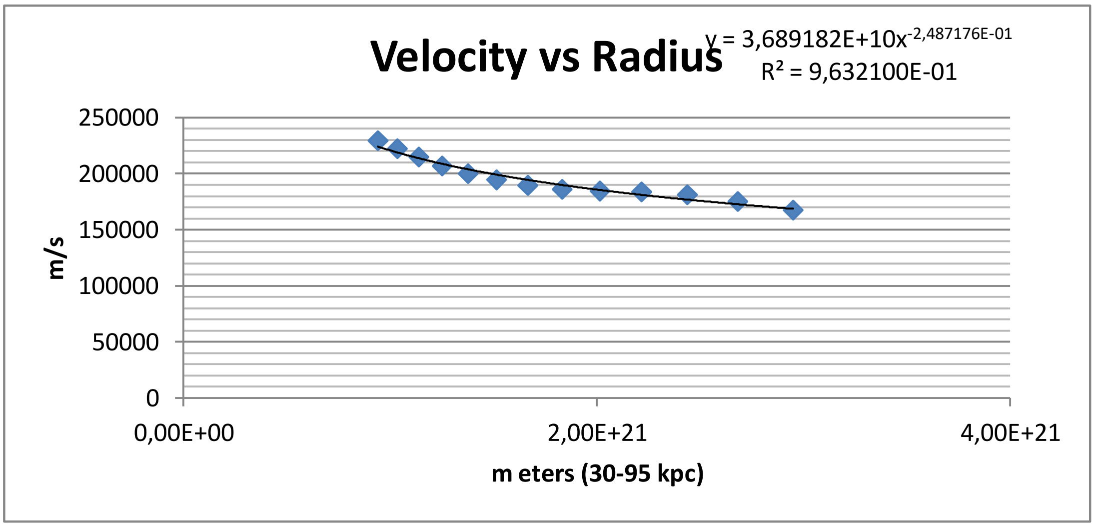
As it is known, Bernoulli solution is only right into the halo region, where the baryonic mass is negligible.
So it is needed to consider a radius where the ratio baryonic matter versus DM density is negligible. To calculate baryonic volume density has been used model provided by Sofue [6] for baryonic disc.

This Table 3 comes from Sofue [6] ,see table 3, page 12. Parameters for baryonic matter at disc in Milky Way.

Table 3. Best-fit parameters of the direct SMD by NFW halo and exponential disk.

Component	Parameter	Fitted Value	χ^2
Expo. disk	a_d	4.38 ± 0.35 kpc	
	Σ_0	$(1.28 \pm 0.09) \times 10^3 M_\odot \mathrm{pc}^{-2}$	

With this parameters is possible to calculate baryonic volume density at 30,5 kpc $D_{BARYONIC}$(30,5Kpc) = 1,33E-24 kg/m^3. Using Direct $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ and having calculated L with a-optimal shown in the following page a-optimal = 3,90E+10 it is got D_{DM}(30,5 Kpc) = 3,35E-23 kg/m^3 . So ratio Baryonic versus DM density at 30,5 kpc is 0,04. Therefore at 30,5 kpc is possible to considerate negligible Baryonic matter and it is right to consider the set of data that ranges from 30,5 kpc to 95,5 kpc.



According the statistical procedure $v=a*R^b$ Being a = 3.68918E+10 and b = - 0.248717

Using Buckingham theorem it has been stated b= - 1/4 so it is needed recalculated parameter **a** through the formula as it was made with M31 rotation curve, using the formula for **a** optimal.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}}$$

In table below the fourth column is calculated the numerator values. In the fifth column are denominator values. At the bottom is found parameter **a** optimal =3,90787373E+10

Which is lightly bigger compared with the which one associated to b= - 0.248717

kpc	m	m/s		
30,448	9,40E+20	229600	1,31E+00	3,26245E-11
33,493	1,03E+21	222500	1,24E+00	3,11061E-11
36,842	1,14E+21	215000	1,17E+00	2,96587E-11
40,527	1,25E+21	207100	1,10E+00	2,82781E-11
44,579	1,38E+21	200300	1,04E+00	2,69624E-11
49,037	1,51E+21	194700	9,87E-01	2,57076E-11
53,941	1,66E+21	189800	9,40E-01	2,45111E-11
59,335	1,83E+21	186200	9,00E-01	2,33705E-11
65,268	2,01E+21	184700	8,72E-01	2,2283E-11
71,795	2,22E+21	183900	8,48E-01	2,12459E-11
78,975	2,44E+21	181400	8,16E-01	2,02571E-11
86,872	2,68E+21	175500	7,71E-01	1,93145E-11
95,56	2,95E+21	167700	7,20E-01	1,84156E-11
		a optimal	3,90787373E+10	with b= -1/4
	D= a^(-4/3)	D parameter	7,54072E-15	

According the theory of dark matter by gravitation, each galaxy has two parameters C and D.

Parameter D is similar for similar galaxies, for example D= 5.85E-15 for M31.

New parameters a&b - A&B for Milky Way	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a optimal	$3{,}90787373 * 10^{10}$
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A $4{,}496262 * 10^{-6}$
$D = 8\pi G A = a^{\frac{-4}{3}}$	$7{,}54 * 10^{-15}$

Dark matter by gravitation theory stated that B has to be the same for all galaxies. However parameter **a** depend on each galaxy because it depend on baryonic matter enclosed by the galaxy.

11.3 PARAMETER C FOR MILKY WAY

C is the initial condition associated to a differential equation for the gravitational field in the galaxy whose formula depend on D and Eo.

$$C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

D is a global parameter because it has been calculated statistically, but Eo is the field at a specific radius Ro, so Eo is very dependent of error measure associated at such radius. In table below, it will be calculated parameter C for each data point because each one may be considerate as a initial condition.

This is the reason why in the following epigraph will be calculated Bernoulli mass for every parameter C and afterwards will be calculated the average Bernoulli mass function in order to calculate its parameter C associated.

The method used to calculate parameter C for Milky Way is lightly different to the one used for M31.

Point	kpc	m	m/s	Eo v^2/R	Parameter C
1	30,448	9,40E+20	229600	5,61E-11	-2,8439E-23
2	33,493	1,03E+21	222500	4,79E-11	-2,0251E-23
3	36,842	1,14E+21	215000	4,07E-11	-9,7430E-24
4	40,527	1,25E+21	207100	3,43E-11	3,2002E-24
5	44,579	1,38E+21	200300	2,92E-11	1,1840E-23
6	49,037	1,51E+21	194700	2,51E-11	1,5517E-23
7	53,941	1,66E+21	189800	2,16E-11	1,6475E-23
8	59,335	1,83E+21	186200	1,89E-11	1,2028E-23
9	65,268	2,01E+21	184700	1,69E-11	-9,8889E-25
10	71,795	2,22E+21	183900	1,53E-11	-1,5770E-23
11	78,975	2,44E+21	181400	1,35E-11	-2,2598E-23
12	86,872	2,68E+21	175500	1,15E-11	-1,5434E-23
13	95,56	2,95E+21	167700	9,54E-12	-2,1145E-36

11.4 FITTING PARAMETER C BY BERNOULLI MASS FORMULA

Below are tabulated the different Bernoulli masses for each parameter C calculated in previous epigraph. The radius dominion extend up to 3 Mpc because according Dark matter by gravitation theory, this phenomenon is associated to gravitational field, so its dominion is unlimited.

$$M_{BER}(r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

There are13 parameters C. The column in cyan colour is the average mass function. The column in yellow shows the Bernoulli mass with a fitted parameter C in order to minimize the relative difference between the average mass function and Bernoulli mass function with a parameter C fitted. C= -5E-24 . The last column shows that relative differences are below 0,1% for radius under 100 kpc and are below 0,6 % for radius under 2 Mpc.

Radius kpc	C at 30,4 kpc	C at 33,5 kpc	C at 36,8 kpc	C at 40,5kpc	C at 44,5 kpc	C at 49 kpc	C at 54 kpc	C at 59kpc
30,448	3,729E+11	3,668E+11	3,59E+11	3,502E+11	3,445E+11	3,420E+11	3,414E+11	3,443E+11
33,493	3,918E+11	3,852E+11	3,77E+11	3,673E+11	3,610E+11	3,584E+11	3,577E+11	3,609E+11
36,842	4,117E+11	4,046E+11	3,96E+11	3,851E+11	3,783E+11	3,755E+11	3,748E+11	3,782E+11
40,527	4,327E+11	4,249E+11	4,15E+11	4,038E+11	3,965E+11	3,934E+11	3,927E+11	3,963E+11
44,579	4,548E+11	4,463E+11	4,36E+11	4,234E+11	4,155E+11	4,122E+11	4,114E+11	4,153E+11
49,037	4,780E+11	4,688E+11	4,57E+11	4,440E+11	4,354E+11	4,319E+11	4,309E+11	4,353E+11
53,941	5,024E+11	4,924E+11	4,80E+11	4,656E+11	4,563E+11	4,524E+11	4,514E+11	4,561E+11
59,335	5,281E+11	5,173E+11	5,04E+11	4,882E+11	4,781E+11	4,740E+11	4,729E+11	4,779E+11
65,268	5,552E+11	5,434E+11	5,29E+11	5,119E+11	5,010E+11	4,965E+11	4,954E+11	5,008E+11
71,795	5,837E+11	5,709E+11	5,55E+11	5,367E+11	5,250E+11	5,201E+11	5,189E+11	5,247E+11
78,975	6,137E+11	5,998E+11	5,83E+11	5,628E+11	5,501E+11	5,448E+11	5,435E+11	5,498E+11
86,872	6,453E+11	6,302E+11	6,12E+11	5,901E+11	5,763E+11	5,707E+11	5,692E+11	5,761E+11
95,56	6,787E+11	6,622E+11	6,42E+11	6,187E+11	6,039E+11	5,977E+11	5,961E+11	6,035E+11
770	2,105E+12	1,999E+12	1,88E+12	1,740E+12	1,659E+12	1,626E+12	1,618E+12	1,657E+12
1000	2,440E+12	2,305E+12	2,15E+12	1,980E+12	1,879E+12	1,839E+12	1,829E+12	1,877E+12
2000	3,639E+12	3,380E+12	3,09E+12	2,786E+12	2,610E+12	2,540E+12	2,523E+12	2,606E+12
3000	4,631E+12	4,249E+12	3,83E+12	3,400E+12	3,156E+12	3,061E+12	3,037E+12	3,151E+12

C at 65kpc	C at 72kpc	C at 79 kpc	C at 87 kpc	C at 95,5 kpc	Average mass	Fitted C C= -5E-24	Relative Diff %
3,531E+11	3,635E+11	3,685E+11	3,633E+11	3,524E+11	3,556E+11	3,55896E+11	0,0920024
3,704E+11	3,817E+11	3,871E+11	3,814E+11	3,696E+11	3,730E+11	3,73386E+11	0,09259705
3,885E+11	4,007E+11	4,066E+11	4,004E+11	3,877E+11	3,914E+11	3,91737E+11	0,09305282
4,075E+11	4,207E+11	4,271E+11	4,204E+11	4,066E+11	4,106E+11	4,11E+11	0,09335478
4,274E+11	4,418E+11	4,487E+11	4,414E+11	4,264E+11	4,308E+11	4,31207E+11	0,09348646
4,483E+11	4,639E+11	4,714E+11	4,635E+11	4,473E+11	4,520E+11	4,52417E+11	0,0934304
4,702E+11	4,871E+11	4,952E+11	4,867E+11	4,691E+11	4,742E+11	4,74677E+11	0,09316754
4,932E+11	5,115E+11	5,203E+11	5,111E+11	4,920E+11	4,976E+11	4,98036E+11	0,09267747
5,173E+11	5,371E+11	5,467E+11	5,367E+11	5,160E+11	5,221E+11	5,22549E+11	0,09193812
5,426E+11	5,641E+11	5,745E+11	5,636E+11	5,412E+11	5,478E+11	5,48279E+11	0,09092555
5,691E+11	5,924E+11	6,038E+11	5,919E+11	5,676E+11	5,748E+11	5,75285E+11	0,08961411
5,969E+11	6,222E+11	6,345E+11	6,216E+11	5,953E+11	6,031E+11	6,03625E+11	0,08797638
6,261E+11	6,535E+11	6,669E+11	6,529E+11	6,244E+11	6,328E+11	6,33375E+11	0,08598213
1,782E+12	1,945E+12	2,029E+12	1,941E+12	1,772E+12	1,827E+12	1,82424E+12	-0,1464653
2,032E+12	2,236E+12	2,342E+12	2,231E+12	2,020E+12	2,089E+12	2,08444E+12	-0,22487616
2,879E+12	3,251E+12	3,451E+12	3,242E+12	2,856E+12	2,989E+12	2,97243E+12	-0,54427538
3,530E+12	4,061E+12	4,353E+12	4,047E+12	3,498E+12	3,693E+12	3,66183E+12	-0,83889186

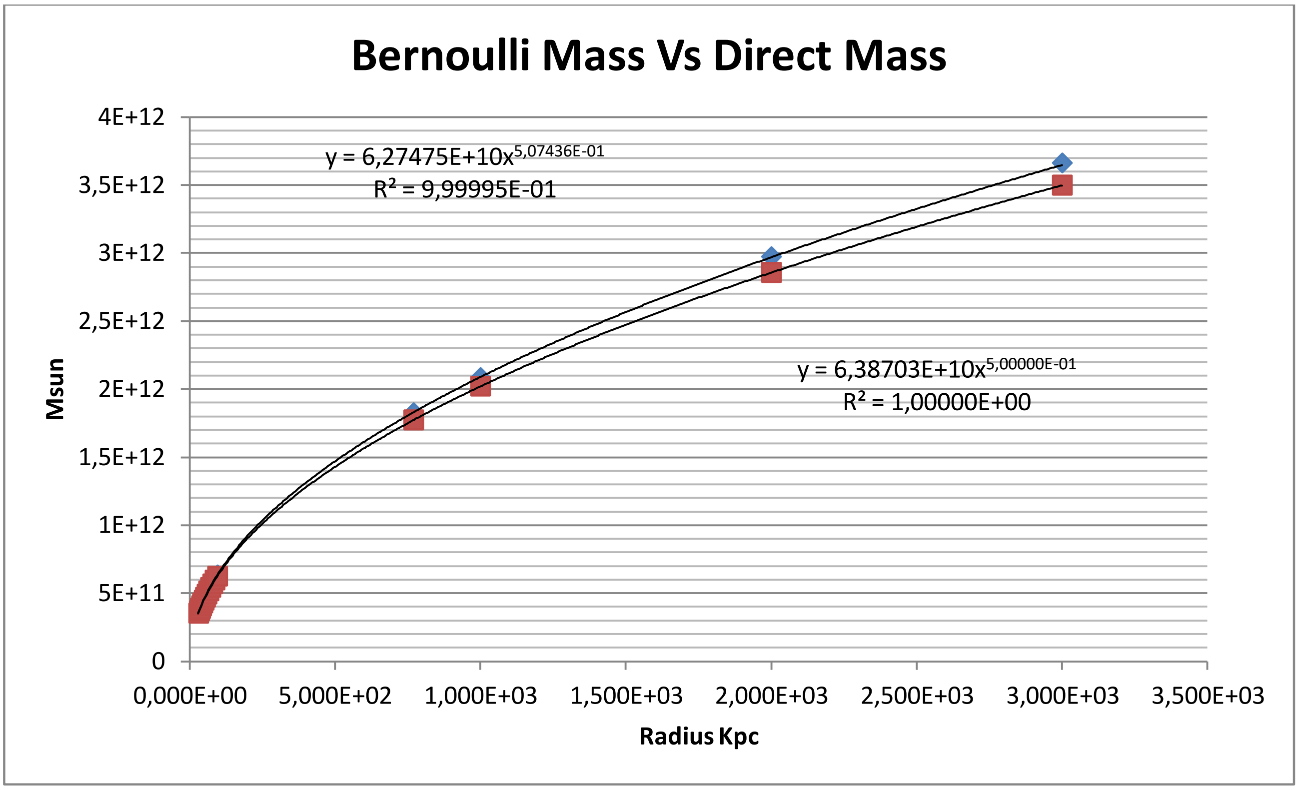
In conclusion, parameter C = -5E-24 is the value that fit better the average mass of the other 13 Bernoulli mass functions. So the parameters for Milky Way are $D = 7{,}54 \cdot 10^{-15}$ and $C = -5 \cdot 10^{-24}$

11.5 BERNOULLI MASS FORMULA VERSUS DIRECT MASS

Below are tabulated and compared both types of formulas for masses. Bernoulli mass is a bit bigger than the other one, but even at 3 Mpc the relative difference is about 4%. , being under 3% for radius under 1Mpc.

Radius kpc	Radius m	Bern Mass	Direct Mas	Rel diff %
3,045E+01	9,40E+20	3,55896E+11	3,52E+11	0,9725524
3,349E+01	1,03E+21	3,73386E+11	3,70E+11	1,00389568
3,684E+01	1,14E+21	3,91737E+11	3,88E+11	1,03624245
4,053E+01	1,25E+21	4,11E+11	4,07E+11	1,06963952
4,458E+01	1,38E+21	4,31207E+11	4,26E+11	1,10409792
4,904E+01	1,51E+21	4,52417E+11	4,47E+11	1,13967098
5,394E+01	1,66E+21	4,74677E+11	4,69E+11	1,17638927
5,934E+01	1,83E+21	4,98036E+11	4,92E+11	1,2142852
6,527E+01	2,01E+21	5,22549E+11	5,16E+11	1,25339682
7,180E+01	2,22E+21	5,48279E+11	5,41E+11	1,29376995
7,898E+01	2,44E+21	5,75285E+11	5,68E+11	1,33544217
8,687E+01	2,68E+21	6,03625E+11	5,95E+11	1,37844797
9,556E+01	2,95E+21	6,33375E+11	6,24E+11	1,42284204
7,700E+02	2,38E+22	1,82424E+12	1,77E+12	2,84565461
1,000E+03	3,09E+22	2,08444E+12	2,02E+12	3,10333024
2,000E+03	6,17E+22	2,97243E+12	2,86E+12	3,90459723
3,000E+03	9,26E+22	3,66183E+12	3,50E+12	4,46534223

Below are plotted both functions, Bernoulli mass and Direct mass.
Blue points belong to Bernoulli mass formula and the Brown points represent to Direct mass formula.



As direct mass is by far simpler than Bernoulli mass, hereafter it will be used to estimate total mass for M33.

12. DIRECT MASS FORMULA FOR M33 GALAXY

radius kpc	Vel km/s	Mdyn Msun
8	116,3	2,514E+10
9	118,7	2,947E+10
10	118,7	3,274E+10
11	118,7	3,601E+10
12	119,3	3,969E+10
13	118,7	4,256E+10
14	119,3	4,630E+10
15	119,88	5,009E+10
16	119,88	5,343E+10
17	119,88	5,677E+10
18	119,88	6,011E+10
19	119,88	6,345E+10
20	119,88	6,679E+10
21	119,88	7,013E+10
22	119,3	7,276E+10
23	119	7,568E+10

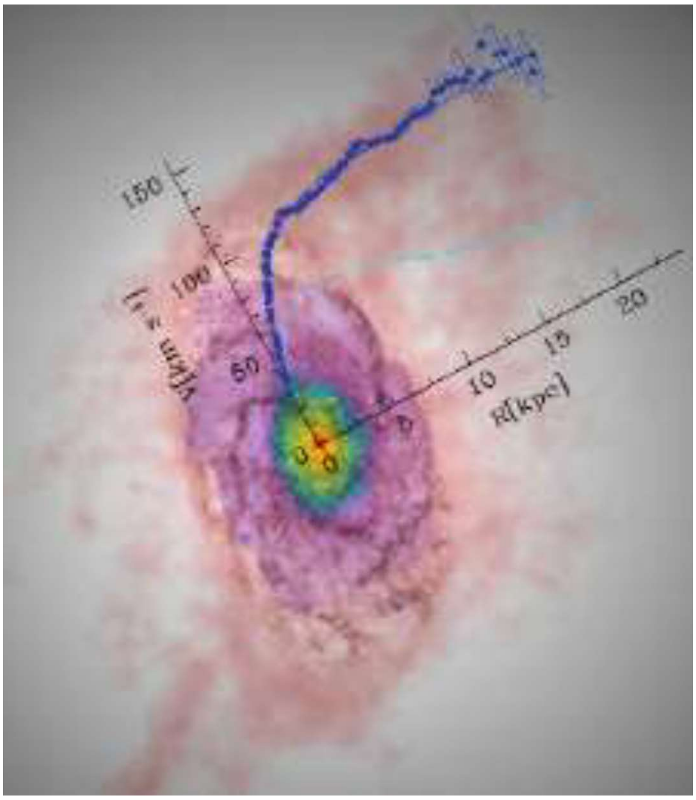
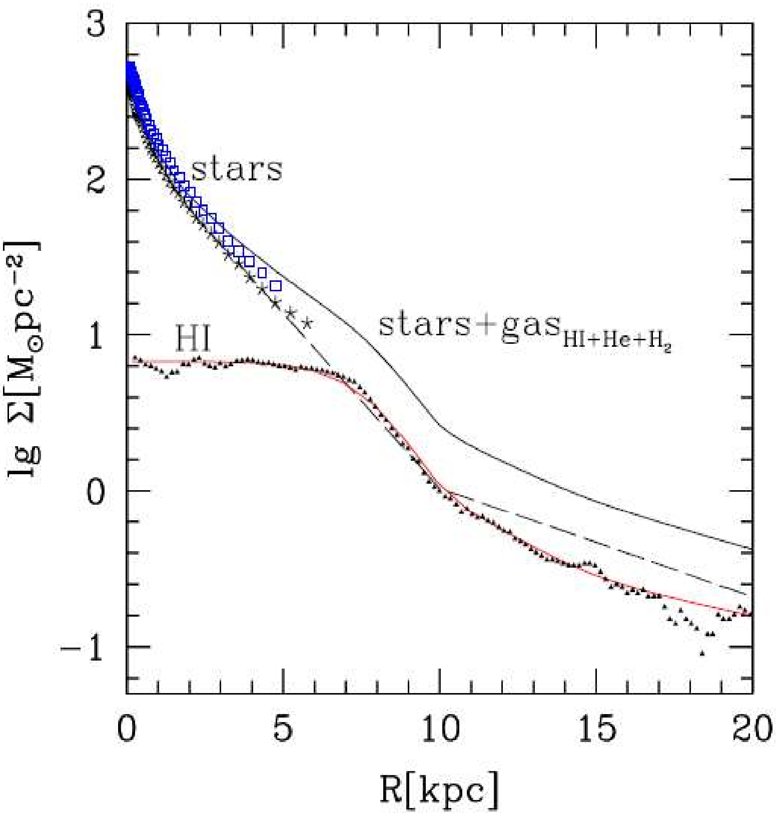


Table and graphic has been taken from [4] Corbelli,E.2014 . The rotation curve from 10 kpc to 23 kpc is quite horizontal. The stellar surface density ranges from red to magenta colours, pink cloud represents the HI gas.



The above Figure and text below has been taken from [4] Corbelli,E.2014 .

According the figure at 20 kpc Superficial density is 0,4 Msun/pc^2 and by linear extrapolation it is got superficial density at 23 kpc equal to Dsup = 0,28 Msun/pc^2

The formula D_{VOLUME} = Dsup/(2R) is a good approximation to calculate the volume density. So at 23 kpc volume density is approximately 6,1E-6 Msun/pc^3 which is $D_{BARYONIC}$ = 4,2E-25 kg/m^3.

Afterwards will see that such density is 2,3 % regarding DM density at 23 kpc. So it is acceptable to consider negligible the baryonic matter at such distance.

Fig. 10. The HI surface density perpendicular to the galactic plane of M33 (small filled triangles) and the function which fits the data (continuous line, **red in the on-line version**) after the 21-cm line intensity has been deconvolved according to tilted ring model-shape. Asterix symbols indicate the stellar mass surface density using the *BVIgi* stellar surface density map. The dashed line is the fit to the stellar surface density and the extrapolation to larger radii. Open squares (**in blue in the on-line version**) show for comparison the surface density using the BVI mass map. The heavy weighted line is the total baryonic surface density, the sum of atomic and molecular hydrogen, helium and stellar mass surface density.

Text beside from [4] Corbelli,E.2014 explains carefully the figure, although the most important line is the black continuous one. i.e. Total baryonic superficial density.

According a new paper, [7] Carignan.2017 Mass dynamic (< 23 kpc) is 8E+10 Msun, which is a bit large mass than value of table above, but as this paper is more recent it is right to consider such date.

As $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ taking the Carignan data it is got **a^2** = 3,987E20 which is rounded to **a^2** = 4E20 (I.S.)

With such parameter direct density DM $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ being $L = \dfrac{a^2}{8 \cdot \pi \cdot G}$ =2,385E+29

Then D_{DM}(23 kpc) = 1,78E-23 kg/m^3. So comparing such data with baryonic density, calculated in previous page, it is right to get that baryonic density is 2,3% regarding DM density. Therefore it is right to consider negligible baryonic matter and so to consider acceptable the value for parameter **a**, calculated with data at 23 kpc.

Beside are calculated some masses at different radii, using Direct mass.

In the following chapter it will be calculated the direct mass function of M31,MW and M33, and it will be shown that M33 add 10 % aprox. of total mass regarding the other two giant galaxies.

Direct Mass M33 function at some radii		
Radius kpc	radius m	Mass Msun
23,00	7,10E+20	8,025E+10
50,00	1,54E+21	1,183E+11
200,00	6,17E+21	2,366E+11
500,00	1,54E+22	3,742E+11
1000,00	3,09E+22	5,291E+11
1500,00	4,63E+22	6,480E+11
2000,00	6,17E+22	7,483E+11

13 THE MASS CALCULUS FOR THE LOCAL GROUP OF GALAXIES

According [5] Sofue, the relative velocity between M31 and Milky Way is 170 km/s. Assuming that both galaxies are linked gravitationally, it is possible to calculate the total mass of the Local group by a simple formula because of the Virial theorem.

$M = \dfrac{v^2 \cdot r}{G}$ As r = 770 kpc and v= 170 km/s then $M_{LOCAL\ GROUP}$ = 5.17 *10^{12} Msun

According [5] Sofue, using the current models of DM, the total mass of M31 and Milky Way is approximately 3*10^{12} Msun, so there is a mass lack of 2*10^{12} Msun which is a considerable amount of matter. Namely read epigraph 4.6 of [5] Sofue paper.

Up to now, in order to do calculus with data of rotation curve, the border of M31 is right to be placed at a half the distance to Milky Way because it is supposed that up to such distance its gravitational field dominates whereas for bigger distances is Milky Way field which dominates.

This hypothesis is right when it is considered rotation curves of different systems bounded to each galaxy i.e. stars or dwarf galaxies. However when it is considered the gravitational interaction between both giant galaxies it is needed to extend their haloes up to 770 kpc, because according DM by gravitation theory the phenomenon of Dark matter is linked to gravitational field, which is unlimited.

Therefore the M31 halo is extend up to 770 kpc and reciprocally the Milky Way halo is extend up to 770 kpc, when it is calculated the gravitational interaction between both galaxies.

13.1 AN ESTIMATION OF TOTAL MASS OF THE LOCAL GROUP AT DIFFERENT RADII

In order to estimate the total mass of Local Group will be considered only M31, M33 and MW. The rest of galaxies have a mass negligible to estimate the total mass of Local Group. In fact M33 add a 10% approximately only. In this version it will be used Direct mass instead Bernoulli mass because it is more simple and both formulas give results quite similar.

The values of parameters $\mathbf{a^2}$ have been got in previous chapters.

Galaxies	Parameter a^2
M31	2,235E+21
MW	1,527E+21
M33	4E+20
Parameter a^2 Local Group	4,16E+21

Radius	Direct mass M31	Direct mass MW	Direct Mass M33	Direct Mass Total
kpc	msun	Msun	Msun	Msun
40,00	5,913E+11	4,040E+11	1,058E+11	1,1011E+12
100,00	9,349E+11	6,388E+11	1,673E+11	1,7410E+12
300,00	1,619E+12	1,106E+12	2,898E+11	3,0155E+12
770,00	2,594E+12	1,773E+12	4,643E+11	4,8310E+12
1000,00	2,956E+12	2,020E+12	5,291E+11	5,5055E+12
1500,00	3,621E+12	2,474E+12	6,480E+11	6,7428E+12
2000,00	4,181E+12	2,857E+12	7,483E+11	7,7859E+12
2500,00	4,674E+12	3,194E+12	8,366E+11	8,7049E+12
3000,00	5,121E+12	3,499E+12	9,165E+11	9,5358E+12

By the formula of Direct mass it is right to get the table of masses using parameters a^2 associated to galaxies.

The fourth column is got by the parameter equivalent or adding the three column masses.

So a^2 for Local Group is 4,16E+21 which is got adding the a^2 parameter associated to the galaxies.

In conclusion adding MW+M31+M33 at 770 kpc the mass reach 4,8E+12 Msun, which is almost the dynamical mass commonly associated to Local Group 5E+12 Msun. In fact, when it is considered the mass of Large Cloud of Magellan which is approximately four times lower than M33 the total mass reach 5E12 Msun.

Another remarkable fact is that at 3 Mpc the mass reach 9,5E+12 Msun, without considering the LCM galaxy. In the summary of paper [14] Azadeh Fattahi, Julio F. Navarro., it is got at the same amount of mass, total mass for LG at 3 Mpc is 1E+13 Msun approximately. However they have used the most sophisticated software, such as APOSTLE Project, whereas in this paper has been used solely the RC of MW, M31 and M33 from Sofue and Corvelli.

Obviously these are magnificent success of Dark *Matter by Gravitation* theory.

When it is considered the universal expansion with a Hubble constant H_o = 70 km/s/Mpc, the local expansion between M31 and MW should be 54 Km/s, therefore to justify an approach velocity equal to 170 km/s it is needed even more mass than 5E+12 Msun. Fortunately, DM by gravitation theory stated that the more distance between galaxies the more mass have both galaxies. As it is shown in table above. For example the total mass if both galaxies would have at 1.5 Mpc distance, would be 6,7E+12Msun.

At 100 Km/s the time to traverse 730 kpc, from 1.5 Mpc to current distance 770 kpc, would be 7.1 Gy which is a half of the Universe age according current cosmology.

This calculus are only an estimation to shows that it is needed much more mass that 5E+12 Msun to justify the current approach velocity of M31 and MW. However DM by gravitation theory may support such extra amount of mass.

To study evolution of cluster dynamics it will be needed a model to consider the universal expansion form the ancient Universe to current universe taking in consideration the total DM associated to the gravitational field.

In chapter 11 of paper [2] Abarca,M.2019. it was made an estimation about the DM in the ancient Universe 9300 million years ago, when the Universe was decreased by 0,5 factor. As in the theory of DM by gravitation, the total DM is related to the distances, it was estimated that DM associated to galaxies was reduced a 30% regarding the current value. It is sure that future studies about dynamic evolution of clusters will be a magnificent test to check the *Theory of dark matter by gravitation.*

14. VIRIAL THEOREM AS A METHOD TO CALCULATE THE DIRECT MASS IN CLUSTERS

Virial theorem states that $M_{DYNAMICAL}(<r) = \dfrac{V^2 \cdot r}{G}$ is a formula right for a cluster of galaxy on condition that velocity and radius are calculated for galaxies in mechanical equilibrium.

If it is considered that the Virial radius is the border of halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then it is possible to apply the formula of $M_{DIRECT}(< R_{VIRIAL})$. Then by equation of both formulas will be possible to clear up a^2.

$M_{VIRIAL}(<r) = \dfrac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G}$ Getting the value for $a^2 = \dfrac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ It is only a way to estimate parameter a^2 because, outside the Virial radius always there will be a fraction of the galaxies belonging to cluster. Anyway, this method may estimate a lower bound of parameter a^2 associated to cluster.

For example, in the Local Group of galaxies, the dynamical data according [5] Sofue, Y.2015, are 770 kpc for distance between M31 and MW and 170 km/s for it relative velocity, getting $M_{LOCAL\text{-}GROUP}$ == 5.17 $*10^{12}$ Msun. = 1.03E+43 kg and by equation with $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G} = 1.03 \cdot 10^{43}$ kg with Radius= 770 kpc. It is got a^2 = 4,45E+21, which is very close to parameter a^2 = 4,162E+21 got in previous chapter adding parameters a^2 associated to M31, M33 and MW and even these values would be closer if it is considered another little galaxies such as Large Cloud of Magellan and others galaxies. Anyway its relative difference is 7%.

This example shows another property of parameter a^2: It is right to think that the cluster parameter a^2 should be the addition of parameter a^2 of every one galaxy inside the cluster.

It is remarkable the fact that the parameter a^2 of M31 and MW were calculated with data in halo whose radius range from 40 kpc up to 300 kpc in M31 case and range from 35 kpc up to 100 kpc in MW case, whereas calculus for parameter a^2 for Local Group has been made with only one data radius 770 kpc and velocity 170 km/s. Both methods have given similar results.

Table 1. The properties of the Coma and Virgo Clusters

	Coma	Virgo	ref.
M_{vir} ($M_\odot$)	1.4×10^{15}	2.19×10^{14}	a,b
R_{vir} (Mpc)	2.9	1.57	a,b
D (Mpc)	105	16.5	c,d
v (km s^{-1})	6930	1138	e,d
σ_v (km s^{-1})	1008	544	e,d

NOTE—a: Lokas & Mamon (2003) b: Mamon et al. (2004) c: Ferrarese et al. (2000) d: Mei et al. (2007) e: Struble & Rood (1999)

With the Virial data for some important clusters such as Virgo or Coma cluster will be calculated its parameter $\mathbf{a^2}$ with formula $a^2 = \dfrac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$. As it has been comment, it is only an estimation for parameter a^2, whose precision also depend on the precision of measures for Virial mass and radius. According [12] Karachentsev I.D. the Virgo cluster, which is 17 Mpc far away from MW, has M_{VIRIAL} = 7E+14 Msun and R_{VIRIAL} = 1.8 Mpc which leads to an approximate parameter $\mathbf{a^2}$ = 3.94E+23

Table beside comes from [13] Joo Heon Yoon, In order to compare with one of the biggest known cluster of galaxies, the Coma cluster has a Virial radius R=2,9 Mpc with a Virial Mass 1,4E+15 Msun, that gives a parameter a^2 = 6,215E+23 .

Using these data for Virgo cluster it is got parameter $\mathbf{a^2}$ = 1.3E+23 that give a value for a^2 three times lower than value got from paper [12] Karachentsev I.D. published in 2014. It is supposed that more recent papers give trustworthy data.

15. DARK MATTER IS COUNTER BALANCED BY DARK ENERGY

The basic concepts about DE on the current cosmology can be studied in [9] Chernin,A.D.

According [11] Biswajit Deb. Plank satellite data (2018) give a new updated, Hubble constant, H = 67.4± 0.5 km/s/Mpc and a new $\Omega_{DE} = 0.6889 \pm 0.0056$. In this paper it will be used $\Omega_{DE} = 0.69$ as the fraction of Universal density of DE.

In the current cosmologic model ΛCDM , dark energy has an effect equivalent to antigravity i.e. the mass of dark energy is negative and the dark energy have a constant density for all the Universe equal to

$\varphi_{DE} = \varphi_C \bullet \Omega_{DE} = -5.865 \cdot 10^{-27} kg/m^3$ being $\Omega_{DE} = 0.69$ and $\rho_C = \dfrac{3H^2}{8\pi G}$ = 8.5E-27 kg/m^3 the critic density of the Universe , updated with Plank satellite data 2018.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According [9] Chernin,A.D. $M_{DE}(<R) = -\dfrac{\rho_{DE} 8\pi R^3}{3}$, and using the values of data Plank 2018, for H = 67.4 Km/s/Mpc and $\Omega_{DE} = 0.69$ a good approximation for total mass of DE is given by the formula $M_{DE}(<R) = -\varphi_{DE} \dfrac{8\pi R^3}{3} = -4.91 \cdot 10^{-26} \cdot R^3\, kg$. [9] Chernin defines gravitating mass $M_G = M_{DE} + M_{TOTAL}$, where M_{TOTAL} is baryonic plus DM mass, and defines R_{ZG} , Radius at zero Gravity as the radius where $M_{DE} + M_{TOTAL} = 0$. When the gravitating mass is zero, this leads to equation $M_{TOTAL} = \varphi_{DE} \dfrac{8\pi R_{ZG}^3}{3}$. As Direct mass $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{R_{ZG}}}{G} \approx M_{TOTAL}$, in the framework of ***DM by gravitation theory*** it is possible to clear up rightly $R_{ZG} = \left[\dfrac{3a^2}{8\pi G \rho_{DE}}\right]^{2/5}$ where the only local parameter is $\mathbf{a^2}$**.** Below it is calculated RZG for some clusters.

For the Local group of galaxies, if $a_{L-G}^2 = 4{,}162 \bullet 10^{21}$ then R_{ZG}=2.25 Mpc. So at that radius the gravitating mass is zero, in other words, for radius under 2,25 Mpc dark matter dominates and for bigger radius dark energy dominates and it is not possible to link by gravitation a galaxy more than 2,25 Mpc far away to centre of mass of Local Group, because there is more dark energy than dark matter.

For Coma cluster with its previously calculated parameter a^2 = 6,215E+23 , it is possible to get rightly R_{ZG}= 16.7 Mpc .In other words 16,7 Mpc is the radius of region where the DM of Coma Cluster dominates versus dark energy.

Similarly for Virgo cluster with parameter $\mathbf{a^2}$ = 3.944E+23 leads to R_{ZG} = 13,9 Mpc. With this simple calculus it is possible to state that the Local Group is placed in the outskirts of gravitational influence of Virgo cluster, as distance to Virgo cluster it is calculated to be 17 Mpc.

With these three important clusters of galaxies, it has been illustrated how the total mass, approximated by $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$, is counter balanced by dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the region size where the cluster has gravitational influence.

16. CONCLUSION

As it has been outlined at the introduction, this work is the consequence of the new set of data for rotation curve in Milky Way halo. With these new data, the fitted function of rotation curve at halo region of Milky Way has exactly the same exponent that fitted function associated to M31 galaxy. This fact back strongly the main hypothesis of Dark gravitation theory i.e. Dark matter is generated according an unknown quantum gravitational mechanism, which depend on the gravitational field, so it is a Universal law.

Through the first ten chapters is developed the theory using M31 rotation curve. This chapters are identical to the previous paper [2] Abarca,M.2019, excepting the process of getting the parameter C , the initial condition of solution for Bernoulli differential equation associated to field E. See chapter 9. This parameter associated to a specific point of rotation curve is quite problematic because of experimental errors. In this work the value got has been C = - 3,77E-24 whereas in the previous work was C = 6,36E-24. Both are little numbers, but with the value negative the field is lightly bigger and consequently DM density is a bit bigger. However, as this parameter is a very little number, it has been highlight that Bernoulli mass formula is very close to data got with Direct mass formula. In fact Bernoulli mass formula becomes mathematically Direct mass when parameter C is equal to zero.

In the chapter 11 has been calculated different parameters associated to Milky Way halo, with them are calculated Bernoulli mass and Direct mass formulas, also it is shown that both formulas have a negligible relative difference even at radius 3 Mpc.

In the chapter 12 is calculated the total mass of M33 at different radii which is approximately a 10 % of masses associated to M31 plus MW. In this calculus has been used Direct mass formula, using only one point data at 23 kpc where it was estimated that baryonic matter was negligible versus DM density.

In the 13 chapter is estimated the mass of Local Group, being $4{,}8 \cdot 10^{12}$ Msun at 770 kpc, without the LCM galaxy, which is equivalent to dynamical mass $5 \cdot 10^{12}$ Msun of LG at such distance. In addition is estimated that total mass for MW+M31+M33 at 3 Mpc being 9,5E+12 Msun which is equivalent to the total mass for LG at 3 Mpc published by [14] Azadeh Fattahi, Julio F. Navarro in 2020 as a results of APOSTLE Project. These calculus are a big success of ***DM by gravitation theory***.

In the chapter 14 it is shown a method to estimate the Direct mass formula for a cluster of galaxies, only with its Virial Mass and radius.

In the chapter 15, it is shown how at 2.25 Mpc the total dark energy is able to counter balance the dark matter contained in the Local group, or in the Virgo cluster the gravitating mass is zero at 13.9 Mpc whereas in Coma cluster the radius zero gravity is 16.7 Mpc. Put in short, DM is counter balanced by dark energy at some mega parsecs of distance according data of current ΛCDM cosmology.

This theory introduces a powerful method to study DM in the halo region of galaxies and cluster of galaxies and conversely measures in galaxies and clusters offer the possibility to check the theory.

In my opinion, it is not possible to develop anymore the theory into the Newtonian framework. However a natural way to develop more in depth ***DM by gravitation theory*** would be to consider General Relativity.
Namely, it is right to get the density of energy associated to DM, multiplying density of dark matter by c^2 i.e. $D_{DM}^{ENERGY} = \frac{a^2 \cdot c^2}{8 \cdot \pi \cdot G} \cdot r^{-5/2}$. So this density of energy would be a new term to consider into the tensor of energy of Einstein's gravitational field equations.

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The missing dwarf galaxies of the Local Group