A DARK MATTER THEORY BY GRAVITATION GOT IN M31 AND MILKY WAY

Author Manuel Abarca Hernandez **email** mabarcaher1@gmail.com

1. ABSTRACT

In this paper is develop carefully an original theory of dark matter, whose main hypothesis is that dark matter is generated by the own gravitational field. In this work is introduced the best version of the theory physical and mathematically, previously published since 2014.

The hypothesis of dark matter by gravitation has two main consequences: the first one is that the law of dark matter generation has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit. Both properties are crucial for the success of this theory. Apparently the hypothesis of unlimited haloes would seem impossible but further cosmological studies may demonstrate that dark energy may cancel the growing of dark matter and this balance happens at cluster scale, at 10 Mpc.

This paper is similar to one previous paper published in 2019, although there is an important newness, which is the new rotation curve of Milky Way published by Sofue,Y.in 2020. Results got with this new data in Milky Way back strongly the universality of dark matter generation law.

Briefly will be explained method followed to develop this work. Rotation curve data come from Sofue,Y.2015 for M31 galaxy and from Sofue 2020 for Milky Way. Thanks these remarkable rotation curves, both regression curves at halo region are fitted with power regression functions whose exponent are the same for both galaxies

By the fitted function is possible to calculate a dark matter density function depending on radius which is transformed into a dark matter density depending on gravitational field. This change is the core of the theory because at this moment it is possible to study the formula of dark matter density by the Buckingham theorem in order to change the statistical calculus by physical formulas which depend on the universal constants G, h and c.

From now on the statistical theory becomes a perfect physical theory that despite it is based on the Newtonian framework allows to get new formulas such as dark matter density, gravitational field and total dark matter mass.

The chapter 10 and 11 are dedicated to apply the theory to M31 and Milky Way galaxies with excellent results.
But it is in the chapter 12 where are published the most brilliant results got in this work.
The total mass of the Local group of galaxies calculated is 4.5E+12 Msun which match perfectly, if it is considered the range of error, with the dynamical mass currently accepted by the Local Group 5E+12Msun.
In addition, the unlimited halo of the theory allows to reach 5.1E12Msun at 1 Mpc and 7.3E+12 Msun at 2 Mpc of distance.

2. INTRODUCTION

Since 2013 up to 2019 I have published several papers studying DM in galactic halos, especially in M31 and Milky Way although also I have published some papers studying DM in Coma cluster [3] Abarca,M.2019. and other galaxies.

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. According [5] Sofue, Y. 2015. Its baryonic masses are $M_{BARYONIC-M31} = 1{,}61\cdot10^{11}\ M_{SUN}$ and $M_{BARYONIC-MILKY\ WAY} = 1{,}4\cdot10^{11}\ M_{SUN}$

The DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* considers that DM is generated by the own gravitational field. Therefore, in order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. i.e. radius bigger than 30 kpc for MW and 40 kpc for M31.

During these years I have developed an original theory based on the hypothesis that it is the gravitational field according an unknown quantum mechanism that generates the dark matter. This hypothesis has two main consequences: the first one is that the law of dark matter generation has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit.

The last version of the theory was published in [2] Abarca,M.2019, and there, it was demonstrated mathematically that total mass goes up proportionally to the root square of distance, so this property may explain how the ratio of dark matter versus baryonic matter at cluster scale is bigger that at galactic scale. This property will be studied in this paper as well. By other side at bigger radius, bigger than 10 Mpc, the growing of the total mass is so slow that Dark energy phenomenon dominates. Precisely, this fact may explain the size of galactic clusters.

The first consequence before mentioned, dark matter generated by a Universal law, has been studied by all my papers, especially inside M31 and Milky Way thanks the paper [5] Sofue, Y.2015.

In fact I could develop rigorously my theory because the rotation curve of M31 at halo region decreased with a power regression fitted curve whose exponent is -1/4. However with data published for Milky Way at the same paper it was not possible to fit rigorously the rotation curve with such exponent.

As experimental errors in Milky Way were so big, it was possible to explain the difficulties of the theory in Milky Way. In addition I was so convinced with the theory because it is physically coherent and mathematically is perfect.

However recently I have known a new paper [6] Sofue,Y.2020, where the author gives a new rotation curve data for Milky Way at halo region whose fitted curve has an exponent -1/4. Such result it is the confirmation of my theory, because it is needed a same law for all the galaxies. So in this paper it is firstly developed all the theory carefully with M31 galaxy up to chapter 10 and chapter 11 is dedicated to apply the theory to Milky Way.

Finally the chapter 12 is dedicated to estimate the total mass of Local Group. The total mass calculated for Milky Way and M31 is $4{,}5*10^{12}$ Msun. The dynamical mass for both galaxies according its velocity of approach is $5*10^{12}$ Msun. However the relative difference between both results may be explained by error measures. So it is possible to state that both results are equivalents when it is considered the range of errors.

The importance of these findings is high because there is not any other theory able to explain such amount of mass. In fact, the current theories of dark matter gives a maximum of dark matter for the Local Group up to $3*10^{12}$ Msun. Therefore, the theory not only may explain the rotation curves in halo region for M31 and MW but also it is the only one that may explain the total mass of the Local group of galaxies. i.e. the theory works perfectly at cluster scale.

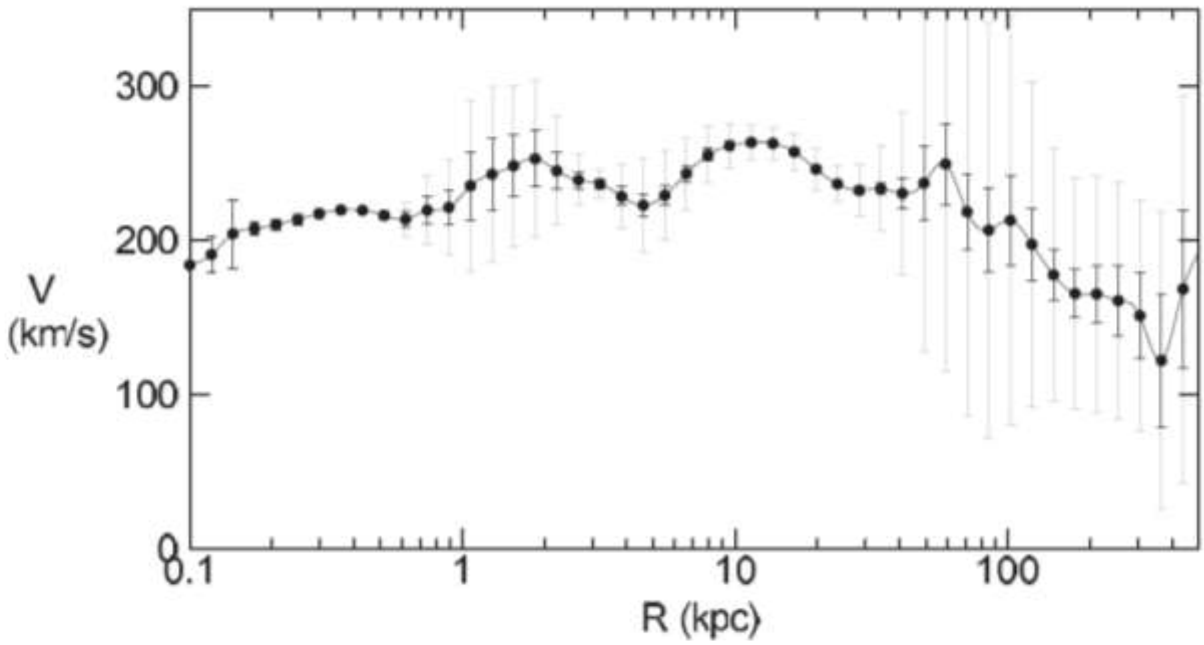
In fact in the paper [3] Abarca,M.2019, it is studied the Coma cluster with the theory and it is shown how works perfectly inside this cluster, which is one of the most massifs between known clusters.

Thanks the possibility to study the gravitational effect of dark matter pure it have been possible to develop a theory mathematically simple. When baryonic mass is mixed with dark matter as it happens inside the galactic disc the mathematical treatment is more complex.

Taking into account that the only ones giant galaxies quite close to be able to study with accuracy the rotation curve at halo region are Milky Way and M31, the coincidence of the same exponent to the fitted function for the rotation curves for both galaxies is crucial in order to state that dark matter is generated according an universal law. In other words the two only ones galaxies where it is possible to develop this theory with simple mathematical methods are M31 and Milky Way.

3. OBSERVATIONAL DATA FOR M31 GALAXY FROM SOFUE. 2015 DATA

Graphic come from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. Although Sofue rotation curve extend from 0,1 kpc up to 352 kpc the range of dominion considered for this work is only the halo region where ratio baryonic matter is negligible. In chapter 6, will be shown that this happens for radius bigger than 40 kpc, despite the fact that disc radius for M31 is accepted to be 35 kpc.



kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8

The last measure at 352 kpc has been rejected because has a velocity too high, so does not match at all with the other measures.

3.1 POWER REGRESSION TO ROTATION CURVE

The measures of rotation curve have a very good fitted curve by power regression.

In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

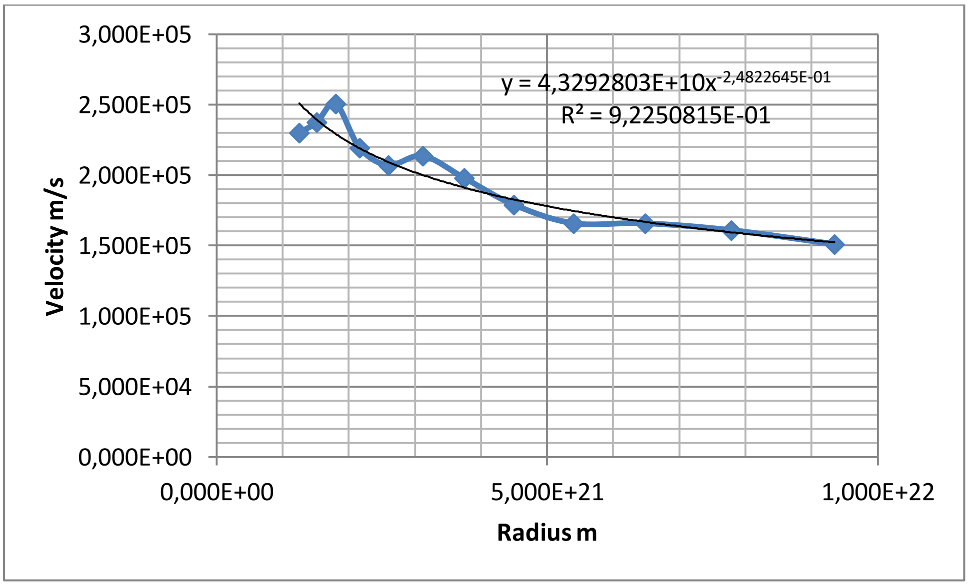
Power regression for M31 rot. curve	
V=a*r^b	
a	$4{,}32928 \cdot 10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Data fitted are in grey columns below.

In fifth column is shown results of fitted velocity and sixth column shows relative difference between measures and fitted results.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Below is shown a graphic with measures data and power regression function.



Correlation coefficient equal to 0,96 whis is a superb result especially when dominion measures is up to 303 kpc. There is not any other galaxy to measure a rotation curve so magnificent. According theory of DM generated by field, galaxy haloes are unlimited although up to a half of distance i.e. 375 kpc toward Milky Way direction is dominated by M31 field whereas the other half distance is dominated by Milky Way.

Furthermore, it has been calculated regression curve with another data placed at 363 kpc, but power regression is -0.28 and correlation coefficient is 0.954. This result shows that such data is not trustworthy because according dimensional analysis power has to be -0.25. As 363 kpc is placed in the border of M31 halo it is possible that such data might be influenced by a different field. Therefore it is better to study data only up to 303 kpc.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a{\cdot}r^b$. As $M(<r) = \frac{v^2{\cdot}R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2{\cdot}R}{G} = \frac{a^2{\cdot}r^{2b+1}}{G}$. Hereafter this formula will be called Direct Mass $M_{DIRECT}(<r) = \frac{a^2{\cdot}r^{2b+1}}{G}$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $d\,m = \frac{a^2\cdot(2b+1){\cdot}r^{2b}dr}{G}$ and $dV = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}{\cdot}r^{2b-2}$$

Writing $L = \frac{a^2\cdot(2b+1)}{4\pi G}\cdot$ results $D_{DM}(r) = L{\cdot}r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L{\cdot}r^{2b-2}$ kg/m^3

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E BY VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula E = v^2/R whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a{\cdot}r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2{\cdot}r^{2b}}{r} = a^2{\cdot}r^{2b-1}$ briefly $E = a^2{\cdot}r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A{\cdot}E^{B}$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by the author in previous papers. [1] Abarca,M. [2] Abarca,M. [8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} {\cdot} r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2 {\cdot} r^{2b}}{r} = a^2 {\cdot} r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

According parameters a & b got in previous chapter, A& B parameters are:

M31 galaxy	$D_{DM} = A{\cdot}E^{B}$
A	$3{,}6559956 \cdot 10^{-6}$
B	1,6682469

As conclusion, in this chapter has been demonstrated that a power law for velocity

$v = a{\cdot}r^{b}$ *is mathematically equivalent to a power law for DM density depending on E.* $D_{DM} = A{\cdot}E^{B}$

5.3 THE IMPORTANCE OF $D_{DM} = A{\cdot}E^{B}$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} {\cdot} r^{2b-2}$ and $E = a^2 {\cdot} r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A{\cdot}E^{B}$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be similar for different galaxies on condition that they were similar. In further chapters will be got that power B is exactly the same for M31 and Milky Way although A will be a bit different.

However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters. As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, for galaxy scale or bigger systems.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy.

[5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r \Sigma(r) dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r) dr$ where $D_{DM}(r) = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$

It is defined ratio function as quotient of both differential quantities $Ratio = \dfrac{dM_{DISK}}{dM_{DM}} = \dfrac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

Radius	Radius	Ratio (r)	$\Sigma(r)$	Direct DM
Kpc	m	Ratio	kg/m^2	kg/m^3
36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23
38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.
This is the reason why in this work dominion for radius begin at 40 kpc.

7. A DIFFERENTIAL EQUATION FOR A GRAVITATIONAL FIELD

7.1 INTRODUCTION

This formula $D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \dfrac{G \cdot M(<r)}{r^2} = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM}

formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A{\cdot}E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G\dfrac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G\dfrac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\dfrac{M'(r){\cdot}r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\dfrac{M(r)}{r^3}$ As

$\varphi_{DM}(r) = A{\cdot}E^B(r)$ it is right to get $E'(r) = 4\pi{\cdot}G{\cdot}A{\cdot}E^B(r) - 2\dfrac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K{\cdot}E^B(r) - 2\dfrac{E(r)}{r}$ being $K = 4\pi{\cdot}G{\cdot}A$

Calling ***y*** to E, the differential equation is written in this simple way $y` = K{\cdot}y^B - \dfrac{2{\cdot}y}{r}$

Bernoulli family equations $y` = K{\cdot}y^B - \dfrac{2{\cdot}y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$. Which is $\dfrac{u`}{1-B} + \dfrac{2u}{r} = K$

The homogenous equation is $\dfrac{u`}{1-B} + \dfrac{2u}{r} = 0$ Whose general solution is $u = C \cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u=z*r then it is got that $z = \dfrac{K\cdot(1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C\cdot r^{2B-2} + z\cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \dfrac{Kr(1-B)}{3-2B}\right)^{\frac{1}{1-B}}$ $with\ B \neq 1\ and\ B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B - 2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B}\right)$ formula may be written as $E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$

Calculus of parameter C through initial conditions R_0 *and* E_0

Suppose R_0 *and* E_0 are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $\varphi_{DM}(r) = A \cdot E^{B}(r)$. Therefore this solution for field works only in the halo region and R_0 *and* E_0 could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E BY BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1,6682469

In this case relative difference between B = 1,6682469 and 10/7 is 16,7 %. A 17% of error in cosmology could be acceptable. However by the end of the chapter it will be found a better solution.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G,h and c =2.99792458$\cdot 10^8$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which can not be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1 \; and \; B \neq 3/2$.

As A is a positive quantity then $2b+1 > 0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ Therefore $B \in (-\infty, 1) \cup (3/2, \infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2, \infty)$.

The main consequence this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is wrong because B=10/7 = 1.428. Therefore formula $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by two pi monomials it is more suitable formula.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3,6559956 \cdot 10^{-6}$
B	1,6682469

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A{\cdot}E^B$ will be 5/3 = 1.666666 , which is the best approximation to B= 1.6682469.

Finally $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

CALCULUS OF DIMENSIONLESS NUMBER INCLUDED IN FORMULA OF DARK MATTER DENSITY

By equation of $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ and $D=A*E^B$

It is right that $A = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}}$ and then $M = A \cdot \sqrt[6]{G^7 \cdot c^5 \cdot h}$

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs less than 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a&b parameters and A&B parameters. Now considering B= 5/3

as $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$. It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Regression for M31 dominion 40-303 kpc	
$V=a*r^b$	
a	$4,32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Due to Buckingham theorem it is needed that b= -1/4 . Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a&b that fit perfectly to experimental measures of rotation curve in M31 halo.

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula V= a*r $^{-0.25}$ on condition that $\sum_e (v - v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \cdot 10^{10}$$

Where r_e represents each radius measure and v_e represents its velocity associated. Consequently the new A= 3.488152*10^{-6}

New parameters a&b and A&B	
B	5/3
$b = \dfrac{B-2}{2B-2}$	b = -1/4
a new	4,727513*10^{10}
$A = \dfrac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A 3.488152*10^{-6}

9.2 FORMULAS OF DIRECT D.M.

With these new parameters recalculated it is going to get the direct formulas got at the beginning of paper.

Function of Density DM depending on radius.

$$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}}$$ being $L = \dfrac{a^2 \cdot (2b+1)}{4\pi G} = \dfrac{a^2}{8 \cdot \pi \cdot G}$ =1,3326*10 ^{30}

Function of E depending on radius $E = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}}$ being a^2 = 2,235*10 ^{21}

Mass enclosed by a sphere of radius r, known as dynamical mass because it is calculated with velocity.

$$M_{DYN}(<r) = \frac{v^2 \cdot R}{G}$$

When velocity is replaced by its fitted function it is got $M_{DIRECT}(<r) = \dfrac{a^2 \cdot r^{2b+1}}{G}$

being $\dfrac{a^2}{G}$ = 3,349*10 ^{31}

9.3 BERNOULLI SOLUTION FOR E IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to get formulas far simple because some parameters are simple fractions.

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$$

being $\alpha = 2B - 2 = \dfrac{4}{3}$ being $\beta = \dfrac{1}{1-B} = \dfrac{-3}{2}$ then, the initial condition C

$C = \dfrac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$ becomes $C = \dfrac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ and $D = \left(\dfrac{4 \cdot \pi \cdot G \cdot A(1-B)}{3-2B}\right) = 8 \cdot \pi \cdot G \cdot A = a^{\frac{-4}{3}}$ so $D = a^{\frac{-4}{3}}$ = 5,85*10^{-15}

Therefore $E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}}$ being C the initial condition of differential equation solution for E and $D=a^{\frac{-4}{3}}$ is a parameter closely related to the global rotation curve at halo region, being parameter **a** =4,7275*10^{10}

9.4 DARK MATTER AT A SPHERICAL CORONA BY BERNOULLI SOLUTION

Formula below express the dark matter contained inside a spherical corona defined by R_1 and R_2 belonging at halo.

$$M_{DM}=\int_{R1}^{R2}4\pi\cdot r^2\cdot\rho(r)dr=\int_{R1}^{R2}4\pi\cdot \mathrm{r}^2 AE^B dr=4\pi A\int_{R1}^{R2}\mathrm{r}^2\left[C\cdot r^{4/3}+D\cdot r\right]^{\frac{-5}{2}}\cdot dr$$

The indefinite integral $I=4\pi A\cdot\int\frac{r^2}{\left(C\cdot r^{4/3}+D\cdot r\right)^{2.5}}=\frac{8\pi A\sqrt{r}}{D\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}=\frac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$

As $\frac{8\pi A}{D}=\frac{1}{G}$. Calling $M(r)=\frac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$ and by the Barrow`s rule, it is got

$M_{DM}{}_{R1}^{R2}$ = M(R2)-M(R1) that provided the DM contained inside the spherical corona defined by R2 and R1.

9.5 CALCULUS OF PARAMETER C

$$C_O=\frac{E_0^{\frac{-2}{3}}-D\cdot R_0}{R_0^{\frac{4}{3}}}$$

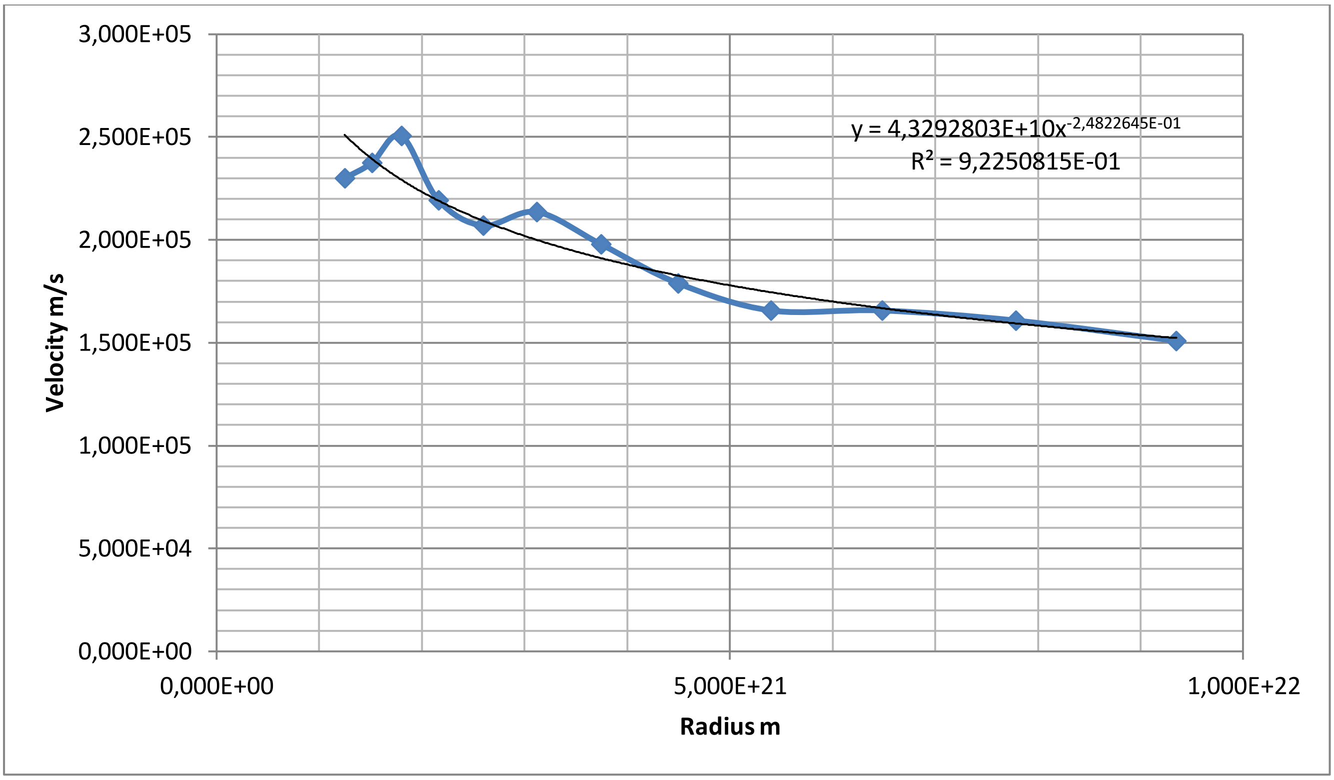
In this formula, E_O is the gravitational field at R_O radius which is V^2_0 / R_O i.e. the centripetal acceleration.

As experimental measures have errors, it will be calculated Co for every data in order to calculate function dark matter mass for all of them.

	Radius	Radius	Velocity	Field Eo	Eo^(-2/3)	Eo^(-2/3)-D*R	Parameters
	kpc	m	m/s	m/s^2			Co
1	40,5	1,250E+21	2,299E+05	4,23E-11	8239256,95	9,27E+05	6,8830E-23
2	49,1	1,515E+21	2,374E+05	3,72E-11	8973615,749	1,11E+05	6,3753E-24
3	58,4	1,802E+21	2,505E+05	3,48E-11	9377625,482	-1,16E+06	-5,3082E-23
4	70,1	2,163E+21	2,192E+05	2,22E-11	12654654,13	1,20E+03	4,2804E-26
5	84,2	2,598E+21	2,069E+05	1,65E-11	15443489,73	2,45E+05	6,8683E-24
6	101,1	3,120E+21	2,135E+05	1,46E-11	16732951,68	-1,52E+06	-3,3317E-23
7	121,4	3,746E+21	1,978E+05	1,04E-11	20928758,45	-9,85E+05	-1,6934E-23
8	145,7	4,496E+21	1,788E+05	7,11E-12	27043372,43	7,42E+05	9,9988E-24
9	175	5,400E+21	1,656E+05	5,08E-12	33846627,1	2,26E+06	2,3822E-23
10	210,1	6,483E+21	1,656E+05	4,23E-12	38232870,38	3,08E+05	2,5446E-24
11	252,3	7,785E+21	1,607E+05	3,32E-12	44959131,06	-5,83E+05	-3,7771E-24

12	302,9	9,347E+21	1,508E+05	2,43E-12	55281485,72	6,02E+05	3,0572E-24

Values in yellow are negatives because these points are above the fitted curve . See graph below. In addition, the more close to curve the point is, the smaller, in absolute value, the parameter C is. The cyan value is the smaller.



FUNCTION DARK MATTER INSIDE SPHERICAL CORONA FOR DIFFERENT PARAMETERS Co

Function of dark matter is $M(r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ being $D = a^{\frac{-4}{3}}$ = 5,85*10^{-15} and parameters C calculated in previous paragrapah.

The row 1 gives the value M(R1) in Msun units. See table below, the grey row.

The row 2 and the followings one give the dark matter inside the spherical corona with radius Rn and R1, where n ranges from 2 to 14. For example, $\underset{R1}{\overset{R2}{M}}{}_{DM}$ = M(R2)-M(R1) that provided the DM contained inside the spherical corona defined by R2 and R1.

The columns 1 to 12 give the 12 dark matter functions inside spherical corona Rn –R1 ,where n ranges from 2 to 12, associated at different values of parameters C showed in previous table.

A good option to minimize error is to considerate the average function mass to try to adopt the most suitable parameter C for M31.

The last column, gives de Average dark matter function considering the 12 functions.

		1	2	3	4	5	6
	kpc	C at 40kpc	C at 49 kpc	C at 58 kpc	C at 70 kpc	C at 84kpc	C at 101kpc
1	40,5	4,9743E+11	5,8461E+11	6,9418E+11	5,9486E+11	5,8382E+11	6,542E+11
	Dark matter inside spherical corona between Rn and R1 with the formulas M(Rn)-M(R1) Dark matter inside spherical corona between Rn and R1 with the formulas M(Rn)-M(R1)						
2	49,1	4,4135E+10	5,8251E+10	7,8343E+10	6,0022E+10	5,8116E+10	7,0714E+10
3	58,4	8,6985E+10	1,1573E+11	1,5717E+11	1,1936E+11	1,1545E+11	1,4137E+11
4	70,1	1,3538E+11	1,8174E+11	2,4957E+11	1,8763E+11	1,8129E+11	2,2357E+11
5	84,2	1,8753E+11	2,5417E+11	3,5325E+11	2,627E+11	2,5352E+11	3,1506E+11
6	101,1	2,4344E+11	3,333E+11	4,6928E+11	3,449E+11	3,3242E+11	4,1656E+11
7	121,4	3,0327E+11	4,1969E+11	5,9926E+11	4,3487E+11	4,1854E+11	5,2918E+11
8	145,7	3,6717E+11	5,1392E+11	7,45E+11	5,3324E+11	5,1246E+11	6,5417E+11
9	175	4,3574E+11	6,173E+11	9,0967E+11	6,4144E+11	6,1548E+11	7,9385E+11
10	210,1	5,0878E+11	7,3E+11	1,095E+12	7,5974E+11	7,2775E+11	9,4917E+11
11	252,3	5,8674E+11	8,5327E+11	1,3047E+12	8,8953E+11	8,5053E+11	1,1227E+12
12	302,9	6,6967E+11	9,8779E+11	1,5422E+12	1,0316E+12	9,8449E+11	1,3164E+12
13	770	1,178E+12	1,8919E+12	3,3875E+12	1,9981E+12	1,884E+12	2,7375E+12
14	1000	1,3479E+12	2,226E+12	4,1909E+12	2,36E+12	2,216E+12	3,3166E+12

	7	8	9	10	11	12	
kpc	C at 121 kpc	C at 145 kpc	C at 175 kpc	C at 210 kpc	C at 252kpc	C at 302 kpc	Average DM mass Msun
40,5	6,2388E+11	5,7887E+11	5,5783E+11	5,9077E+11	6,0119E+11	5,8994E+11	
	Dark matter inside spherical corona between Rn and R1 with formulas M(Rn)-M(R1)						
49,1	6,5156E+10	5,7271E+10	5,3736E+10	5,9313E+10	6,1126E+10	5,917E+10	6,0446E+10
58,4	1,2991E+11	1,1372E+11	1,065E+11	1,1791E+11	1,2162E+11	1,1761E+11	1,2028E+11
70,1	2,0481E+11	1,7848E+11	1,6679E+11	1,8527E+11	1,9131E+11	1,8479E+11	1,8922E+11
84,2	2,8768E+11	2,4946E+11	2,3257E+11	2,5928E+11	2,6805E+11	2,5859E+11	2,6515E+11
101,1	3,7898E+11	3,269E+11	3,0403E+11	3,4025E+11	3,5218E+11	3,3931E+11	3,4846E+11
121,4	4,7959E+11	4,1134E+11	3,8156E+11	4,2878E+11	4,444E+11	4,2755E+11	4,3984E+11
145,7	5,9039E+11	5,0332E+11	4,6558E+11	5,2548E+11	5,4539E+11	5,2392E+11	5,4E+11
175	7,1321E+11	6,0407E+11	5,5712E+11	6,3175E+11	6,5666E+11	6,2979E+11	6,5051E+11
210,1	8,4859E+11	7,1372E+11	6,5619E+11	7,4779E+11	7,7853E+11	7,4537E+11	7,7172E+11
252,3	9,9841E+11	8,3347E+11	7,6371E+11	8,7494E+11	9,1248E+11	8,7199E+11	9,0521E+11
302,9	1,164E+12	9,6392E+11	8,801E+11	1,014E+12	1,0594E+12	1,0104E+12	1,052E+12
770	2,331E+12	1,8348E+12	1,6391E+12	1,9551E+12	2,0665E+12	1,9465E+12	2,0708E+12
1000	2,7857E+12	2,1543E+12	1,9104E+12	2,3056E+12	2,4468E+12	2,2947E+12	2,4629E+12

Comparing the Average DM mass placed in the last column with the others columns, it can be checked that ,relatives differences between values of mass at column C at 252 kpc and Average mass are under 1% . See green columns.

So the most suitable parameter C is at 252 kpc C= -3,7771E-24.

In my opinion, this is the way to get the best parameter C for M31 galaxy. $C_{M31} = -3,777*10^{-24}$ Because there is no way to know which experimental data is more reliable, so taking the average dark matter mass function it will possible to reduce errors.

P.6 STUDYING CASE WHEN C = 0

Now It will be investigated the conditions to get C= 0. Then formula $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ leds to

$E_0^{\frac{-2}{3}} = D \cdot R_0 = a^{\frac{-4}{3}} \cdot R_0$ and as $E = a^2 \cdot r^{2b-1}$ then $E_0^{\frac{-2}{3}} = a^{\frac{-4}{3}} \cdot R_0^{\frac{2-4b}{3}} = D * R_0 = a^{\frac{-4}{3}} \cdot R_0$ and by equation of power of R_0 $\frac{2-4b}{3} = 1$ it is got b= -1/4

At the beginning of chapter was shown that B= 5/3 leds rightly to b = -1/4. So b= -1/4 is rigorously the power of radius on the rotation curve of galaxy in the halo region, where there is not any baryonic matter. Namely formula is $V = a*r^{-0.25}$. Therefore C= 0 for every point belonging to regression curve whose power is -1 /4.

In the epigraph 9.7 it will be shown that for C= 0 the Bernoulli solution for field becomes direct formula for field, and the same happens with Bernoulli DM density and mass formulas.

9.6 GETTING DIRECT FORMULAS BY BERNOULLI FIELD WHEN PARAMETER C = 0

Thanks demonstration made in previous epigraph it is trustworthy to consider C= 0 in halo región.

FOR FIELD E

When in formula $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}}$ C= 0 then it is got $E = a^2 \cdot r^{\frac{-3}{2}}$ being $a^2 = 2.235*10^{21}$ which is precisely direct formula for E.

FOR DM DENSITY

As $D_{DM} = A*E^B$ Using field got by Bernoulli solution it is right to get

$$D_{DM}(r) = A\left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-5}{2}}$$

Being **A=** $3,488*10^{-6}$ **D**=$5,85*10^{-15}$ if **C** = 0 then formula becomes

$$D_{DM}(r) = A \cdot D^{\frac{-5}{2}} \cdot r^{\frac{-5}{2}} = L \cdot r^{\frac{-5}{2}}$$

being $L = \frac{a^2}{8 \cdot \pi \cdot G} = 1.3326*10^{30}$ which is direct DM density formula.

FOR TOTAL MASS INSIDE A SPHERICAL CORONA

$$M_{DM} = \int_{R1}^{R2} 4\pi r^2 \cdot \rho(r) dr = \int_{R1}^{R2} 4\pi r^2 A E^B dr = 4\pi A \int_{R1}^{R2} r^2 [D \cdot r]^{\frac{-5}{2}} \cdot dr$$

whose indefinite integral is $M(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$

which is direct mass $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ Being $\frac{a^2}{G} = 3.349 \cdot 10^{31}$.

Such formula is only right for radius belonging to halo. Therefore it is only possible to calculate the DM inside a spherical corona defined by two radius R_1 and R_2 so $R_1 < M_{DM} < R_2 = \frac{a^2}{G} \cdot \left[\sqrt{R_2} - \sqrt{R_1}\right]$

10. MASSES IN M31

In this chapter, It will be calculated some different types of masses related to M31.

10.1 DYNAMICAL MASSES, DIRECT MASS AND DYNAMICAL FITTED MASSES

10.1.1 DYNAMICAL MASSES VERSUS DIRECT MASS

As it is known, dynamical mass represents the mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius.

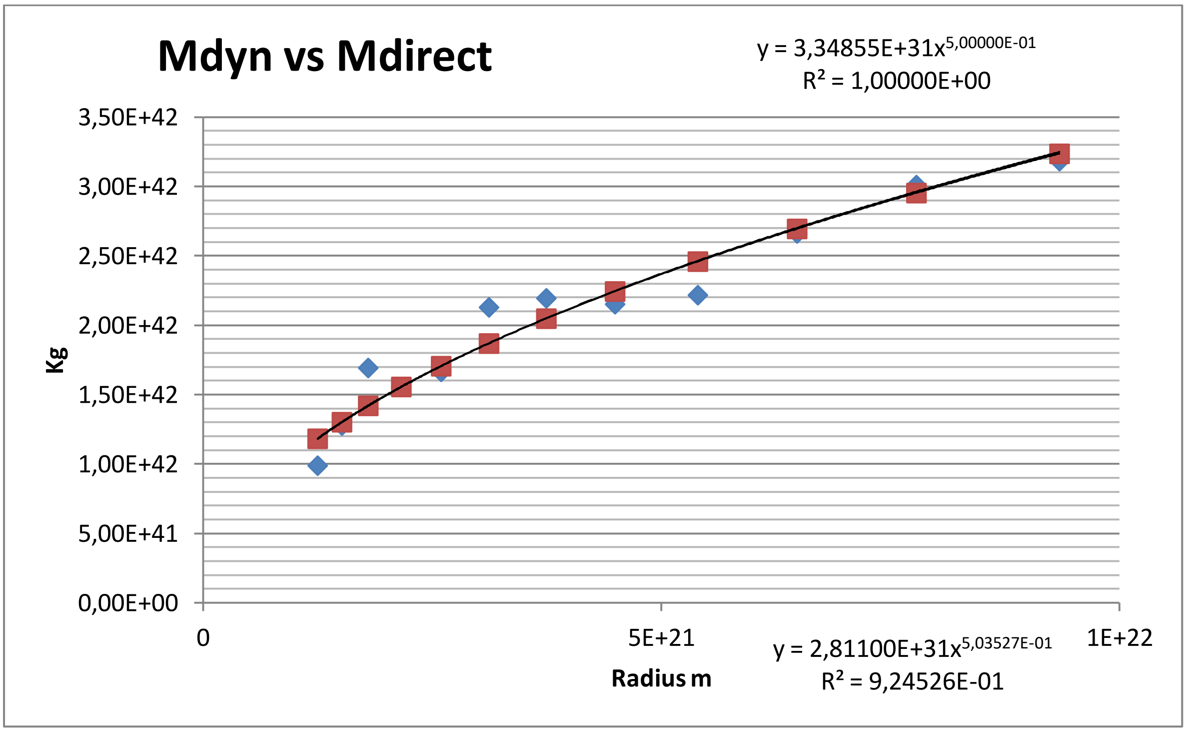
The formula of dynamical mass is $M_{DYN}(<r) = \frac{V^2 \cdot r}{G}$. and $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3{,}35.10^{31}$

			Dyn Mass	Direct mass	Rel diff
kpc	m	m/s	Msun	Msun	%
40,5	1,250E+21	2,299E+05	4,974E+11	5,95E+11	1,639E+01
49,1	1,515E+21	2,374E+05	6,429E+11	6,55E+11	1,849E+00
58,4	1,802E+21	2,505E+05	8,514E+11	7,14E+11	-1,919E+01
70,1	2,163E+21	2,192E+05	7,825E+11	7,83E+11	1,419E-02
84,2	2,598E+21	2,069E+05	8,373E+11	8,58E+11	2,373E+00
101,1	3,120E+21	2,135E+05	1,071E+12	9,40E+11	-1,392E+01
121,4	3,746E+21	1,978E+05	1,103E+12	1,03E+12	-7,143E+00
145,7	4,496E+21	1,788E+05	1,082E+12	1,13E+12	4,087E+00
175	5,400E+21	1,656E+05	1,115E+12	1,24E+12	9,833E+00
210,1	6,483E+21	1,656E+05	1,339E+12	1,35E+12	1,204E+00
252,3	7,785E+21	1,607E+05	1,514E+12	1,48E+12	-1,951E+00
302,9	9,347E+21	1,508E+05	1,600E+12	1,63E+12	1,629E+00

In the fifth column is tabulated the direct masses in order to be compared with dynamical masses.

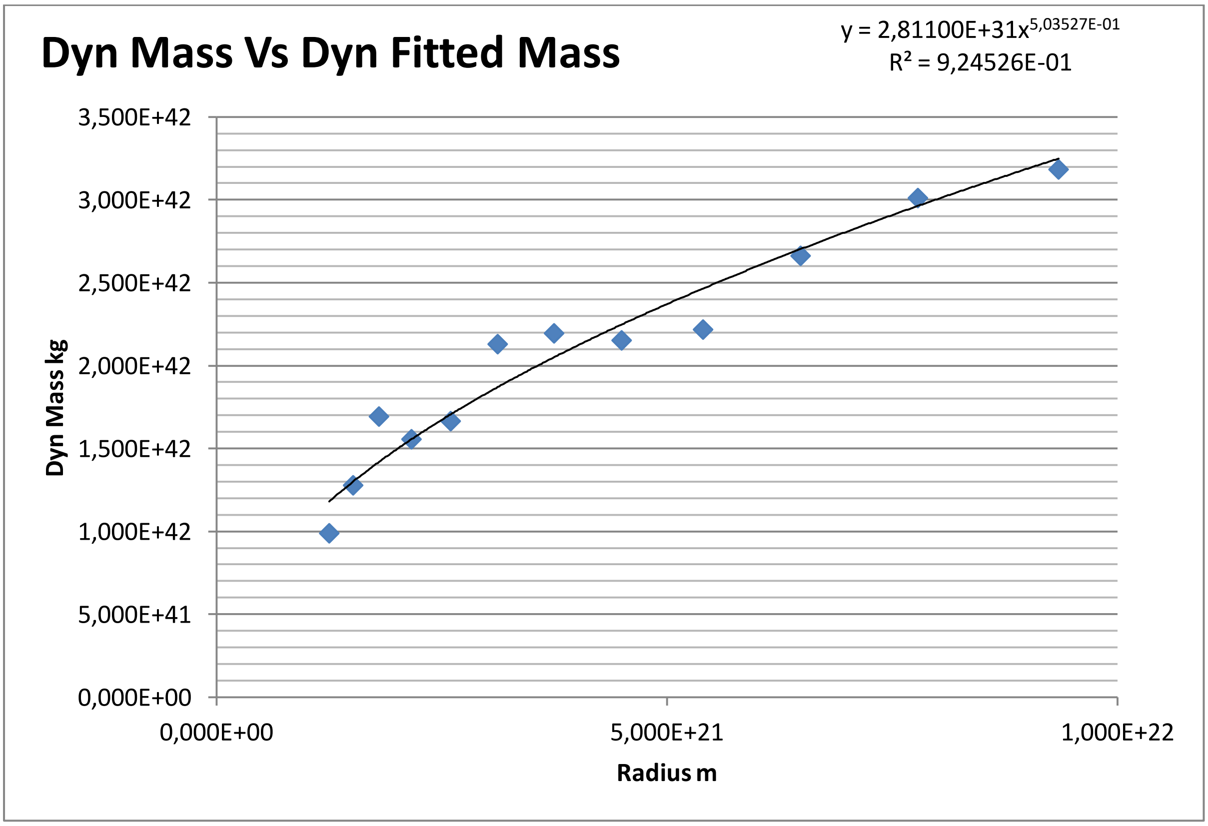
Below in the graph are plotted both functions, blue points are dynamical masses and brown point are direct masses.

The first and third points have the maximum difference regarding fitted curve whereas relative differences decreased as radius increased.



10.1.2 DIRECT MASS VERSUS DYNAMICAL FITTED MASS

Below is plotted dynamical mass. In the table above this paragraph, data are in the grey column. Below in the graph is written the dynamical fitted mass formula as well.



	Direct mass	DynFitted	Relative Diff

kpc	Msun		
40,5	5,95E+11	5,928E+11	-3,681E-01
49,1	6,55E+11	6,530E+11	-3,000E-01
58,4	7,14E+11	7,126E+11	-2,387E-01
70,1	7,83E+11	7,812E+11	-1,742E-01
84,2	8,58E+11	8,568E+11	-1,094E-01
101,1	9,40E+11	9,395E+11	-4,481E-02
121,4	1,03E+12	1,030E+12	1,969E-02
145,7	1,13E+12	1,129E+12	8,403E-02
175	1,24E+12	1,238E+12	1,486E-01
210,1	1,35E+12	1,358E+12	2,129E-01
252,3	1,48E+12	1,489E+12	2,773E-01
302,9	1,63E+12	1,632E+12	3,416E-01

$M_{DYN}^{FIT}(<r) = \frac{v^2 \cdot R}{G} = \frac{\alpha^2 \cdot r^\beta}{G}$ being $\frac{\alpha^2 \cdot}{G} = 2{,}811*10^{31}$ and $\beta = 0{,}503527$ This function of mass is lightly different regarding $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ Being $\frac{a^2}{G} = 3.349 \cdot 10^{31}$.

Because parameter **a** was recalculated using exponent b = - 1/4. See epigraph 9.1.

Relative differences are below 0,4 % throughout dominion, so hereafter it will be used Direct mass instead Fitted dynamical mass function in order to approximate dynamical fitted mass function. For example the dynamical mass at 40,5 kpc is $M_{DYN}(<40{,}5kpc) = 5{,}95*10^{11} Msun$ which may be considerate as the total mass of M31 at radius 40,5 kpc, despite the fact that $M_{MEASURED}(<40{,}5kpc) = 4{,}97*10^{11} Msun$ and its relative difference is 16,4%. As it is well known, the first and third point of rotation curve have the biggest relative differences regarding the fitted function as it is seen at graphic.

Obviously, dominion of dynamic fitted mass is 40,5 kpc up to 303 kpc, as well as for direct mass, because parameter **a** was got with a statistical procedure at such dominion. However in following chapter will be shown that direct mass is a very good approximation for Bernoulli mass at halo region, even for cluster scale (3 Mpc).

10.2 BERNOULLI MASS VERSUS DIRECT MASS

Below are both function formulas.

$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3{,}35.10^{31}$. As was pointed in previous paragraph, will be used to calculate dynamical mass $M_{DYN}(<40{,}5kpc) = 5{,}95*10^{11} Msun$ may be considerate the total mass of M31 at 40,5 kpc because direct mass is equivalent to dynamical fitted mass.

Hereafter $M_{DYN}(<40{,}5kpc) = 5{,}95*10^{11} Msun$ will be renamed M_{TOTAL} (<40,5kpc)

According the theory, carefully developed in previous chapters, Bernoulli mass formula is the primitive to calculate the dark matter in the halo region by the Barrow`s rule.

$$M(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

M31 PARAMETERS C & D
C_{M31} = -3,777*10^-24
$D_{M31} = a^{\frac{-4}{3}}$ = 5,85*10^-15

Despite the fact that Bernoulli mass function works only at halo region, at 40,5 kpc differs only 1,04 % regarding direct mass at 40,5 kpc.

Below are tabulated both function and its relative difference. It is remarkable that even at 3 Mpc its difference is only 4,35 %, despite the fact that its dominion has been extended 30 times¡

		Direct mass	Bernoulli mass	Rel diff
kpc	m	Msun	Msun	%
40,5	1,250E+21	5,949E+11	6,011E+11	1,04E+00
60	1,851E+21	7,240E+11	7,327E+11	1,19E+00
80	2,469E+21	8,361E+11	8,471E+11	1,31E+00
100	3,086E+21	9,347E+11	9,481E+11	1,41E+00
200	6,171E+21	1,322E+12	1,346E+12	1,77E+00
385	1,188E+22	1,834E+12	1,875E+12	2,20E+00
500	1,543E+22	2,090E+12	2,142E+12	2,40E+00
770	2,376E+22	2,594E+12	2,668E+12	2,77E+00
1000	3,086E+22	2,956E+12	3,048E+12	3,02E+00
1500	4,629E+22	3,620E+12	3,750E+12	3,46E+00
2000	6,171E+22	4,180E+12	4,346E+12	3,80E+00
3000	9,257E+22	5,120E+12	5,353E+12	4,35E+00

10.3 DM MASS OF SPHERICAL CORONA IN HALO BY BERNOULLI FORMULA AND TOTAL MASS

According theory developed, using Barrow`s rule to Bernoulli formula it will be calculated DM inside a spherical corona with Radius R1 and R2 belonging at halo region i.e. DM (R1,R2) = Bernoulli (R2)- Bernoulli(R1)

Being $M_{BER}(<r) = \dfrac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$ the Bernoulli formula for M31 whose parameters are:

M31 PARAMETERS C & D
$C_{M31} = -3{,}777*10^{-24}$
$D_{M31} = a^{\frac{-4}{3}} = 5{,}85*10^{-15}$

Therefore, if it is considerate R1 = 40,5 kpc then DM-Spherical Corona (R1,R) = M_{BER}(<R) (Cyan data)- 6,011E+11 (grey data) and total mass at R radius will be got adding M_{TOTAL} (<40,5kpc) = 5,95E+11 (pink data) to the DM inside spherical corona defined by 40,5 kpc and R (green data). See table below. Obviously the differences between total mass (yellow column) and Bernoulli mass (cyan column) are negligible because the difference between grey and pink values is 1% only.

Radius	Radius	Bernoulli Mass	DM-SphCorona	Total Mass M(<R)	Relative diff % Bernoulli Mass Versus Total Mass
kpc	m	Msun	M(R)-M<40,5	Msun	
40,5	1,250E+21	6,011E+11	0,000E+00	5,95E+11	1 %
60	1,851E+21	7,327E+11	1,316E+11	7,266E+11	8,4E-1

80	2,469E+21	8,471E+11	2,460E+11	8,410E+11	7,2E-1
100	3,086E+21	9,481E+11	3,470E+11	9,420E+11	6,4E-1
200	6,171E+21	1,346E+12	7,446E+11	1,340E+12	4,5E-1
385	1,188E+22	1,875E+12	1,274E+12	1,869E+12	3,3E-1
500	1,543E+22	2,142E+12	1,540E+12	2,135E+12	2,9E-1
770	2,376E+22	2,668E+12	2,067E+12	2,662E+12	2,3E-1
1000	3,086E+22	3,048E+12	2,447E+12	3,042E+12	2E-1
1500	4,629E+22	3,750E+12	3,149E+12	3,744E+12	1,6E-1
2000	6,171E+22	4,346E+12	3,744E+12	4,339E+12	1,4E-1
3000	9,257E+22	5,353E+12	4,751E+12	5,346E+12	1,1E-1

In conclusion, Bernoulli mass formula may be considerate as a very good estimation for total mass at a specific radius R belonging to halo region i.e. for R > 40,5 kpc as according the theory the halo region is unlimited.

Hereafter Bernoulli mass function will be considerate as synonymous of total mass at a specific radius.

11. DARK GRAVITATION THEORY IN MILKY WAY

In [17] Abarca M.2019. was introduced the extended halo of M31 whose radius was defined as the distance between M31 and Milky Way. Similarly the extended halo for Milky Way is the same distance. The reason to consider this extension is backed by the concept of DM nature according with theory of gravitational field as generator of DM.

This concept will be discussed widely at the end of the chapter.

11. 1 ROTATION CURVE OF MILKY WAY BY SOFUE 2020 DATA

This table of rotation curve of Milky Way comes from [2] Sofue.

kpc	km/s	radius m	vel m/s
30,448	229,6	9,40E+20	229600
33,493	222,5	1,03E+21	222500
36,842	215	1,14E+21	215000
40,527	207,1	1,25E+21	207100
44,579	200,3	1,38E+21	200300
49,037	194,7	1,51E+21	194700
53,941	189,8	1,66E+21	189800
59,335	186,2	1,83E+21	186200
65,268	184,7	2,01E+21	184700
71,795	183,9	2,22E+21	183900
78,975	181,4	2,44E+21	181400
86,872	175,5	2,68E+21	175500
95,56	167,7	2,95E+21	167700

This new set of Sofue data is very important for the theory of Dark gravitation theory because gives a rotation curve at halo region with a power for radius very close to -1/4 which is the same for M31. This fact back strongly the hypothesis of this theory.

11.2 FITTED FUNCTION VELOCITY VERSUS RADIUS AND PARAMETER a AT HALO REGION

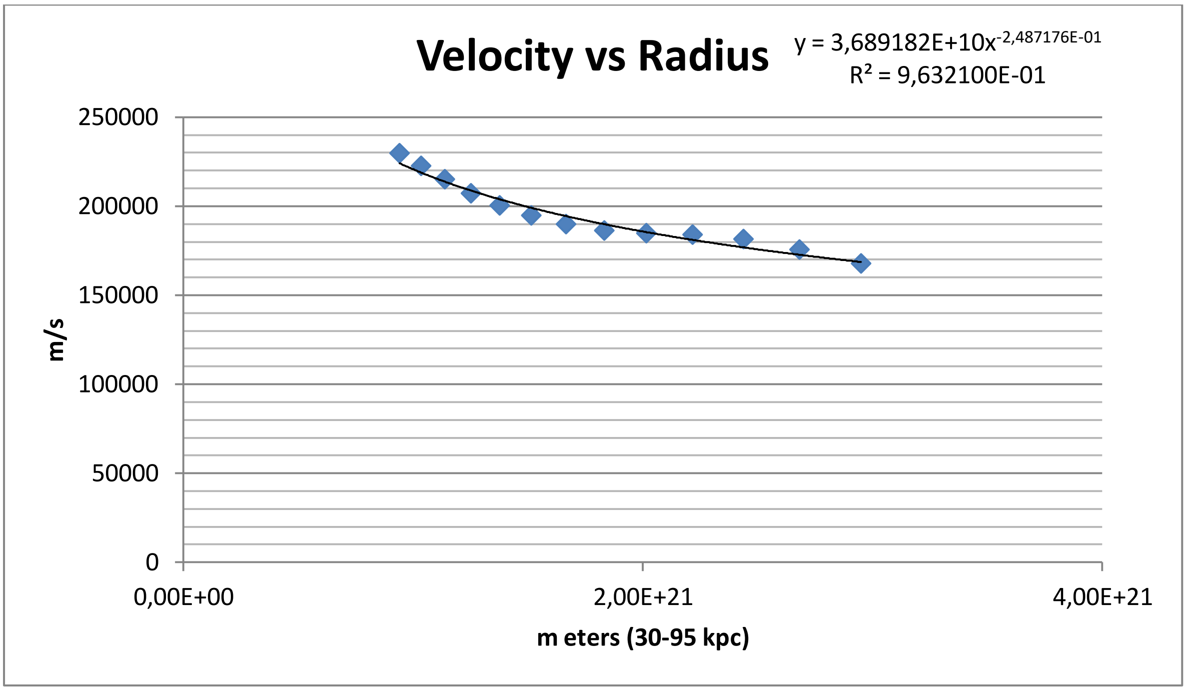
As it is known, Bernoulli solution is only right into the halo region, where the baryonic mass is negligible.
So it is needed to consider a radius where the ratio baryonic matter versus DM density is negligible. To calculate baryonic volume density has been used model provided by Sofue [6] for baryonic disc.

See pg 12. Table 3 below. Parameter for baryonic matter at disc in Milky Way.

Table 3. Best-fit parameters of the direct SMD by NFW halo and exponential disk.

Component	Parameter	Fitted Value	χ^2
Expo. disk	a_d	4.38 ± 0.35 kpc	
	Σ_0	$(1.28 \pm 0.09) \times 10^3 M_{\odot}\text{pc}^{-2}$	

With this parameters is possible to calculate baryonic volume density at 30,5 kpc $D_{BARYONIC}$(30,5Kpc) = 1,5E-24 kg/m^3. With the Direct DM density D_{DM}(30,5 Kpc) = 3,3E-23 kg/m^3. So ratio Baryonic versus DM at 30,5 kpc is 0,045. Therefore at 30,5 kpc is possible to considerate negligible Baryonic matter.



According the statistical procedure $v=a*R^b$ Being a = 3.68918E+10 and b = - 0.248717

Using Buckingham theorem it has been stated b= - 1/4 donc it is needed recalculated parameter **a** through the formula as it was made with M31 rotation curve.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}}$$

In table below the fourth column is calculated the numerator values. In the fifth column are denominator values. At the bottom is found parameter **a** optimal **a**=3,90787373E+10

Which is lightly bigger compared with which one associated to b= - 0.248717

kpc	m	m/s	Parameter **a** optimal when b=-1/4	
30,448	9,40E+20	229600	1,31E+00	3,26245E-11
33,493	1,03E+21	222500	1,24E+00	3,11061E-11
36,842	1,14E+21	215000	1,17E+00	2,96587E-11
40,527	1,25E+21	207100	1,10E+00	2,82781E-11
44,579	1,38E+21	200300	1,04E+00	2,69624E-11
49,037	1,51E+21	194700	9,87E-01	2,57076E-11
53,941	1,66E+21	189800	9,40E-01	2,45111E-11
59,335	1,83E+21	186200	9,00E-01	2,33705E-11
65,268	2,01E+21	184700	8,72E-01	2,2283E-11
71,795	2,22E+21	183900	8,48E-01	2,12459E-11
78,975	2,44E+21	181400	8,16E-01	2,02571E-11
86,872	2,68E+21	175500	7,71E-01	1,93145E-11
95,56	2,95E+21	167700	7,20E-01	1,84156E-11
		a optimal	3,90787373E+10	with b= -1/4
	D= a^(-4/3)	D parameter	7,54072E-15	

According the theory of dark matter by gravitation, each galaxy has two parameters C and D.

Parameter D is similar for similar galaxies, for example D= 5.85E-15 for M31.

New parameters a&b - A&B for Milky Way	
B	5/3
$b = \dfrac{B-2}{2B-2}$	b = -1/4
a optimal	$3{,}90787373 * 10^{10}$
$A = \dfrac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A $4{,}496262 * 10^{-6}$
$D = 8\pi G A = a^{\frac{-4}{3}}$	$7{,}54 * 10^{-15}$

Dark matter by gravitation theory stated that B has to be the same for all galaxies. However parameter **a** depend on each galaxy because it depend on baryonic matter enclosed by the galaxy.

11.3 PARAMETER C FOR MILKY WAY

C is the initial condition associated to a differential equation for the gravitational field in the galaxy whose formula depend on D and Eo.

$$C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

D is a global parameter because it has been calculated statistically, but Eo is the field at a specific radius Ro, so Eo is very dependent of error measure associated at such radius. In table below, it will be calculated parameter C for each data point because each one may be considerate as a initial condition.

This is the reason why in the following epigraph will be calculated Bernoulli mass for every parameter C and afterwards will be calculated the average Bernoulli mass function in order to calculate its parameter C associated.

Point	kpc	m	m/s	Eo v^2/R	Parameter C
1	30,448	9,40E+20	229600	5,61E-11	-2,8439E-23
2	33,493	1,03E+21	222500	4,79E-11	-2,0251E-23
3	36,842	1,14E+21	215000	4,07E-11	-9,7430E-24
4	40,527	1,25E+21	207100	3,43E-11	3,2002E-24
5	44,579	1,38E+21	200300	2,92E-11	1,1840E-23
6	49,037	1,51E+21	194700	2,51E-11	1,5517E-23
7	53,941	1,66E+21	189800	2,16E-11	1,6475E-23
8	59,335	1,83E+21	186200	1,89E-11	1,2028E-23
9	65,268	2,01E+21	184700	1,69E-11	-9,8889E-25
10	71,795	2,22E+21	183900	1,53E-11	-1,5770E-23
11	78,975	2,44E+21	181400	1,35E-11	-2,2598E-23
12	86,872	2,68E+21	175500	1,15E-11	-1,5434E-23
13	95,56	2,95E+21	167700	9,54E-12	-2,1145E-36

11.4 FITTING PARAMETER C BY BERNOULLI MASS FORMULA

Below are tabulated the different Bernoulli masses for each parameter C calculated in previous epigraph. The radius dominion extend up to 3 Mpc because according Dark matter by gravitation theory, this phenomenon is associated to gravitational field, so its dominion is unlimited.

$$M_{BER}(r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

There are13 parameters C. The column in cyan colour is the average mass function. The column in yellow shows the Bernoulli mass with a fitted parameter C in order to minimize the relative difference between the average mass function and Bernoulli mass function with a parameter C fitted. C= -5E-24 . The last column shows that relative differences are below 0,1%

Radius kpc	C at 30,4 kpc	C at 33,5 kpc	C at 36,8 kpc	C at 40,5kpc	C at 44,5 kpc	C at 49 kpc	C at 54 kpc	C at 59kpc
30,448	3,729E+11	3,668E+11	3,59E+11	3,502E+11	3,445E+11	3,420E+11	3,414E+11	3,443E+11
33,493	3,918E+11	3,852E+11	3,77E+11	3,673E+11	3,610E+11	3,584E+11	3,577E+11	3,609E+11
36,842	4,117E+11	4,046E+11	3,96E+11	3,851E+11	3,783E+11	3,755E+11	3,748E+11	3,782E+11
40,527	4,327E+11	4,249E+11	4,15E+11	4,038E+11	3,965E+11	3,934E+11	3,927E+11	3,963E+11
44,579	4,548E+11	4,463E+11	4,36E+11	4,234E+11	4,155E+11	4,122E+11	4,114E+11	4,153E+11
49,037	4,780E+11	4,688E+11	4,57E+11	4,440E+11	4,354E+11	4,319E+11	4,309E+11	4,353E+11
53,941	5,024E+11	4,924E+11	4,80E+11	4,656E+11	4,563E+11	4,524E+11	4,514E+11	4,561E+11
59,335	5,281E+11	5,173E+11	5,04E+11	4,882E+11	4,781E+11	4,740E+11	4,729E+11	4,779E+11
65,268	5,552E+11	5,434E+11	5,29E+11	5,119E+11	5,010E+11	4,965E+11	4,954E+11	5,008E+11
71,795	5,837E+11	5,709E+11	5,55E+11	5,367E+11	5,250E+11	5,201E+11	5,189E+11	5,247E+11
78,975	6,137E+11	5,998E+11	5,83E+11	5,628E+11	5,501E+11	5,448E+11	5,435E+11	5,498E+11

86,872	6,453E+11	6,302E+11	6,12E+11	5,901E+11	5,763E+11	5,707E+11	5,692E+11	5,761E+11
95,56	6,787E+11	6,622E+11	6,42E+11	6,187E+11	6,039E+11	5,977E+11	5,961E+11	6,035E+11
770	2,105E+12	1,999E+12	1,88E+12	1,740E+12	1,659E+12	1,626E+12	1,618E+12	1,657E+12
1000	2,440E+12	2,305E+12	2,15E+12	1,980E+12	1,879E+12	1,839E+12	1,829E+12	1,877E+12
2000	3,639E+12	3,380E+12	3,09E+12	2,786E+12	2,610E+12	2,540E+12	2,523E+12	2,606E+12
3000	4,631E+12	4,249E+12	3,83E+12	3,400E+12	3,156E+12	3,061E+12	3,037E+12	3,151E+12

C at 65kpc	C at 72kpc	C at 79 kpc	C at 87 kpc	C at 95,5 kpc	Average mass	Fitted C C= -5E-24	Relative Diff %
3,531E+11	3,635E+11	3,685E+11	3,633E+11	3,524E+11	3,556E+11	3,55896E+11	0,0920024
3,704E+11	3,817E+11	3,871E+11	3,814E+11	3,696E+11	3,730E+11	3,73386E+11	0,09259705
3,885E+11	4,007E+11	4,066E+11	4,004E+11	3,877E+11	3,914E+11	3,91737E+11	0,09305282
4,075E+11	4,207E+11	4,271E+11	4,204E+11	4,066E+11	4,106E+11	4,11E+11	0,09335478
4,274E+11	4,418E+11	4,487E+11	4,414E+11	4,264E+11	4,308E+11	4,31207E+11	0,09348646
4,483E+11	4,639E+11	4,714E+11	4,635E+11	4,473E+11	4,520E+11	4,52417E+11	0,0934304
4,702E+11	4,871E+11	4,952E+11	4,867E+11	4,691E+11	4,742E+11	4,74677E+11	0,09316754
4,932E+11	5,115E+11	5,203E+11	5,111E+11	4,920E+11	4,976E+11	4,98036E+11	0,09267747
5,173E+11	5,371E+11	5,467E+11	5,367E+11	5,160E+11	5,221E+11	5,22549E+11	0,09193812
5,426E+11	5,641E+11	5,745E+11	5,636E+11	5,412E+11	5,478E+11	5,48279E+11	0,09092555
5,691E+11	5,924E+11	6,038E+11	5,919E+11	5,676E+11	5,748E+11	5,75285E+11	0,08961411
5,969E+11	6,222E+11	6,345E+11	6,216E+11	5,953E+11	6,031E+11	6,03625E+11	0,08797638
6,261E+11	6,535E+11	6,669E+11	6,529E+11	6,244E+11	6,328E+11	6,33375E+11	0,08598213
1,782E+12	1,945E+12	2,029E+12	1,941E+12	1,772E+12	1,827E+12	1,82424E+12	-0,1464653
2,032E+12	2,236E+12	2,342E+12	2,231E+12	2,020E+12	2,089E+12	2,08444E+12	-0,22487616
2,879E+12	3,251E+12	3,451E+12	3,242E+12	2,856E+12	2,989E+12	2,97243E+12	-0,54427538
3,530E+12	4,061E+12	4,353E+12	4,047E+12	3,498E+12	3,693E+12	3,66183E+12	-0,83889186

In conclusion, parameter C = -5E-24 is the value that fit better the average mass of the other 13 Bernoulli mass functions.

So the parameters for Milky Way are D = $7{,}54 \cdot 10^{-15}$ and C= $-5 \cdot 10^{-24}$

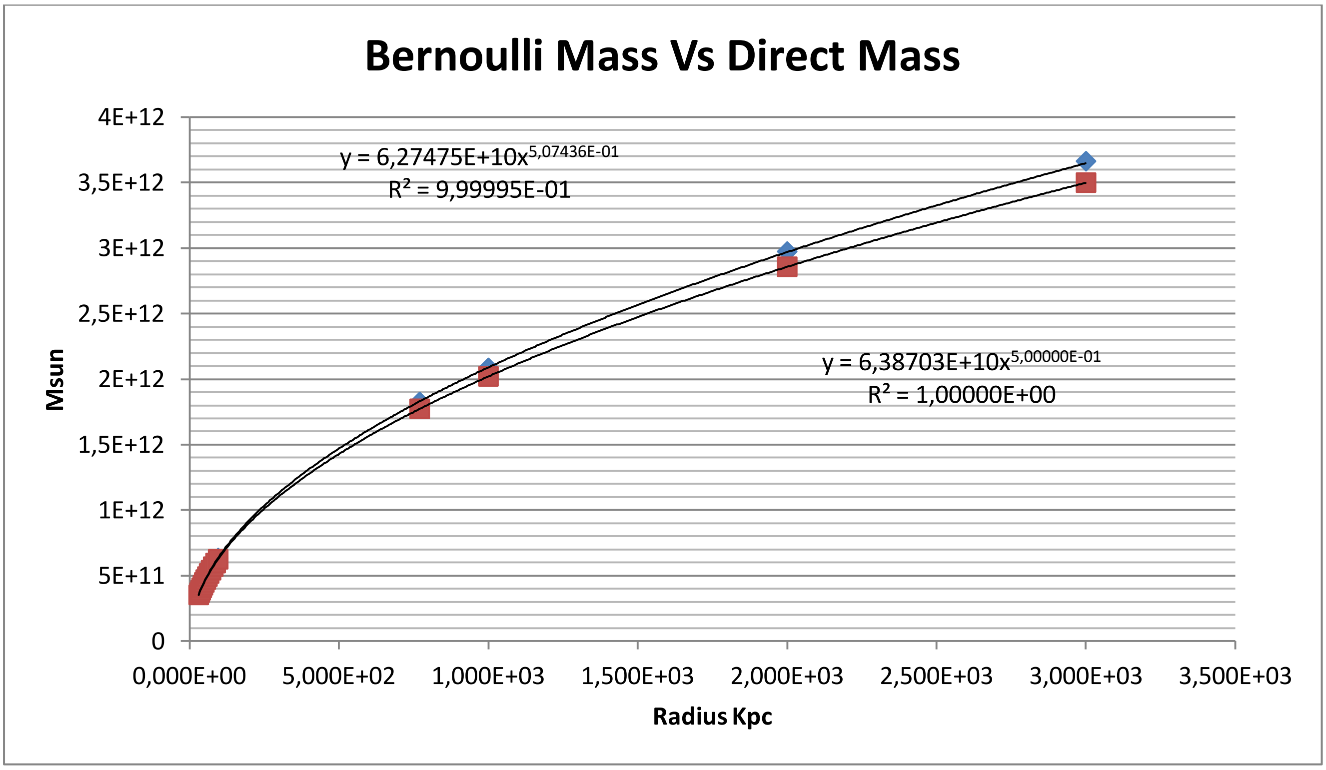
Parameter for Milky Way
C= $-5 \cdot 10^{-24}$
D = $7{,}54 \cdot 10^{-15}$

11.5 BERNOULLI MASS FORMULA VERSUS DIRECT MASS

Below are tabulated and compared both types of formulas for masses. As it was expected Bernoulli mass is a bit bigger than the other one, but even at 3 Mpc the relative difference is about 4%.

Radius kpc	Radius m	Bern Mass	Direct Mas	Rel diff %
3,045E+01	9,40E+20	3,55896E+11	3,52E+11	0,9725524
3,349E+01	1,03E+21	3,73386E+11	3,70E+11	1,00389568
3,684E+01	1,14E+21	3,91737E+11	3,88E+11	1,03624245

4,053E+01	1,25E+21	4,11E+11	4,07E+11	1,06963952
4,458E+01	1,38E+21	4,31207E+11	4,26E+11	1,10409792
4,904E+01	1,51E+21	4,52417E+11	4,47E+11	1,13967098
5,394E+01	1,66E+21	4,74677E+11	4,69E+11	1,17638927
5,934E+01	1,83E+21	4,98036E+11	4,92E+11	1,2142852
6,527E+01	2,01E+21	5,22549E+11	5,16E+11	1,25339682
7,180E+01	2,22E+21	5,48279E+11	5,41E+11	1,29376995
7,898E+01	2,44E+21	5,75285E+11	5,68E+11	1,33544217
8,687E+01	2,68E+21	6,03625E+11	5,95E+11	1,37844797
9,556E+01	2,95E+21	6,33375E+11	6,24E+11	1,42284204
7,700E+02	2,38E+22	1,82424E+12	1,77E+12	2,84565461
1,000E+03	3,09E+22	2,08444E+12	2,02E+12	3,10333024
2,000E+03	6,17E+22	2,97243E+12	2,86E+12	3,90459723
3,000E+03	9,26E+22	3,66183E+12	3,50E+12	4,46534223



12 THE MASS CALCULUS FOR THE LOCAL GROUP OF GALAXIES

According [5] Sofue the relative velocity between M31 and Milky Way is 170 km/s. Assuming that both galaxies are bounded gravitationally it is possible to calculate the total mass of the Local group by a simple formula because of the Virial theorem.

$M = \frac{v^2 \cdot r}{G}$ As r = 770 kpc and v= 170 km/s then $M_{LOCAL\ GROUP}$ = 5.17 *10^{12} Msun

According [5] Sofue, the total mass of M31 and Milky Way is approximately 3*10^{12} Msun, so there is a mass lack of 2*10^{12} Msun which is a considerable amount of matter. Namely read epigraph 4.6 of [5] Sofue paper.

Up to now, in order to do calculus with data of rotation curve, the border of M31 is right to be placed at a half the distance to Milky Way because it is supposed that up to such distance its gravitational field dominates whereas for bigger distances is Milky Way field which dominates.

This hypothesis is right when it is considered rotation curves of different systems bounded to each galaxy i.e. stars or dwarf galaxies. However when it is considered the gravitational interaction between both giant galaxies it is needed to extend their haloes up to 770 kpc, because according Dark gravitation theory the phenomenon of Dark matter is linked to gravitational field, which is unlimited.

Therefore the M31 halo extend up to 770 kpc and reciprocally the Milky Way halo extend up to 770 kpc, when it is calculated the gravitational interaction between both galaxies.

12.1 AN ESTIMATION OF TOTAL MASS OF THE LOCAL GROUP AT DIFFERENT RADIUS

Here are the parameters of M31 and Milky Way got in previous chapters.
With them it is possible to estimate quite well the total mass of Local Group, at different radius.

M31 PARAMETERS C & D
C_{M31} = -3,777*10^{-24}
$D_{M31} = a^{\frac{-4}{3}}$ = 5,85*10^{-15}

Parameters for Milky Way
C= -5*10^{-24}
D = 7,54*10^{-15}

In table below is tabulated the mass of both galaxies at different radius.

The green row shows the mass of M31 and MW at 770 kpc. In other words, 1,8E12 Msun is the mass of MW that M31 feels, and 2,7E12 Msun is the mass of M31 that MW feels. Both together add 4,5E+12 Msun which is very close to 5,1E+12 which represents the dynamical mass of both galaxies according its current relative velocity.

As the Bernoulli mass have two parameter got by measures in rotation curves with experimental errors, it is right to understand that both calculus 4,5E+12Msun and 5,1E+12 Msun are equivalents, because its relative difference is 12% and experimental errors should be bigger.

Radius	Tot mass M31	Tot m MW	M31+MW
kpc	Msun	Msun	Msun
4,00E+01	5,974E+11	4,084E+11	1,006E+12

5,00E+01	6,684E+11	4,569E+11	1,125E+12
1,00E+02	9,481E+11	6,482E+11	1,596E+12
2,00E+02	1,346E+12	9,201E+11	2,266E+12
3,00E+02	1,652E+12	1,130E+12	2,782E+12
5,00E+02	2,142E+12	1,465E+12	3,606E+12
7,70E+02	2,668E+12	1,825E+12	4,492E+12
1,00E+03	3,048E+12	2,085E+12	5,133E+12
1,50E+03	3,750E+12	2,565E+12	6,315E+12
2,00E+03	4,345E+12	2,973E+12	7,318E+12
2,50E+03	4,873E+12	3,334E+12	8,207E+12
3,00E+03	5,352E+12	3,662E+12	9,015E+12

When it is considered the universal expansion with a Hubble constant H_o = 70 km/s/Mpc, the local expansion between M31 and MW should be 54 Km/s, therefore to justify an approach velocity equal to 170 km/s it is needed even more mass than 5E+12 Msun. However dark gravitation theory stated that the more distance between galaxies the more mass have both galaxies. For example the total mass if both galaxies would have at 1,5 Mpc distance, would be 6,3E+12Msun.

At 100 Km/s the time to traverse 730 kpc, from 1,5 Mpc to current distance 770 kpc, would be 7,1 Gy which is a half of the Universe age according current cosmology.

This calculus are only an estimation to shows that it is needed much more mass that 5E+12 Msun to justify the current approach velocity of M31 and MW. However Dark gravitation theory may support such extra amount of mass.

To study evolution of cluster dynamics it will be needed a model to consider the universal expansion form the ancient Universe to current universe taking in consideration the total DM associated to the gravitational field.

In chapter 11 of paper [2] Abarca,M.2019. it was made an estimation about the DM in the ancient Universe 9300 million years ago, when the Universe was decreased by 0,5 factor. As in the theory of DM by gravitation, the total DM is related to the distances, it was estimated that DM associated to galaxies was reduced a 30% regarding the current value.

It is sure that such studies will be a magnificent test to check the *Theory of dark matter by gravitation.*

12.2 RATIO BARYONIC MASS VERSUS TOTAL MASS IN THE LOCAL GROUP

No.] Dark Halo of M31 and the Galaxy 5

Table 2. The best fit dynamical parameters for M31 and the Galaxy†

Component	Parameter	M31	Milky Way
Bulge	a_b (kpc)	1.35 ± 0.02	0.87 ± 0.07
	$M_b(10^{11} M_\odot)$	0.35 ± 0.004	0.25 ± 0.02
Disk	a_d (kpc)	5.28 ± 0.25	5.73 ± 1.23
	$M_d(10^{11} M_\odot)$	1.26 ± 0.08	1.12 ± 0.40
NFW Halo	h (kpc)	34.6 ± 2.1	10.7 ± 2.9
	$\rho_0(10^{-3} M_\odot \mathrm{pc}^{-3})$	2.23 ± 0.24	18.2 ± 7.4
	$\rho_{8\ \mathrm{kpc}}(10^{-3} M_\odot \mathrm{pc}^{-3})$	6.36 ± 0.70	7.93 ± 3.24
	= (in energy density: GeV cm^{-3})	0.24 ± 0.03	0.30 ± 0.12
	$M_{h:200}(10^{11} M_\odot)$	12.3 ± 2.6	5.7 ± 5.1
	$M_{h:385}(10^{11} M_\odot)$	18.3 ± 3.9	7.3 ± 6.7
Total Mass	$M_{tot:200}(10^{11} M_\odot)$	13.9 ± 2.6	7.0 ± 5.1
	$M_{tot:385}(10^{11} M_\odot)$	19.9 ± 3.9	8.7 ± 5.1
Bulge	χ_b^2/N ($R_1 - R_2$ kpc)	0.36 (0.0-20.0)	3.5 (0.0-20.0)
Disk	χ_d^2/N ($R_1 - R_2$ kpc)	0.33 (0.0-40.0)	3.6 (0.0-40.0)
Halo	χ_h^2/N ($R_1 - R_2$ kpc)	0.25 (0.0-385.0)	3.0 (0.0-385.0)

† $M_{h:200}$, $M_{h:385}$,$M_{tot:200}$ and $M_{tot:385}$ are dark halo and total masses within $R = 200$ and 385 kpc, respectively; ρ_{8kpc} is a local value at $R = 8$ kpc both in mass and energy densities; R_1 and R_2 are start and end radii for fitting.

The table placed above come from Sofue,Y. S. 2015. Data from Milky Way related to its rotation curve has been updated in[6] Sofue,Y.2020. However M31 does not change.

Mb represents the mass of bulge and Md the mass of disc, so adding both masses results Baryonic Mass,

BM_{M31}= 1,6E+11 Msun and BM_{MW}= 1,4E11 Msun. So it is a very good estimation for total baryonic mass in the Local group $BM_{LOCAL\ GROUP}$ = 3E+11Msun

In the table below, the grey columns represent the Bernoulli mass function for both galaxies and its addition mass.

The cyan columns represent the ratio baryonic mass versus total mass at different radius for both galaxies and its addition mass function.

For example at 200 kpc the ratios are11,9 % for M31 and 15,2 % for MW and at 375 kpc ratios go down up to 8,6 % for M31 and 11,1 % for MW. Finally at 3 Mpc ratios are 3% for M31 and 3,8 % for MW.

Radius kpc	Total-Mass M31 Msun	Ratio M31 Baryon-Vs-Tot %	Total-Mass Milky Way Msun	Ratio MW Baryon-Vs-Tot %	Total-M31+MW Msun	Ratio M31+MW Baryon-Vs-Total %
40	5,974E+11	2,678E+01	4,084E+11	3,43E+01	1,006E+12	2,983E+01
50	6,684E+11	2,394E+01	4,569E+11	3,06E+01	1,125E+12	2,666E+01
100	9,481E+11	1,688E+01	6,482E+11	2,16E+01	1,596E+12	1,879E+01
200	1,346E+12	1,189E+01	9,201E+11	1,52E+01	2,266E+12	1,324E+01
375	1,850E+12	8,646E+00	1,265E+12	1,11E+01	3,116E+12	9,628E+00
500	2,142E+12	7,471E+00	1,465E+12	9,56E+00	3,606E+12	8,319E+00
770	2,668E+12	5,998E+00	1,825E+12	7,67E+00	4,492E+12	6,678E+00
1000	3,048E+12	5,249E+00	2,085E+12	6,72E+00	5,133E+12	5,845E+00
1500	3,750E+12	4,267E+00	2,565E+12	5,46E+00	6,315E+12	4,751E+00
2000	4,345E+12	3,682E+00	2,973E+12	4,71E+00	7,318E+12	4,099E+00
2500	4,873E+12	3,283E+00	3,334E+12	4,20E+00	8,207E+12	3,655E+00
3000	5,352E+12	2,989E+00	3,662E+12	3,82E+00	9,015E+12	3,328E+00

Also it is remarkable the coincidence between data Sofue for total mass at 200 kpc and 375 kpc and Bernoulli mass calculated at such distances for M31 galaxy. The coincidence is perfect when it is considered the error range.

M31 Galaxy	Total Mass at 200 kpc Msun	Total Mass at 375 kpc Msun
Sofue by NFW halo	13,9E+11	19,9E+11
Bernoulli Mass	13,46E+11	18,5E+11

12.3 RATIO BARYONIC MASS VERSUS TOTAL MASS DECREASE AT CLUSTER SCALE

Below is shown the ratio between baryonic mass and total mass at different radius. It is remarkable the fact that the more radius is considered the less ratio the baryonic versus total mass mass is.

Radius kpc	Total mass M31+MW Msun	MB/Mtotal %
4,00E+01	1,006E+12	2,983E+01
5,00E+01	1,125E+12	2,666E+01
1,00E+02	1,596E+12	1,879E+01
2,00E+02	2,266E+12	1,324E+01
3,00E+02	2,782E+12	1,078E+01
5,00E+02	3,606E+12	8,319E+00
7,70E+02	4,492E+12	6,678E+00
1,00E+03	5,133E+12	5,845E+00
1,50E+03	6,315E+12	4,751E+00
2,00E+03	7,318E+12	4,099E+00
2,50E+03	8,207E+12	3,655E+00
3,00E+03	9,015E+12	3,328E+00

It have been measured widely that at cluster scale the ratio of baryonic mass is lower than at galactic halo scale.

This table shows that Dark gravitation theory explain perfectly this fact.

For example at 300 kpc the ratio is 10,7% whereas at 3 Mpc the ratio go down to 3,3 %

The concept of unlimited halos applied to galaxy cluster may explain the reason why the proportion of DM inside a cluster is bigger than inside a galaxy. The gravitational interaction inside the intergalactic space produces an extra of DM.

For example Baryonic mass of Local Group is approximately $3*10^{11}$ Msun and total mass is $4,5*10^{12}$ Msun at 770 kpc so the proportion of baryonic mass versus total mass is 6,7% whereas such proportion was 8,6 % for M31 at 375 kpc and 11,1% for Milky Way at 375 kpc.

However for bigger scales the effect of DM growing is compensated by the Dark energy. See [9] Abarca. A study about Coma cluster. Some years ago was checked that super clusters are the biggest structures gravitationally bounded because the universal expansion dominates over gravitational forces between super clusters.

13. CONCLUSION

As it has been outlined at the introduction, this work is the consequence of the new set of data for rotation curve in Milky Way halo. With these new data, the fitted function of rotation curve at halo region of Milky Way has exactly the same exponent that fitted function associated to M31 galaxy. This fact back strongly the main hypothesis of Dark gravitation theory i.e. Dark matter is generated according an unknown quantum gravitational mechanism, which depend on the gravitational field, so is a universal law.

Through the firsts ten chapters it is developed the theory using M31 rotation curve. This chapters are identical to the previous paper [2] Abarca,M.2019, excepting the getting of parameter C , the initial condition of solution got for Bernoulli differential equation associated to field E. See chapter 9. This parameter associated to a specific point of rotation curve is quite problematic because of experimental errors. In this work the value got has been C = - 3,77E-24 whereas in the previous work was C = 6,36E-24. Both are little numbers, but with the value negative the field is lightly bigger and consequently DM density is a bit bigger.

However this parameter has a low influence in the result of DM calculus because the most important parameter for DM calculus is D. Therefore results of DM calculus in [2] Abarca,M.2019 are similar to results got in this paper.

The new results published in this work are in chapter 10 because there are calculated parameters C & D associated to Milky Way halo. With them is calculated total mass function in Milky Way halo and in the chapter 11 is calculated the mass associated to Local group of galaxies.

The mass of Local Group calculated by Dark matter by gravitation is $4{,}5*10^{12}$ Msun which is equivalent to dynamical mass $5*10^{12}$ Msun if it is considered the range of errors of experimental measures.

The current theories of dark matter give a mass for Local group about $3*10^{12}$ Msun which is very different to dynamical mass of Local Group.

In addition it is shown how the theory gives a total mass 5.1E+12Msun at 1Mpc of radius , 6.3E+12Msun at 1.5 Mpc, and 7.3E+12 Msun at 2 Mpc. In brief, that theory works quite well at cluster scale, as it was widely developed in previous paper [3] Abarca,M.2019 where it was studied dark matter in Coma cluster.

In my opinion, this theory can no be developed any more from Newtonian framework. It will be needed to considered General Relativity to introduce into the tensor energy T, the energy associated to dark matter.
A natural way to do it will be to consider the density of dark matter multiplied by c^2. This step will have to be done by some one expert in general relativity, obviously.

14. BIBLIOGRAPHYC REFERENCES

[1] Abarca,M.2014,viXra:1410.0200. *Dark matter model by quantum vacuum*

[2] Abarca,M.2019. viXra:1909.0594
A dark matter formula for Milky Way and M31 halo got by Buckingham theorem.V4

[3] Abarca,M.2019. viXra:1912.0233
Dark Gravitation Theory for M31 and Coma Cluster

[4] Abarca, M.2016.viXra. 1601.0014
A New Dark Matter Density Profile for M33 Galaxy to Demonstrate that Dark Matter is Generated by Gravitational Field

[5] Sofue, Y.2015. arXiv:1504.05368v1 *Dark halos of M31 and the Milky Way.*

[6] Sofue,Y.2020.MDPI/Galaxies
Rotation curve of the Milky Way and the dark matter density.

[7] Abarca, M.2016. viXra:1606.0007
A new Dark matter density profile for Milky Way which depend on gravitational field.

[8] Abarca,M.2016. viXra:1606.0279v1
Dark matter density on big galaxies depend on gravitational field as Universal law.

[9] Abarca,M.2016.vixra:1607.0427
A New Dark Matter Density Profile as Power of Gravitational Field for Coma Cluster.

[10] Abarca,M.2016.vixra:1601.0242
A New Dark Matter Density Profile for M31 Galaxy to Demonstrate that Dark Matter is Generated by Gravitational Field

[11] Nieuwenhuizen,T.M. 2010 .arXiv:1011.2530v1
Do micro brown dwarf detections explain the galactic dark matter?

[12] Nieuwenhuizen,T.M. 2012. arXiv:1210.0489v2
Do the Herschel cold clouds in the Galactic halo embody its dark matter?

[13] Nieuwenhuizen,T.M. 2010 arXiv:1003.0453v1
Gravitational hydrodynamics versus observations of voids, jeans clusters and MACHO dark matter.

[14] Wyrzykowski,L.2010 .arXiv:1012.1154v2
The OGLE View of Microlensing towards the Magellanic Clouds. III. Ruling out sub-solar MACHOs with the OGLE-III LMC data.

[15] Hawkins M.R.S. 2015. arXiv: 1503.01935v1
A new look at microlensing limits on dark matter in the Galactic halo.

[16] Alfred L. Tiley arXiv:1811.05982v3

The Shapes of the Rotation Curves of Star-forming Galaxies Over the Last 10 Gyr