Unification of Fundamental Forces

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April 16, 2023

Abstract

In this paper, I found a mathematical method to unify fundamental forces. In this method, without using any quantum gravity theories such as string theory and loop quantum gravity I created a field equation for the unification between fundamental forces. Also, I found out the acceleration equation of our universe.

1 Introduction

In mathematical methods in physics textbooks you can see the equation below:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha} \quad (1)$$

$U^{\alpha\beta}$is a tensor field. For unifying the forces I assume that the tensor field has four elements:

$$U^{\alpha\beta} = \eta G^{\alpha\beta} + \lambda F^{\alpha\beta} + \mu G^{\alpha\beta} + \gamma Z^\alpha W_\beta \quad (2)$$

As you see, the fundamental forces are tensor rank2. For unifying them we have to put a coefficient for each force to have the same degree of freedom. $Z^\alpha W_\beta$ is electro weak, $G^{\alpha\beta}$ is the gluon field, $F^{\alpha\beta}$ is the electromagnetism field, and $G^{\alpha\beta}$ is the gravitation field. This research explains the effects of fundamental forces on the curvature of our universe. Finally, I achieved an equation that describes all fundamental forces in one equation. Another point I would like to mention is that the acceleration of our universe affects the final equation which means fundamental forces have a great impact on the acceleration of the universe.
Final field equation:

$$(8\kappa^2(\mathrm{A}_\alpha - \frac{q}{m}F_{\lambda\alpha}\frac{dx^\lambda}{d\tau}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)U^{\alpha\beta} = 0 \quad (3)$$

The equation above is the modern version of F=ma. In fact, my model is generalized for gravity and quantum and classical physics.

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2 Proving Field Equation

Let's get started to do the math:

$$\mathcal{L} = \frac{1}{2\kappa}\sqrt{-g}\mathcal{R} = \sqrt{g_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}} \tag{4}$$

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha} \tag{5}$$

$$\begin{aligned}
&\tfrac{1}{\sqrt{g}}\partial_\alpha(\tfrac{2\kappa\mathcal{L}}{i\mathcal{R}})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha},\\
&\tfrac{\partial\mathcal{L}}{\partial\mathcal{X}^\alpha} = \tfrac{1}{2\mathcal{L}}\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau},\\
&\tfrac{2\kappa}{\sqrt{g}}(\tfrac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}}{-\mathcal{R}^2})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha},\\
&2\kappa(\tfrac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}}{-\mathcal{R}^2})U^{\alpha\beta} = -\sqrt{-g}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha},\\
&2\kappa(\tfrac{1}{2}\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \sqrt{-g}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{L}\mathcal{R}^2,\\
&\mathcal{L} = \tfrac{1}{2\kappa}\sqrt{-g}\mathcal{R} \to \sqrt{-g} = \tfrac{2\kappa\mathcal{L}}{\mathcal{R}},\\
&2\kappa(\tfrac{1}{2}\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{2\kappa\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2\\
&\nabla_\alpha g_{\alpha\beta} = 0\\
&\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha} - \Gamma^d_{\alpha\alpha}g_{d\beta} - \Gamma^d_{\alpha\beta}g_{\alpha d} = 0\\
&\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha} = \Gamma^d_{\alpha\alpha}g_{d\beta} + \Gamma^d_{\alpha\beta}g_{\alpha d}
\end{aligned} \tag{6}$$

$$\begin{aligned}
&(\tfrac{1}{2}(\Gamma^d_{\alpha\alpha}g_{d\beta} + \Gamma^d_{\alpha\beta}g_{\alpha d})\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2\\
&(\tfrac{1}{2}\Gamma^d_{\alpha\alpha}g_{d\beta}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} + \tfrac{1}{2}\Gamma^d_{\alpha\beta}g_{\alpha d}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\Gamma^d_{\alpha\beta}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau} = \tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d\\
&(\tfrac{1}{2}\Gamma^d_{\alpha\alpha}g_{d\beta}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} + \tfrac{1}{2}\tfrac{d^2x^d}{d\tau^2}g_{\alpha d}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2\\
&((\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \mathcal{L}^2U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}\\
&\mathcal{L}^2 = -\tfrac{1}{4\kappa^2}g\mathcal{R}^2
\end{aligned} \tag{8}$$

$$\begin{aligned}
&(4\kappa^2(\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d}\mathcal{R} + \tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}^2)U^{\alpha\beta} = -g\mathcal{R}^3U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\\
&\Gamma^\gamma{}_{\gamma\alpha} = \tfrac{1}{2g}\tfrac{\partial g}{\partial x^\alpha}\\
&(4\kappa^2(\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d}\mathcal{R} + \tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}^2)U^{\alpha\beta} = -g\mathcal{R}^3U^{\alpha\beta}\tfrac{1}{2g}\tfrac{\partial g}{\partial x^\alpha}\\
&(8\kappa^2(\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d} + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R})U^{\alpha\beta} + \tfrac{\partial g}{\partial x^\alpha}U^{\alpha\beta}\mathcal{R}^2 = 0\\
&U^{\alpha\beta} = \eta G^{\alpha\beta} + \lambda F^{\alpha\beta} + \mu G^{\alpha\beta} + \gamma Z^\alpha W_\beta\\
&\mathrm{A}^d = \tfrac{d^2x^d}{d\tau^2}\\
&(8\kappa^2(\mathrm{A}_\alpha + \mathrm{X}^d g_{\alpha d}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R})U^{\alpha\beta} + \tfrac{\partial g}{\partial x^\alpha}U^{\alpha\beta}\mathcal{R}^2 = 0
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\mathrm{X}^d = -\tfrac{q}{m}F^{d\gamma}\tfrac{dx^\lambda}{d\tau}g_{\lambda\gamma}\\
&(8\kappa^2(\mathrm{A}_\alpha - \tfrac{q}{m}F^{d\gamma}\tfrac{dx^\lambda}{d\tau}g_{\lambda\gamma}g_{\alpha d}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R})U^{\alpha\beta} + \tfrac{\partial g}{\partial x^\alpha}U^{\alpha\beta}\mathcal{R}^2 = 0
\end{aligned} \tag{10}$$

$$(8\kappa^2(\mathrm{A}_\alpha - \tfrac{q}{m}F_{\alpha\lambda}\tfrac{dx^\lambda}{d\tau}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \tfrac{\partial g}{\partial x^\alpha}\mathcal{R}^2)U^{\alpha\beta} = 0$$

$$(8\kappa^2(\mathrm{A}_\alpha - \tfrac{q}{m}F_{\alpha\lambda}\tfrac{dx^\lambda}{d\tau}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \tfrac{\partial g}{\partial x^\alpha}\mathcal{R}^2)(\eta G^{\alpha\beta} + \lambda F^{\alpha\beta} + \mu G^{\alpha\beta} + \gamma Z^\alpha W_\beta) = 0 \tag{11}$$

For finding the coefficients we have to use numerical methods or experiments.

3 One Example of The Equation Field

For example, I want to consider only the electromagnetic field in the equation above:

$$U^{\alpha\beta} = F^{\alpha\beta} \tag{12}$$

We will see a familiar equation. I have to mention that in this example I consider the flat space-time; however, we can calculate for the curved space-time, but for simplicity, I consider the flat space-time. We chose Beta as zero for finding an equation.

$$\begin{aligned}
&U^{\alpha\beta} = F^{\alpha\beta}\\
&\mathrm{A}^d = \frac{d^2x^d}{d\tau^2}\\
&\beta = 0\\
&8\kappa^2(\mathrm{A}_\alpha F^{\alpha\beta} - \frac{q}{m}F_{\alpha\lambda}\frac{dx^\lambda}{d\tau}F^{\alpha\beta}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}F^{\alpha\beta} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2F^{\alpha\beta} = 0\\
&8\kappa^2(\mathrm{A}_1F^{10} + \mathrm{A}_2F^{20} + \mathrm{A}_3F^{30} - \frac{q}{m}(F_{\alpha0}\frac{dx^0}{d\tau} + F_{\alpha1}\frac{dx^1}{d\tau} + F_{\alpha2}\frac{dx^2}{d\tau} + F_{\alpha3}\frac{dx^3}{d\tau})F^{\alpha0})\\
&+2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}F^{\alpha0} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2F^{\alpha0} = 0\\
&For flatspacetime\\
&2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}F^{\alpha0} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2F^{\alpha0} = 0
\end{aligned} \tag{13}$$

$$\begin{aligned}
&8\kappa^2(\mathrm{A}_1F^{10} + \mathrm{A}_2F^{20} + \mathrm{A}_3F^{30} - \frac{q}{m}(F_{\alpha0}F^{\alpha0}\frac{dx^0}{d\tau} + F_{\alpha1}F^{\alpha0}\frac{dx^1}{d\tau} + F_{\alpha2}F^{\alpha0}\frac{dx^2}{d\tau} + F_{\alpha3}F^{\alpha0}\frac{dx^3}{d\tau}) = 0\\
&F_{\alpha0}F^{\alpha0} = F_{10}F^{10} + F_{20}F^{20} + F_{30}F^{30} = -(E_x^2 + E_y^2 + E_z^2) = -E^2\\
&F_{\alpha1}F^{\alpha0} = F_{21}F^{20} + F_{31}F^{30} = B_zE_y - B_yE_z\\
&F_{\alpha2}F^{\alpha0} = F_{12}F^{10} + F_{32}F^{30} = -B_zE_x + B_xE_z\\
&F_{\alpha3}F^{\alpha0} = F_{13}F^{10} + F_{23}F^{20} = B_yE_x - B_xE_y\\
&\mathrm{A}_1F^{10} + \mathrm{A}_2F^{20} + \mathrm{A}_3F^{30} - \frac{q}{m}(F_{\alpha0}F^{\alpha0}\frac{dx^0}{d\tau} + F_{\alpha1}F^{\alpha0}\frac{dx^1}{d\tau} + F_{\alpha2}F^{\alpha0}\frac{dx^2}{d\tau} + F_{\alpha3}F^{\alpha0}\frac{dx^3}{d\tau}) = 0\\
&\vec{\mathrm{A}}.\vec{E} - \frac{q}{m}(\gamma E^2 + (B_zE_y - B_yE_z)\frac{dx^1}{d\tau} + (-B_zE_x + B_xE_z)\frac{dx^2}{d\tau} + (B_yE_x - B_xE_y)\frac{dx^3}{d\tau}) = 0\\
&\vec{\mathrm{A}}.\vec{E} - \frac{q}{m}(\gamma E^2 + (\vec{B}\times\vec{E}).\vec{v}) = 0
\end{aligned} \tag{14}$$

This is familiar $\vec{\mathrm{A}}.\vec{E} - \frac{q}{m}(\gamma E^2 + (\vec{B}\times\vec{E}).\vec{v}) = 0$ for us (Lorentz force equation).

4 Acceleration of Our Universe

For every diagonal metric and diagonal field tensor, such as the Robertson-walker metric and gravitational field, and considering for the gravity as only force in our equation we have:

$$U^{\alpha\beta} = G^{\alpha\beta} = 8\pi GT^{\alpha\beta} \tag{15}$$

$$\begin{aligned}
&(8\kappa^2(\mathrm{A}_\alpha - \frac{q}{m}F_{\alpha\lambda}\frac{dx^\lambda}{d\tau}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)8\pi GT_{\alpha\beta} = 0\\
&Assume:\\
&F_{\alpha\lambda} = 0\\
&(8\kappa^2\mathrm{A}_\alpha + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)T^{\alpha\beta} = 0\\
&8\kappa^2\mathrm{A}_\alpha + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2 = 0\\
&\mathrm{A}_\alpha = -\frac{1}{8\kappa^2}(2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)
\end{aligned} \tag{16}$$

The acceleration is positive because R in the terms is squared, and g is negative; hence, The acceleration is positive. Therefore, the acceleration of the universe is positive. this result is compatible with other cosmological assumptions.
If we want to find the accurate rate of acceleration expansion, we have to consider all fundamental forces in the equation above.

5 Conclusion

I do not feel like calculating field equations for all fundamental forces, and all kinds of metrics such as black holes and the universes, but you can calculate with the equation below.

$$(8\kappa^2(\mathrm{A}_\alpha - \frac{q}{m}F_{\lambda\alpha}\frac{dx^\lambda}{d\tau}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)U^{\alpha\beta} = 0 \tag{17}$$