Unification of fundamental forces

Mohammad Hadi Mohammadi [*]*Department of Physics , Faculty of Science , Bu-Ali Sina University, Hamedan, Iran*

April 8, 2023

Abstract

In this paper, I found a mathematical method to unify fundamental forces. In this method, without using any quantum gravity theories such as string theory and loop quantum gravity I created a field equation for the unification between fundamental forces. Also, I found out the acceleration equation of our universe.

1 Introduction

In mathematical methods in physics textbooks you can see the equation below:

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^{\gamma}{}_{\gamma\alpha} \tag{1}$$

$U^{\alpha\beta}$is a tensor field. For unifying the forces I assume that the tensor field has four elements:

$$U^{\alpha\beta} = \eta G^{\alpha\beta} + \lambda F^{\alpha\beta} + \mu G^{\alpha\beta} + \gamma Z^\alpha W_\beta \tag{2}$$

As you see, the fundamental forces are tensor rank2. For unifying them we have to put a coefficient for each force to have the same degree of freedom. $Z^\alpha W_\beta$ is electroweak,$G^{\alpha\beta}$ is the gluon field, is the electromagnetism field, and is the gravitation field. This research explains the effects of fundamental forces on the curvature of our universe. Finally, I achieved an equation that describes all fundamental forces in one equation. Another point I would like to mention is that the acceleration of our universe affects the final equation which means fundamental forces have a great impact on the acceleration of the universe.
Final field equation:

$$(8\kappa^2(\mathrm{A}_\alpha - \frac{q}{m}F_{\lambda\alpha}\frac{dx^\lambda}{d\tau}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)U^{\alpha\beta} = 0 \tag{3}$$

The equation above is the modern version of F=ma. In fact, my model is generalized for gravity and quantum and classical physics.

[*]Emai:hadi.symbol@gmail.com

2 Proving Field Equation

Let‘s get started to do the math:

$$\mathcal{L} = \frac{1}{2\kappa}\sqrt{-g}\mathcal{R} = \sqrt{g_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}} \tag{4}$$

$$\frac{1}{\sqrt{g}}\partial_\alpha(\sqrt{g})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha} \tag{5}$$

$$\begin{aligned}
&\tfrac{1}{\sqrt{g}}\partial_\alpha(\tfrac{2\kappa\mathcal{L}}{i\mathcal{R}})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha},\\
&\tfrac{\partial\mathcal{L}}{\partial\mathcal{X}^\alpha} = \tfrac{1}{2\mathcal{L}}\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau},\\
&\tfrac{2\kappa}{\sqrt{g}}(\tfrac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}}{-\mathcal{R}^2})U^{\alpha\beta} = U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha},\\
&2\kappa(\tfrac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial x^\alpha}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}}{-\mathcal{R}^2})U^{\alpha\beta} = -\sqrt{-g}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha},\\
&2\kappa(\tfrac{1}{2}\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \sqrt{-g}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{L}\mathcal{R}^2,\\
&\mathcal{L} = \tfrac{1}{2\kappa}\sqrt{-g}\mathcal{R} \rightarrow \sqrt{-g} = \tfrac{2\kappa\mathcal{L}}{\mathcal{R}},\\
&2\kappa(\tfrac{1}{2}\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{2\kappa\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2\\
&\nabla_\alpha g_{\alpha\beta} = 0\\
&\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha} - \Gamma^d_{\alpha\alpha}g_{d\beta} - \Gamma^d_{\alpha\beta}g_{\alpha d} = 0\\
&\tfrac{\partial g_{\alpha\beta}}{\partial x^\alpha} = \Gamma^d_{\alpha\alpha}g_{d\beta} + \Gamma^d_{\alpha\beta}g_{\alpha d}
\end{aligned} \tag{6}$$

$$\begin{aligned}
&(\tfrac{1}{2}(\Gamma^d_{\alpha\alpha}g_{d\beta} + \Gamma^d_{\alpha\beta}g_{\alpha d})\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2\\
&(\tfrac{1}{2}\Gamma^d_{\alpha\alpha}g_{d\beta}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} + \tfrac{1}{2}\Gamma^d_{\alpha\beta}g_{\alpha d}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\Gamma^d_{\alpha\beta}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau} = \tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d\\
&(\tfrac{1}{2}\Gamma^d_{\alpha\alpha}g_{d\beta}\tfrac{dx^\alpha}{d\tau}\tfrac{dx^\beta}{d\tau}\mathcal{R} + \tfrac{1}{2}\tfrac{d^2x^d}{d\tau^2}g_{\alpha d}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \tfrac{\mathcal{L}^2}{\mathcal{R}}U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}^2\\
&((\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d}\mathcal{R} - \tfrac{\partial\mathcal{R}}{\partial x^\alpha}\mathcal{L}^2)U^{\alpha\beta} = \mathcal{L}^2U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\mathcal{R}\\
&\mathcal{L}^2 = -\tfrac{1}{4\kappa^2}g\mathcal{R}^2
\end{aligned} \tag{8}$$

$$\begin{aligned}
&(4\kappa^2(\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d}\mathcal{R} + \tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}^2)U^{\alpha\beta} = -g\mathcal{R}^3U^{\alpha\beta}\Gamma^\gamma{}_{\gamma\alpha}\\
&\Gamma^\gamma{}_{\gamma\alpha} = \tfrac{1}{2g}\tfrac{\partial g}{\partial x^\alpha}\\
&(4\kappa^2(\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d}\mathcal{R} + \tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}^2)U^{\alpha\beta} = -g\mathcal{R}^3U^{\alpha\beta}\tfrac{1}{2g}\tfrac{\partial g}{\partial x^\alpha}\\
&(8\kappa^2(\tfrac{d^2x^d}{d\tau^2} + \mathrm{X}^d)g_{\alpha d} + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R})U^{\alpha\beta} + \tfrac{\partial g}{\partial x^\alpha}U^{\alpha\beta}\mathcal{R}^2 = 0\\
&U^{\alpha\beta} = \eta G^{\alpha\beta} + \lambda F^{\alpha\beta} + \mu G^{\alpha\beta} + \gamma Z^\alpha W_\beta\\
&\mathrm{A}^d = \tfrac{d^2x^d}{d\tau^2}\\
&(8\kappa^2(\mathrm{A}_\alpha + \mathrm{X}^d g_{\alpha d}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R})U^{\alpha\beta} + \tfrac{\partial g}{\partial x^\alpha}U^{\alpha\beta}\mathcal{R}^2 = 0
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\mathrm{X}^d = -\tfrac{q}{m}F^{d\gamma}\tfrac{dx^\lambda}{d\tau}g_{\lambda\gamma}\\
&(8\kappa^2(\mathrm{A}_\alpha - \tfrac{q}{m}F^{d\gamma}\tfrac{dx^\lambda}{d\tau}g_{\lambda\gamma}g_{\alpha d}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R})U^{\alpha\beta} + \tfrac{\partial g}{\partial x^\alpha}U^{\alpha\beta}\mathcal{R}^2 = 0
\end{aligned} \tag{10}$$

$$(8\kappa^2(\mathrm{A}_\alpha - \tfrac{q}{m}F_{\alpha\lambda}\tfrac{dx^\lambda}{d\tau}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \tfrac{\partial g}{\partial x^\alpha}\mathcal{R}^2)U^{\alpha\beta} = 0$$

$$(8\kappa^2(\mathrm{A}_\alpha - \tfrac{q}{m}F_{\alpha\lambda}\tfrac{dx^\lambda}{d\tau}) + 2\tfrac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \tfrac{\partial g}{\partial x^\alpha}\mathcal{R}^2)(\eta G^{\alpha\beta} + \lambda F^{\alpha\beta} + \mu G^{\alpha\beta} + \gamma Z^\alpha W_\beta) = 0 \tag{11}$$

For finding the coefficients we have to use numerical methods or experiments.

3 One Example of The Equation Field

For example, I want to consider only the electromagnetic field in the equation above:

$$U^{\alpha\beta} = F^{\alpha\beta} \tag{12}$$

We will see a familiar equation. I have to mention that in this example I consider the flat space-time; however, we can calculate for the curved space-time, but for simplicity, I consider the flat space-time. We chose Beta as zero for finding an equation.

$$\begin{aligned}
&U^{\alpha\beta} = F^{\alpha\beta}\\
&\mathrm{A}^d = \frac{d^2x^d}{d\tau^2}\\
&\beta = 0\\
&8\kappa^2(\mathrm{A}_\alpha F^{\alpha\beta} - \frac{q}{m}F_{\alpha\lambda}\frac{dx^\lambda}{d\tau}F^{\alpha\beta}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}F^{\alpha\beta} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2F^{\alpha\beta} = 0\\
&8\kappa^2(\mathrm{A}_1F^{10} + \mathrm{A}_2F^{20} + \mathrm{A}_3F^{30} - \frac{q}{m}(F_{\alpha 0}\frac{dx^0}{d\tau} + F_{\alpha 1}\frac{dx^1}{d\tau} + F_{\alpha 2}\frac{dx^2}{d\tau} + F_{\alpha 3}\frac{dx^3}{d\tau})F^{\alpha 0})\\
&+2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}F^{\alpha 0} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2F^{\alpha 0} = 0\\
&For flat space time\\
&2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R}F^{\alpha 0} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2F^{\alpha 0} = 0
\end{aligned} \tag{13}$$

$$\begin{aligned}
&8\kappa^2(\mathrm{A}_1F^{10} + \mathrm{A}_2F^{20} + \mathrm{A}_3F^{30} - \frac{q}{m}(F_{\alpha 0}F^{\alpha 0}\frac{dx^0}{d\tau} + F_{\alpha 1}F^{\alpha 0}\frac{dx^1}{d\tau} + F_{\alpha 2}F^{\alpha 0}\frac{dx^2}{d\tau} + F_{\alpha 3}F^{\alpha 0}\frac{dx^3}{d\tau}) = 0\\
&F_{\alpha 0}F^{\alpha 0} = F_{10}F^{10} + F_{20}F^{20} + F_{30}F^{30} = -(E_x^2 + E_y^2 + E_z^2) = -E^2\\
&F_{\alpha 1}F^{\alpha 0} = F_{21}F^{20} + F_{31}F^{30} = B_zE_y - B_yE_z\\
&F_{\alpha 2}F^{\alpha 0} = F_{12}F^{10} + F_{32}F^{30} = -B_zE_x + B_xE_z\\
&F_{\alpha 3}F^{\alpha 0} = F_{13}F^{10} + F_{23}F^{20} = B_yE_x - B_xE_y\\
&\mathrm{A}_1F^{10} + \mathrm{A}_2F^{20} + \mathrm{A}_3F^{30} - \frac{q}{m}(F_{\alpha 0}F^{\alpha 0}\frac{dx^0}{d\tau} + F_{\alpha 1}F^{\alpha 0}\frac{dx^1}{d\tau} + F_{\alpha 2}F^{\alpha 0}\frac{dx^2}{d\tau} + F_{\alpha 3}F^{\alpha 0}\frac{dx^3}{d\tau}) = 0\\
&\vec{\mathrm{A}}.\vec{E} - \frac{q}{m}(\gamma E^2 + (B_zE_y - B_yE_z)\frac{dx^1}{d\tau} + (-B_zE_x + B_xE_z)\frac{dx^2}{d\tau} + (B_yE_x - B_xE_y)\frac{dx^3}{d\tau}) = 0\\
&\vec{\mathrm{A}}.\vec{E} - \frac{q}{m}(\gamma E^2 + (\vec{B}\times\vec{E}).\vec{v}) = 0
\end{aligned} \tag{14}$$

This is familiar $\vec{\mathrm{A}}.\vec{E} - \frac{q}{m}(\gamma E^2 + (\vec{B}\times\vec{E}).\vec{v}) = 0$ for us (Lorentz force equation).

4 Acceleration of Our Universe

For every diagonal metric, such as the Robertson-walker metric :

$$\begin{aligned}
&(8\kappa^2(\mathrm{A}_\alpha - \frac{q}{m}F_{\alpha\lambda}\frac{dx^\lambda}{d\tau}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)8\pi GT_{\alpha\beta} = 0\\
&Assume:\\
&F_{\alpha\lambda} = 0\\
&(8\kappa^2\mathrm{A}_\alpha + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)T^{\alpha\beta} = 0\\
&8\kappa^2\mathrm{A}_\alpha + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2 = 0\\
&\mathrm{A}_\alpha = -\frac{1}{8\kappa^2}(2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)
\end{aligned} \tag{15}$$

The acceleration is positive because R in the terms is squared, and g is negative; hence, The acceleration is positive. Therefore, the acceleration of the universe is positive. this result is compatible with other assumptions.

5 Conclusion

I do not feel like calculating field equations for all fundamental forces, and all kinds of metrics such as black holes and the universe, but you can calculate with this equation.

$$(8\kappa^2(\mathrm{A}_\alpha - \frac{q}{m}F_{\lambda\alpha}\frac{dx^\lambda}{d\tau}) + 2\frac{\partial\mathcal{R}}{\partial x^\alpha}g\mathcal{R} + \frac{\partial g}{\partial x^\alpha}\mathcal{R}^2)U^{\alpha\beta} = 0 \tag{16}$$