High-Energy Particle Physics the Helmholtzian Factorization Methodology

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Abstract

Quantum field theory flows from the Helmholtzian factorization approach, including the weak interaction. The Helmholtzian factorization approach (including the Covariant) yields unification and more without tricks or obfuscations. The Helmholtzian factorization approach is superior to the gauge theory/combined symmetry group/symmetry breaking/Higgs mechanism patchwork quilt approach (which doesn't even include gravitation).

Theoretical physics field theory essentially began with Maxwell's equations of an electromagnetic field and the wave equation (which the **E** and **B** field strengths satisfy with the speed of light parameter) and the d'Alembert operator.

The cnuclear force holding atomic nuclei together was found to satisfy the Yukawa potential:

$$V_{Yukawa} = -g^2 \frac{e^{-\mu m r}}{r} \Rightarrow \left[\Box + (\mu m)^2\right] V_{Yukawa} - 0 \quad , \quad \left(\mu = \frac{c}{\hbar}\right)$$

This Klein–Gordon/Helmholtzian operator has been factored by Dirac producing the Dirac equation.

(i.e.: the d'Alembert operator is a "massless" Klein–Gordon/Helmholtzian operator.)

(which all fermions - leptons & quarks satisfy)

This Klein–Gordon/Helmholtzian operator may more ganerally be factored [1]:

$$\mathbf{J} \equiv D_B D_A \mathbf{f} = ((\Box - |m|^2))\mathbf{f}$$

where:

$$D_B \equiv \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \quad \& \quad D_A \equiv \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}$$

and:

$$D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} , \; D_i^{\Updownarrow} \equiv \begin{pmatrix} D_j^- & 0 \\ 0 & D_i^+ \end{pmatrix} , \; D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} , \; D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_j^- & 0 \end{pmatrix}$$

and:

$$|m|^2 \equiv \sum_{j=0}^{4-1} m_j^2$$

and:

$$\mathbf{f} \equiv \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} \quad , \quad f^j \equiv \begin{pmatrix} f_+^j \\ f_-^j \end{pmatrix}$$

$$f_+ \equiv \begin{pmatrix} \begin{pmatrix} f_+^1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^2 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^3 \\ 0 \end{pmatrix} \\ \begin{pmatrix} f_+^0 \\ 0 \end{pmatrix} \end{pmatrix} , \; f_- \equiv \begin{pmatrix} \begin{pmatrix} 0 \\ f_-^1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^2 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^3 \end{pmatrix} \\ \begin{pmatrix} 0 \\ f_-^0 \end{pmatrix} \end{pmatrix} , \; f \equiv \begin{pmatrix} \begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} \\ \begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} \\ \begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} \\ \begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \end{pmatrix} = f_+ + f_-$$

$$\Rightarrow \begin{pmatrix} -D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = D_B D_A \mathbf{f} = ((\Box - |m|^2))\mathbf{f}$$

Which yields:

$\mathbf{0} = (\partial_0 - m_0)\vec{\mathbf{B}} + (\vec{\nabla} + \vec{\mathbf{m}}) \times \vec{\mathbf{E}}$;	$0 = (\vec{\nabla} + \vec{\mathbf{m}}) \cdot \vec{\mathbf{B}}$	; Homgeneous
$\vec{\mathbf{J}} = (\partial_0 + m_0)\vec{\mathbf{E}} - (\vec{\nabla} - \vec{\mathbf{m}}) \times \vec{\mathbf{B}}$;	$\rho = (\vec{\nabla} - \vec{\mathbf{m}}) \cdot \vec{\mathbf{E}}$	; Inhomgeneous

where:

$$\mathbf{E} = \mathbf{w}^{4;1}\left(-D_0^{\Updownarrow}f^1 - D_1f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Updownarrow}f^2 - D_2f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Updownarrow}f^3 - D_3f^0\right)$$

$$\mathbf{B} = \mathbf{w}^{4;1}(D_2f^3 - D_3f^2) + \mathbf{w}^{4;2}(-D_1f^3 + D_3f^1) + \mathbf{w}^{4;3}(D_1f^2 - D_2f^1)$$

$$\mathbf{E}_{\Updownarrow} = \mathbf{w}^{4;1}\left(-D_0^{\Leftrightarrow\Updownarrow}f^1 - D_1^{\Leftrightarrow}f^0\right) + \mathbf{w}^{4;2}\left(-D_0^{\Leftrightarrow\Updownarrow}f^2 - D_2^{\Leftrightarrow}f^0\right) + \mathbf{w}^{4;3}\left(-D_0^{\Leftrightarrow\Updownarrow}f^3 - D_3^{\Leftrightarrow}f^0\right)$$

$$\mathbf{B}_{\Updownarrow} = \mathbf{w}^{4;1}(D_2^{\Leftrightarrow}f^3 - D_3^{\Leftrightarrow}f^2) + \mathbf{w}^{4;2}(-D_1^{\Leftrightarrow}f^3 + D_3^{\Leftrightarrow}f^1) + \mathbf{w}^{4;3}(D_1^{\Leftrightarrow}f^2 - D_2^{\Leftrightarrow}f^1)$$

where:

$\mathbf{f} \equiv \mathbf{w}^{4;1}f^\mu$	$f^\mu \equiv \begin{pmatrix} f_+^\mu \\ f_-^\mu \end{pmatrix}$

Just as the Dirac equation predicts a four-dimensional matter-antimatter field, so does this Helmholtzian operator factorization (as is easily seen, here).

However, the lepton & quark architecture naturally flows from this paradigm [1][2].

$e^- = e(1) = \overline{(E^1,E^2,E^3)}_1$	$\mu^- = e(2) = \overline{(E^1,E^2,E^3)}_2$	$\tau^- = e(3) = \overline{(E^1,E^2,E^3)}_3$
$\nu_e = \nu(1) = \left(B^1_{\Updownarrow},B^2_{\Updownarrow},B^3_{\Updownarrow}\right)_1$	$\nu_\mu = \nu(2) = \left(B^1_{\Updownarrow},B^2_{\Updownarrow},B^3_{\Updownarrow}\right)_2$	$\nu_\tau = \nu(3) = \left(B^1_{\Updownarrow},B^2_{\Updownarrow},B^3_{\Updownarrow}\right)_3$
$u_R = u_1(1) = \left(B^1_{\Updownarrow},E^2,E^3\right)_1$	$c_R = u_1(2) = \left(B^1_{\Updownarrow},E^2,E^3\right)_2$	$t_R = u_1(3) = \left(B^1_{\Updownarrow},E^2,E^3\right)_3$
$u_G = u_2(1) = \left(E^1,B^2_{\Updownarrow},E^3\right)_1$	$c_G = u_2(2) = \left(E^1,B^2_{\Updownarrow},E^3\right)_2$	$t_G = u_2(3) = \left(E^1,B^2_{\Updownarrow},E^3\right)_3$
$u_B = u_3(1) = \left(E^1,E^2,B^3_{\Updownarrow}\right)_1$	$c_B = u_3(2) = \left(E^1,E^2,B^3_{\Updownarrow}\right)_2$	$t_B = u_3(3) = \left(E^1,E^2,B^3_{\Updownarrow}\right)_3$
$d_R = d_1(1) = \overline{\left(E^1,B^2_{\Updownarrow},B^3_{\Updownarrow}\right)}_1$	$s_R = d_1(2) = \overline{\left(E^1,B^2_{\Updownarrow},B^3_{\Updownarrow}\right)}_2$	$b_R = d_1(3) = \overline{(E^1,B^2,B^3)}_3$
$d_G = d_2(1) = \overline{\left(B^1_{\Updownarrow},E^2,B^3_{\Updownarrow}\right)}_1$	$s_G = d_2(2) = \overline{\left(B^1_{\Updownarrow},E^2,B^3\right)}_2$	$b_G = d_2(3) = \overline{\left(E^1,B^2_{\Updownarrow},B^3_{\Updownarrow}\right)}_3$
$d_B = d_3(1) = \overline{\left(B^1_{\Updownarrow},B^2_{\Updownarrow},E^3\right)}_1$	$s_B = d_3(2) = \overline{\left(B^1_{\Updownarrow},B^2_{\Updownarrow},E^3\right)}_2$	$b_B = d_3(3) = \overline{\left(B^1_{\Updownarrow},B^2_{\Updownarrow},E^3\right)}_3$

or:

$\nu_{e_R} = \nu(1) = f\left(-1,\ 1,-1,-1\right)$	$\nu_\mu = \nu(2) = f\left(-1,\ 1,-1,\ 0\right)$	$\nu_\tau = \nu(3) = f\left(\ 1,\ 1,-1,\ 1\right)$
$e^- = e(1) = f\left(-1,-1,\ 0,-1\right)$	$\mu^- = e(2) = f\left(-1,-1,\ 0,\ 0\right)$	$\tau^- = e(3) = f\left(\ 1,-1,\ 0,\ 1\right)$
$\nu_{e_L} = \nu(1) = f\left(-1,\ 1,\ 1,-1\right)$	$\nu_\mu = \nu(2) = f\left(-1,\ 1,\ 1,\ 0\right)$	$\nu_\tau = \nu(3) = f\left(\ 1,\ 1,\ 1,\ 1\right)$
$u_R = u_1(1) = f\left(\ 1,-1,-1,-1\right)$	$c_R = u_1(2) = f\left(\ 1,-1,-1,\ 0\right)$	$t_R = u_1(3) = f\left(\ 1,-1,-1,\ 1\right)$
$u_G = u_0(1) = f\left(\ 1,-1,\ 0,-1\right)$	$c_G = u_0(2) = f\left(\ 1,-1,\ 0,\ 0\right)$	$t_G = u_0(3) = f\left(\ 1,-1,\ 0,\ 1\right)$
$u_B = u_{-1}(1) = f\left(\ 1,-1,\ 1,-1\right)$	$c_B = u_{-1}(2) = f\left(\ 1,-1,\ 1,\ 0\right)$	$t_B = u_{-1}(3) = f\left(\ 1,-1,\ 1,\ 1\right)$
$d_R = d_1(1) = f\left(\ 1,\ 1,-1,-1\right)$	$s_R = d_1(2) = f\left(\ 1,\ 1,-1,\ 0\right)$	$b_R = d_1(3) = f\left(\ 1,\ 1,-1,\ 1\right)$
$d_G = d_0(1) = f\left(\ 1,\ 1,\ 0,-1\right)$	$s_G = d_0(2) = f\left(\ 1,\ 1,\ 0,\ 0\right)$	$b_G = d_0(3) = f\left(\ 1,\ 1,\ 0,\ 1\right)$
$d_B = d_{-1}(1) = f\left(\ 1,\ 1,\ 1,-1\right)$	$s_B = d_{-1}(2) = f\left(\ 1,\ 1,\ 1,\ 0\right)$	$b_B = d_{-1}(3) = f\left(\ 1,\ 1,\ 1,\ 1\right)$

where: $f(x_1,x_2,x_3,x_4)$:

$x_1 = \begin{cases} -1 : & \text{lepton} \\ 1 : & \text{quark} \end{cases}$	$x_2 = \begin{cases} -1 : & \text{up} \\ 1 : & \text{down} \end{cases}$
$x_3 = \begin{cases} \underset{(x_1=1)}{color} = \begin{cases} -1 : & \text{R} \\ 0 : & \text{G} \\ 1 : & \text{B} \end{cases} \\ \underset{(x_1=0)}{right/left} = \begin{cases} -1 : & \text{Right} \\ 1 : & \text{Left} \end{cases} \end{cases}$	$x_4 = generation = \begin{cases} -1 : \\ 0 : \\ 1 : \end{cases}$

and: :

$\nu(h,i) \equiv \left(\dfrac{i + T_0(i)(-1)^i}{2\left[2h + (-1)^h\right]^2(hi^2+2) + (h+1)(h-i)^2} \right)^{T_0(h)T_0(i-1)}$
$u(h,i) \equiv \left(\dfrac{\left[2i + (-1)^i\right]^2 - 2T_0(i)}{\left[2i + (-1)^i\right]^2} \right)^{\frac{\delta_1^{(-1)^{T_0(h)}}\delta_{-1}^{(-1)^h}}{2^{T_0(i-1)}}}$
$f(h) \equiv (h^2+1)\left(\tfrac{1}{2}\lambda_h(h^2+1)^{2h}\right)^{T_0(T_0(h+1))}$
$g(h,i) \equiv u(h,i)\left[\nu(h,i)\left(\left[2h + (-1)^h 2^{\frac{1}{2}\left[1+(-1)^h\right]}\right]k\right)^{(i-1)[T_0(h+1)-T_0(T_0(h+1))]}\right]^{(i-1)^{T_0(h)}}$
$T_0(j) = \tfrac{1}{2}\left[j - 1 + \delta^1_{(-1)^j}\right]$
$\left(h,i \in \mathbb{N} \; ; \; 0 \le h \le 3 \; , \; 1 \le i \le 3\right)$

$\Rightarrow$

$\dfrac{m(h,1)}{m\left(\left[h + \delta_{-1}^{(-1)^{T_0(h+1)}}\right]\delta_{-1}^{(-1)^{T_0(h+1)}},1\right)} = f(h)$
$\dfrac{m(h,i)}{m\left(h + (-1)^{h+1}\delta_{-1}^{(-1)^{T_0(h+1)}}(1-\delta_1^i),1\right)} = g(h,i)$

$$\Rightarrow \begin{array}{|l|} \hline m(h,1) = \prod_{n=h}^{3}\left[(h^2+1)\left(\tfrac{1}{2}\lambda_h (h^2+1)^{2h}\right)^{T_0(T_0(h+1))} \right] m(0,1) \\ \hline \qquad = \left(\tfrac{1}{2}\right)^{T_0(T_0(n+1))} \prod_{n=h}^{3}\left[\lambda_h (h^2+1)\left((h^2+1)^{2h}\right)^{T_0(T_0(h+1))} \right] m(0,1) \\ \hline \end{array}$$

$$m_e = \frac{1}{10}\left[\frac{15}{8} + \frac{1}{4000}\left(\frac{486}{25}\right) \right] e\ MeV/c^2 = 0.5109989278047020776144390005897\ MeV/c^2$$

$$\Rightarrow \left\{ \begin{array}{|l|} \hline m_e = m(3,1) = 0.5109989278047020776144390005897 \\ \hline m_u = m(2,1) = 5m(3,1) = 2.5549946390235103880721950029485 \\ \hline m_d = m(1,1) = 2m(2,1) = 5.1099892780470207761443900058 97 \\ \hline \end{array} \right.$$

and:

$\frac{m(0,2)}{m(0,1)} = \lambda_2$	$\frac{m(0,3)}{m(0,1)} = \lambda_3$
$\frac{m(1,2)}{m(2,1)} = \left(\frac{23}{25}\right) \cdot (k)$	$\frac{m(1,3)}{m(2,1)} = \left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (k)^2$
$\frac{m(2,2)}{m(1,1)} = 1 \cdot (6k)$	$\frac{m(2,3)}{m(1,1)} = 1 \cdot \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2$
$\frac{m(3,2)}{m(3,1)} = 1 \cdot (5k)$	$\frac{m(3,3)}{m(3,1)} = 1 \cdot \left[\left(\frac{2}{1450}\right)(5k)^2\right]^2$

Yielding:

$$\begin{aligned} k &= \frac{m(1,2)}{m(2,1)}\left(\frac{25}{23}\right) = \frac{1}{6}\left[\frac{m(2,2)}{m(1,1)}\right] = \frac{1}{5}\left[\frac{m(3,2)}{m(3,1)}\right] \\ &= \sqrt{\frac{m(1,3)}{m(2,1)}\sqrt{\frac{25}{23}}} = \frac{1}{6}\sqrt{\frac{1004}{3}\sqrt{\frac{m(2,3)}{m(1,1)}}} = \frac{1}{5}\sqrt{\frac{1450}{2}\sqrt{\frac{m(3,3)}{m(3,1)}}} \\ &= 41.353655699595529713433202094743 \\ &= 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\sum_{k=0}^{\infty}\left(-\frac{1}{20}\right)^k = 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\left(\frac{20}{21}\right) \end{aligned}$$

$\Rightarrow$

(in Mev/c^2)	
Calculated	**Measured**
$m_d = m(1,1) = 5.1099892780470207761443900058 97$	$m_d = m(1,1) \approx 5.0(0.5)$
$m_u = m(2,1) = 2.5549946390235103880721950029485$	$m_u = m(2,1) \approx 2.4(0.6)$
$m_e = m(3,1) = 0.5109989278047020776144390005897$	$m_e = m(3,1) \approx 0.510998928(11)$
$m_{\nu_e} = m(0,1) = 0.10219978556094041552288780011794 \times 10^{-6}$	$m_{\nu_e} = m(0,1) < 10^{-6} \times 2.2$
$m_s = m(1,2) = 97.205699127171364309497389896187$	$m_s = m(1,2) \approx 95(5)$
$m_c = m(2,2) = 1267.9004233978873605586616073416$	$m_c = m(2,2) \approx 1275(25)$
$m_\mu = m(3,2) = 105.65836861649061337988846727846$	$m_\mu = m(3,2) \approx 105.6583715(35)$
$m_{\nu_{\mu e}} = m(0,2) = 0.21131673723298122675977693455693 \times 10^{-6}$	$m_{\nu_{\mu e}} = m(0,2) < 10^{-6} \times 0.17$
$m_b = m(1,3) = 4190.9426907545271186849743851983$	$m_b = m(1,3) \approx 4180(30)$
$m_t = m(2,3) = 172924.17191486611744398343538627$	$m_t = m(2,3) \approx 172970(620)$
$m_\tau = m(3,3) = 1776.9680674108457768918379570944$	$m_\tau = m(3,3) \approx 1,776.82(16)$
$m_{\nu_\tau} = m(0,3) = 81.1.3697452227612011492344608 1431 \times 10^{-6}$	$m_{\nu_\tau} = m(0,3) < 10^{-6} \times 18.2$

This empirically-based treatment has uncovered/revealed the unitless/dimensionless constant:

$$\begin{aligned} k &= 41.353655699595529713433202094743 \\ &= 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\sum_{k=0}^{\infty}\left(-\frac{1}{20}\right)^k = 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\left(\frac{20}{21}\right) \end{aligned}$$

$$\Rightarrow \left\{ \begin{array}{|l|l|l|} \hline m_e = m_e & m_\mu = 5km_e & m_\tau = \left(\frac{1}{9}\right)^2 k^4 m_e \\ \hline m_u = 5m_e & m_c = 6km_d & m_t = \left(\frac{27}{251}\right)^2 k^4 m_d \\ \hline m_d = 10m_e & m_s = \frac{23}{25}km_u & m_b = \sqrt{\frac{23}{25}}\,k^2 m_u \\ \hline \end{array} \right\}$$

$$\Updownarrow$$

$$\Rightarrow \left\{ \begin{array}{|l|l|l|} \hline m_e = m_e & m_\mu = 5km_e & m_\tau = \left(\frac{k}{3}\right)^4 m_e \\ \hline m_u = 5m_e & m_c = 12km_u & m_t = 30\left(\frac{9k^2}{251}\right)^2 m_e \\ \hline m_d = 10m_e & m_s = \frac{1}{2}\frac{23}{25}km_d & m_b = \frac{1}{2}\sqrt{\frac{23}{25}}\,k^2 m_d \\ \hline \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{|l|l|l|} \hline m_e = m_e & m_u = 5m_e & m_d = 10m_e \\ \hline m_\mu = 5km_e & m_c = 12km_u & m_s = \frac{1}{2}\frac{23}{25}km_d \\ \hline m_\tau = \left(\frac{1}{9}\right)^2 k^4 m_e & m_t = 2\left(\frac{27}{251}\right)^2 k^4 m_u & m_b = \frac{1}{2}\sqrt{\frac{23}{25}}\,k^2 m_d \\ \hline \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{|c|} \hline \frac{m_b}{\left[\sqrt{23}\,k^2\right]} = \frac{m_s}{\left[\frac{23}{5}k\right]} = \frac{m_t}{\left[30\left(\frac{9k^2}{251}\right)^2\right]} = \frac{m_c}{60k} = \frac{m_\tau}{\left[\left(\frac{k}{3}\right)^4\right]} = \frac{m_\mu}{5k} = \frac{m_d}{10} = \frac{m_u}{5} = m_e \\ \hline \end{array} \right\}$$

$$\Rightarrow \left\{ \begin{array}{|l|l|l|l|} \hline \frac{m_d}{m_u} = 2 & \frac{m_b}{m_s} = \left(\frac{5}{\sqrt{23}}\right)k & \frac{m_\tau}{m_\mu} = \left(\frac{1}{5 \cdot 3^4}\right)k^3 & \frac{m_t}{m_c} = \left[\frac{1}{2}\left(\frac{9}{251}\right)^2\right]k^3 \\ \hline \end{array} \right\}$$

The Helmholtzian factorization yields the Helmholtz operator:

$D_B D_A = (\Box + |m|^2)$; $m = (m_1, m_2, m_3, m_0)$, $|m|^2 = m_1^2 + m_2^2 + m_3^2 + m_0^2$

This Klein–Gordon/Helmholtzian operator may more ganerally be factored [1]:

The Dirac equation shows that for heavy leptons (electrons, muons, taons; and their anti-particles) the Klein–Gordon/Helmholtzian operator may be factored via a single mass parameter; but quarks and light leptons include three color parameters.

Thus, specify heavy leptons by: $(0,0,0,m_0)$, and quarks by: $(m_1, m_2, m_3, 0)$.

Further, $|m|^2$ are pythagorean quadruples,such as: $(4,10,28,30)$ leading to $\left(\frac{1}{3},\frac{5}{6},\frac{7}{3},\frac{5}{2}\right)$ and $\left(\frac{2}{3},\frac{5}{3},\frac{14}{3},5\right)$; further leading to the quark mass, color, and charge architecture.

$$(2^2 + 5^2 + 14 = 4 + 25 + 196 = 225 = 15^2)$$

$$\left(\left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2 + \left(\frac{14}{3}\right)^2 = \left(\frac{15}{3}\right)^2 = 5^2 \right)$$

$$\left(\left(\frac{1}{3}\right)^2 + \left(\frac{5}{6}\right)^2 + \left(\frac{7}{3}\right)^2 = \left(\frac{15}{3}\right)^2 = \left(\frac{5}{2}\right)^2 \right)$$

Now, the Weak Force/Interaction has been theorized as a half-generalized electroweak phenomenology in conjunction with the Higgs Lagrangian to predict boson particles of certain masses.

As the Higgs is unnecessary for establishing the fermion architecture, nor their charges, color, nor masses (via the fundamental dimensionless physical constant k , as above) ; neither the electroweak nor the Higgs theory are necessary for the weak interaction:

$$q + \bar{q} \leftrightarrow Z^0 \quad , \quad q \in \{d,s,t,u,c,b\}$$

$$q\left[\left(\pm\frac{1}{3},\pm\frac{5}{6},\pm\frac{7}{3}\right)\lambda_1\right] \leftrightarrow W^{\pm} + q\left[\left(\mp\frac{1}{3},\mp\frac{5}{6},\mp\frac{7}{3}\right)\lambda_2\right]$$

(where allowed by energy conservation)

where:

$$|m_W| = 4^3 m_c = 4^3 \times 12 k m_u = 4^3 \times 60 k m_e = 81,145.627097464791075754342869 86$$

$$|m_Z| = \frac{1}{2}\left(\frac{3}{2}\right)^2 4^3 m_c = \frac{1}{2}\left(\frac{3}{2}\right)^2 4^3 \times 12 k m_u = 4^2 \times 54 k m_u = \frac{1}{2} \times (12)^2 \times 60 k m_e$$

$$= \frac{5}{2}(12)^3 k m_e = 91,288.8304846478899602236357285 93$$

So:

$$\frac{m_W}{m_u}\left(\frac{1}{4^3 \times 12}\right) = \frac{81130}{2.5549946390235103880721950029485}\left(\frac{1}{4^3 \times 12}\right) = 41.3457$$

$$\frac{m_Z}{m_u}\left(\frac{1}{4^2 \times 54}\right) = \frac{91890}{2.5549946390235103880721950029485}\left(\frac{1}{4^2 \times 54}\right) = 41.6259$$

(apparently the actual mass of the Z^0 is much closer to the lower measure than the higher)

$$\frac{m_Z}{m_u}\left(\frac{1}{4^2 \times 54}\right) = \frac{91300}{2.5549946390235103880721950029485}\left(\frac{1}{4^2 \times 54}\right) = 41.358715467474582378365794645146$$

Note:

$$m_u = (m_1, m_2, m_3, m_0) = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3},0\right)\lambda = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3},0\right)2\lambda$$

$$\Rightarrow \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{5}{6}\right)^2 + \left(\frac{7}{3}\right)^2} \times 2\lambda = \frac{5}{2}2\lambda = |m_u| = 5|m_e| \Rightarrow \lambda = |m_e| \Rightarrow m_u = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3},0\right)|m_e|$$

$$m_d = (m_1, m_2, m_3, m_0) = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)\lambda = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)\lambda$$

$$\Rightarrow \frac{5}{2}\lambda = |m_d| \Rightarrow \lambda = \frac{|m_d|}{\left(\frac{5}{2}\right)} = \frac{2|m_u|}{\left(\frac{5}{2}\right)} = \frac{10|m_e|}{\left(\frac{5}{2}\right)} = \frac{20}{5}|m_e| = 4|m_e| \Rightarrow m_u = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)4|m_e|$$

$$m_{q_u} = (m_1, m_2, m_3, m_0) = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3},0\right)\lambda = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3},0\right)2\lambda$$

$$\Rightarrow \frac{5}{2}2\lambda = 5\lambda = |m_{q_u}| \Rightarrow \lambda = \frac{1}{5}|m_{q_u}| \Rightarrow m_{q_u} = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3},0\right)\frac{1}{5}|m_{q_u}| = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3},0\right)\frac{|m_{q_u}|}{5}$$

$$m_{q_d} = (m_1, m_2, m_3, m_0) = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)\lambda = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)\lambda$$

$$\Rightarrow \frac{5}{2}\lambda = |m_{q_d}| \Rightarrow \lambda = \frac{2}{5}|m_{q_d}| \Rightarrow m_{q_d} = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)\frac{2}{5}|m_{q_d}| = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3},0\right)\frac{|m_{q_d}|}{\left(\frac{5}{2}\right)}$$

(The fourth element of a pythagorean quadruple is, of course, the square root of the sum of the squares of first three elements. However, the fourth element of a quark primitive is 0 ; this square root of the sum of the squares of first three elements becomes the divisor of the quark mass as the primitive multiplier λ)

And:

$$m_{W^{\pm}} = (m_1, m_2, m_3, m_0) = \left(\pm\frac{1}{3},\pm\frac{5}{6},\pm\frac{7}{3},0\right)\lambda + \left(0,0,0,\pm\frac{2}{3}\right)\lambda$$

$$\Rightarrow |m_{W^{\pm}}| = \sqrt{\left[\left(\pm\frac{1}{3}\lambda\right)^2 + \left(\pm\frac{5}{6}\lambda\right)^2 + \left(\pm\frac{7}{3}\lambda\right)^2\right] + \left(\pm\frac{2}{3}\lambda\right)^2}$$

$$= \sqrt{\left(\frac{4}{6^2} + \frac{25}{6^2} + \frac{196}{6^2}\right)\lambda^2 + \left(\frac{2}{3}\lambda\right)^2} = \sqrt{\left(\frac{5}{2}\lambda\right)^2 + \left(\frac{2}{3}\lambda\right)^2}$$

$$= \sqrt{\frac{(5 \cdot 3)^2 + (2 \cdot 2)^2}{6^2}}\lambda = \sqrt{\frac{15^2 + 4^2}{6^2}}\lambda = \frac{\sqrt{225 + 16}}{6}\lambda = \frac{\sqrt{241}}{6}\lambda$$

$$= |m_{W^{\pm}}| = 4^3 \times 60 k m_e \Rightarrow \lambda = 4^3 \times \frac{6 \times 60}{\sqrt{241}} k m_e$$

$$\Rightarrow m_{W^{\pm}} = (m_1, m_2, m_3, m_0) = \left(\pm\frac{1}{3},\pm\frac{5}{6},\pm\frac{7}{3},\pm\frac{2}{3}\right)(4^3)\left(\frac{360}{\sqrt{241}}\right) k m_e$$

$$m_Z = (m_1, m_2, m_3, m_0) = \left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3},0\right)\lambda + \left(0,0,0,-\frac{1}{3}\right)\lambda$$

$$\Rightarrow |m_Z| = \sqrt{\left[\left(\pm\frac{1}{3}\lambda\right)^2 + \left(\pm\frac{5}{6}\lambda\right)^2 + \left(\pm\frac{7}{3}\lambda\right)^2\right] + \left(\pm\frac{1}{3}\lambda\right)^2}$$

$$= \sqrt{\left(\frac{4}{6^2} + \frac{25}{6^2} + \frac{196}{6^2}\right)\lambda^2 + \left(\frac{1}{3}\lambda\right)^2} = \sqrt{\left(\frac{5}{2}\lambda\right)^2 + \left(\frac{1}{3}\lambda\right)^2}$$

$$= \sqrt{\frac{(5 \cdot 3)^2 + (1 \cdot 2)^2}{6}}\lambda = \frac{\sqrt{229}}{6}\lambda$$

$$= |m_Z| = \frac{5}{2}(12)^3 km_e \Rightarrow \lambda = \frac{30}{\sqrt{229}}(12)^3 km_e$$

$$\Rightarrow m_Z = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, -\frac{1}{3}\right)\left(\frac{30}{\sqrt{229}}\right)(12)^3 km_e$$

Note:

Because neutrinos have and interact color-wise and have zero charge, they have architecture like Z^0 particles:

$$\Rightarrow m_\nu = (m_1, m_2, m_3, m_0) = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, 0\right)\lambda_\nu + \left(0, 0, 0, -\frac{1}{3}\right)\lambda_\nu$$

$$\Rightarrow |m_\nu| = \sqrt{\left[\left(\pm\frac{1}{3}\lambda_\nu\right)^2 + \left(\pm\frac{5}{6}\lambda_\nu\right)^2 + \left(\pm\frac{7}{3}\lambda_\nu\right)^2\right] + \left(\pm\frac{1}{3}\lambda_\nu\right)^2}$$

$$= \sqrt{\left(\frac{4}{6^2} + \frac{25}{6^2} + \frac{196}{6^2}\right)\lambda_\nu^2 + \left(\frac{1}{3}\lambda_\nu\right)^2} = \sqrt{\left(\frac{5}{2}\lambda_\nu\right)^2 + \left(\frac{1}{3}\lambda_\nu\right)^2}$$

$$= \sqrt{\frac{(5 \cdot 3)^2 + (1 \cdot 2)^2}{6}}\lambda_\nu = \frac{\sqrt{229}}{6}\lambda_\nu$$

$$\Rightarrow \lambda_\nu = \frac{6}{\sqrt{229}}|m_\nu|$$

$$\Rightarrow m_\nu = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, -\frac{1}{3}\right)\left(\frac{6}{\sqrt{229}}\right)|m_\nu|$$

Note:

although the mass constituents may be in any order the addition is equivalent to aligning/rearranging the constituents from least to largest and adding and adding the resultant tuple , constituent-wise

so:

the primitive triples of the quark mass/color constituent pythagorean quadruples add up to $0 \pmod 3$, as **color** , and

the first/least of the primitive triples of the quark mass/color constituent pythagorean quadruples add up to $0 \pmod 2$ as **charge** , and

the total sum of the squares of the primitive triples is the square of the quark **mass magnitude** (the last/largest constituent pythagorean quadruple/quintuple)

Thus, the quark mass/color/charge constituent pythagorean quadruple/quintuple is fully determinant of of the quark mass, color, and charge.

Further:

Just as the electromanetic force is given by:

$$F_e = \lambda_e \frac{e_1 e_2}{r^2}$$

and the gravitational force is given by: (at least to first aproximation):

$$F_g = \lambda_g \frac{m_1 m_2}{r^2}$$

The Yukawa color force between quarks may be given by:

$$F_q = \lambda_q \frac{q_1 \circ q_2}{r^2} e^{\mu r}$$

as follows:

Let: $q_c \equiv \boldsymbol{\sigma}_c$, where:

$\boldsymbol{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\boldsymbol{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\boldsymbol{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$q_R \equiv k_R\boldsymbol{\sigma}^1$	$q_G \equiv k_G\boldsymbol{\sigma}^2$	$q_B \equiv k_B\boldsymbol{\sigma}^3$	$k_R, k_G, k_B \in \mathbb{R}$

$\det(\boldsymbol{\sigma}^j) = 1$	$\det(\boldsymbol{\sigma}^0) = \det(\mathbf{I}_2) = 1$

$A \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow kA = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

$$\det A = ad - bc \Rightarrow \det(kA) = kakd - kbkc = k^2(ad - bc) = k^2 \det A$$

$$\Downarrow$$

$\boldsymbol{\sigma}^1\boldsymbol{\sigma}^1 = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = \mathbf{I}_2$
$\boldsymbol{\sigma}^1\boldsymbol{\sigma}^2 = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}i&0\\0&-i\end{pmatrix} = i\begin{pmatrix}1&0\\0&-1\end{pmatrix} = i\boldsymbol{\sigma}^3$
$\boldsymbol{\sigma}^1\boldsymbol{\sigma}^3 = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}0&-1\\1&0\end{pmatrix} = -i\begin{pmatrix}0&-i\\i&0\end{pmatrix} = -i\boldsymbol{\sigma}^2$
$\boldsymbol{\sigma}^2\boldsymbol{\sigma}^1 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}-i&0\\0&i\end{pmatrix} = i\begin{pmatrix}1&0\\0&-1\end{pmatrix} = -i\boldsymbol{\sigma}^3$
$\boldsymbol{\sigma}^2\boldsymbol{\sigma}^2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = \mathbf{I}_2$
$\boldsymbol{\sigma}^2\boldsymbol{\sigma}^3 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}0&i\\i&0\end{pmatrix} = i\begin{pmatrix}0&1\\1&0\end{pmatrix} = i\boldsymbol{\sigma}^1$
$\boldsymbol{\sigma}^3\boldsymbol{\sigma}^1 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}0&1\\-1&0\end{pmatrix} = i\begin{pmatrix}0&-i\\i&0\end{pmatrix} = i\boldsymbol{\sigma}^2$
$\boldsymbol{\sigma}^3\boldsymbol{\sigma}^2 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}0&-i\\-i&0\end{pmatrix} = -i\begin{pmatrix}0&1\\1&0\end{pmatrix} = -i\boldsymbol{\sigma}^1$
$\boldsymbol{\sigma}^3\boldsymbol{\sigma}^3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = \mathbf{I}_2$

and, define: $q_{C_1} \circ q_{C_2} = \det(q_{C_1} q_{C_2})$

$\Downarrow$

$q_R \circ q_R = \det(q_R q_R) = \det(k_R\boldsymbol{\sigma}^1 k_R\boldsymbol{\sigma}^1) = \det(k_R k_R \boldsymbol{\sigma}^1\boldsymbol{\sigma}^1) = k_R^2 \det(\mathbf{I}_2) = k_R^2 > 0$
$q_R \circ q_G = \det(q_R q_G) = \det(k_R\boldsymbol{\sigma}^1 k_G\boldsymbol{\sigma}^2) = \det(k_R k_G \boldsymbol{\sigma}^1\boldsymbol{\sigma}^2) =$
$= \det(k_R k_G[i\boldsymbol{\sigma}^3]) = (ik_R k_G)^2 \det(\boldsymbol{\sigma}^3) = -(k_R k_G)^2 < 0$
$q_R \circ q_B = \det(q_R q_B) = \det(k_R\boldsymbol{\sigma}^1 k_B\boldsymbol{\sigma}^3) = \det(k_R k_B \boldsymbol{\sigma}^1\boldsymbol{\sigma}^3) =$
$= \det(k_R k_B[-i\boldsymbol{\sigma}^2]) = (-ik_R k_B)^2 \det(\boldsymbol{\sigma}^2) = -(k_R k_B)^2 < 0$
$q_G \circ q_R = \det(q_G q_R) = \det(k_G\boldsymbol{\sigma}^2 k_R\boldsymbol{\sigma}^1) = \det(k_G k_R \boldsymbol{\sigma}^2\boldsymbol{\sigma}^1) =$
$= \det(k_G k_R[-i\boldsymbol{\sigma}^3]) = (-ik_G k_R)^2 \det(\boldsymbol{\sigma}^3) = -(k_G k_R)^2 < 0$
$q_G \circ q_G = \det(q_G q_G) = \det(k_G\boldsymbol{\sigma}^2 k_G\boldsymbol{\sigma}^2) = \det(k_G k_G \boldsymbol{\sigma}^2\boldsymbol{\sigma}^2) = \det(k_G^2 \mathbf{I}_2) = k_G^2 > 0$
$q_G \circ q_B = \det(q_G q_B) = \det(k_G\boldsymbol{\sigma}^2 k_B\boldsymbol{\sigma}^3) = \det(k_G k_B \boldsymbol{\sigma}^2\boldsymbol{\sigma}^3) =$
$= \det(k_G k_B[i\boldsymbol{\sigma}^1]) = (ik_G k_B)^2 \det(\boldsymbol{\sigma}^1) = -(k_G k_B)^2 < 0$
$q_B \circ q_R = \det(q_B q_R) = \det(k_B\boldsymbol{\sigma}^3 k_R\boldsymbol{\sigma}^1) = \det(k_B k_R \boldsymbol{\sigma}^3\boldsymbol{\sigma}^1) =$
$= \det(k_B k_R[i\boldsymbol{\sigma}^2]) = (ik_B k_R)^2 \det(\boldsymbol{\sigma}^2) = -(k_B k_R)^2 < 0$
$q_B \circ q_G = \det(q_B q_G) = \det(k_B\boldsymbol{\sigma}^3 k_G\boldsymbol{\sigma}^2) = \det(k_B k_G \boldsymbol{\sigma}^3\boldsymbol{\sigma}^2) =$
$= \det(k_B k_G[-i\boldsymbol{\sigma}^1]) = (-ik_B k_G)^2 \det(\boldsymbol{\sigma}^1) = -(k_B k_G)^2 < 0$
$q_B \circ q_B = \det(q_B q_B) = \det(k_B\boldsymbol{\sigma}^3 k_B\boldsymbol{\sigma}^3) = \det(k_B k_B \boldsymbol{\sigma}^3\boldsymbol{\sigma}^3) = k_B^2 \det(\mathbf{I}_2) = k_B^2 > 0$

$\Downarrow$

$$q_{C_1} \circ q_{C_2} = \begin{cases} k_{C_1}^2 > 0 & , \ C_1 = C_2 \\ -(k_{C_1} k_{C_2})^2 < 0 & , \ C_1 \neq C_2 \end{cases}$$

Thus, differing color quarks attract, alike color quarks repel.

And, define: $\overline{q_{C_1}} = iq_{C_1} \Rightarrow \overline{q_{C_1}} \circ q_{C_1} = q_{C_1} \circ \overline{q_{C_1}} = \det(iq_{C_1} q_{C_1}) = -k_{C_1}^2 < 0$

$\Rightarrow$ quark/anti-quark attraction

vis.:

$q_R \circ \overline{q_R} = \det(q_R \overline{q_R}) = \det(k_R\boldsymbol{\sigma}^1 ik_R\boldsymbol{\sigma}^1) = \det(ik_R k_R \boldsymbol{\sigma}^1\boldsymbol{\sigma}^1) = (ik_R)^2 \det(\mathbf{I}_2) = -k_R^2 < 0$
$q_R \circ \overline{q_G} = \det(q_R \overline{q_G}) = \det(k_R\boldsymbol{\sigma}^1 ik_G\boldsymbol{\sigma}^2) = \det(ik_R k_G \boldsymbol{\sigma}^1\boldsymbol{\sigma}^2) =$
$= \det(ik_R k_G[i\boldsymbol{\sigma}^3]) = (i^2 k_R k_G)^2 \det(\boldsymbol{\sigma}^3) = (-k_R k_G)^2 = (k_R k_G)^2 > 0$

In this way, meson & baryon color force attraction/repulsion is manifested with a force field similar to that of the electric and gravitational.

(In fact, since: $\alpha \in R$ is in a same equivalence class as $\alpha\begin{pmatrix}1&0\\0&1\end{pmatrix}$, is indistinguishable from it.)

Thus, the above forms may equivalenty written:

$$F_e = (e_1 \circ e_2)\left(\frac{\lambda_e}{r^2}\right) \ , \ F_g = (m_1 \circ m_2)\left(\frac{\lambda_g}{r^2}\right) \ , \ F_q = (q_1 \circ q_2)\left(\lambda_q \frac{e^{\mu r}}{r^2}\right)$$

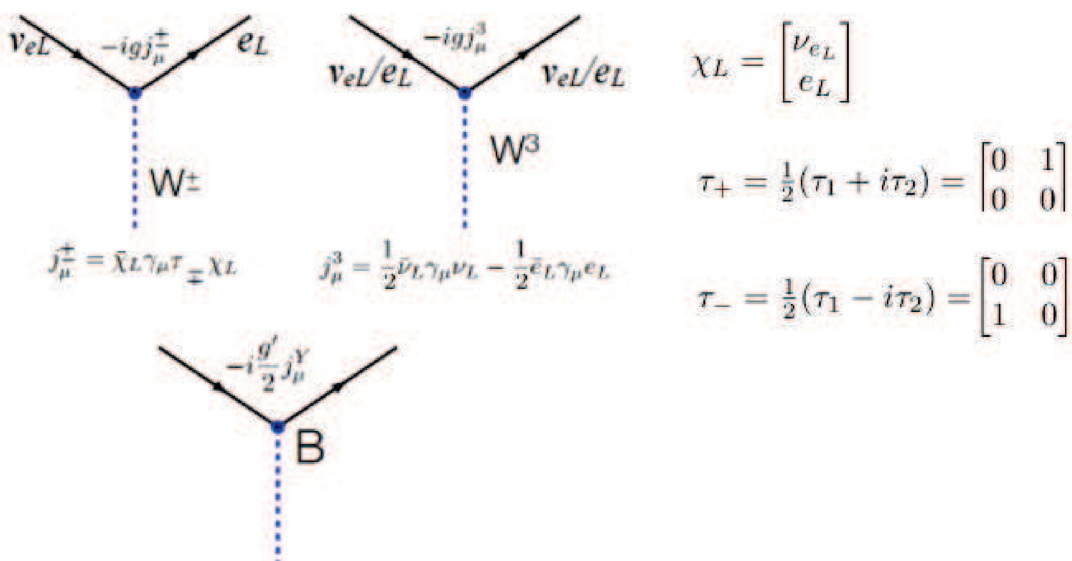
(the potentials being solutions of $(\Box - \lambda_\mu)\phi_\mu = J_\mu$: (using: $L_j = \partial_j$)
(the first two being of the d'Alembert, and the third being of the
(Klein-Gordon/ Helmholtzian operator, respectively.
(actually all the same, understanding that the first two have zero constant term;
(and that the space-time of the first and third are flat euclidean, the gravitational

(is curvature of Swarzschild/Eddington/Kerr-Newman/Kerr-Schild/Gibbons-Hawking
(metric and coordinates, as appropriate
$\Rightarrow F_\xi = (\rho_1 \circ \rho_2)\lambda_\xi \phi_\xi$
where:

$$(\xi, \rho_{\xi j}, \lambda_\xi, \phi_\xi, J_\xi) \in \left\{ \left(e, e_j, \lambda_e, \frac{1}{r^2}, J_e\right), \left(g, m_j, \lambda_g, \frac{1}{r^2}, J_g\right), \left(q, q_j, \lambda_q, \frac{e^{\mu r}}{r^2}, J_q\right) \right\}$$

NOTE:
(Using the Klein-Gordon/ Helmholtzian with space-time coorinates with metric and
(curvature as appropriate or using the Covariant Helmholtzian rather than:
($G^{\alpha\gamma} = R^{\alpha\gamma} - \frac{1}{2}g^{\alpha\gamma}R = -\frac{8\pi\kappa}{c^2}T^{\alpha\gamma}$, in effect separates the field from the space curvature -
(even with the same result.
(So, the field is expressed using fortuitive coordinates as is often done
((such as using polar coordinates in analysis of a pendulum simple harmonic oscillator)
(Thus, notions on wormholes and such become fictional speculations.

The Weak force/interaction may be considered empirically-based on (vertex) interaction currents:



$$j^Y_\mu = (-\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L) + (-\bar{e}_R\gamma_\mu e_R - \bar{\nu}_L\gamma_\mu\nu_L)$$
$$j^3_\mu = \tfrac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \tfrac{1}{2}\bar{e}_L\gamma_\mu e_L$$
$$j^+_\mu = (\bar{\nu}_L\,\bar{e}_L)\gamma_\mu \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \bar{e}^+_L\gamma_\mu\nu_L$$
$$j^-_\mu = (\bar{\nu}_L\,\bar{e}_L)\gamma_\mu \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \bar{\nu}_L\gamma_\mu e^-_L$$

$$J^Y_\mu = (-\bar{e}_R\gamma_\mu e_R - \bar{e}_L\gamma_\mu e_L) + (-\bar{e}_R\gamma_\mu e_R - \bar{\nu}_L\gamma_\mu\nu_L)$$
$$J^3_\mu = -\frac{1}{2}\bar{e}_L\gamma_\mu e_L$$
$$J^+_\mu = -\frac{1}{2}\bar{e}^+_L\gamma_\mu\nu_L$$
$$J^-_\mu = -\frac{1}{2}\bar{\nu}_L\gamma_\mu e^-_L$$
$$\Downarrow$$

$B_\mu \equiv -\frac{i}{2}g_e[-(e_L\gamma_\mu e_L + \bar{e}_R\gamma_\mu e_R) - (\bar{\nu}_L\gamma_\mu\nu_L + \bar{e}_R\gamma_\mu e_R)]$
$W^3_\mu \equiv -\frac{i}{2}g_e[(\bar{\nu}_L\gamma_\mu\nu_L - \bar{e}_L\gamma_\mu e_L)]$

and define:

$A_\mu = B_\mu\cos\theta_W + W^3_\mu\sin\theta_W$
$Z_\mu \equiv -B_\mu\sin\theta_W + W^3_\mu\cos\theta_W$

$(\sin^2\theta_W \approx 0.22)$

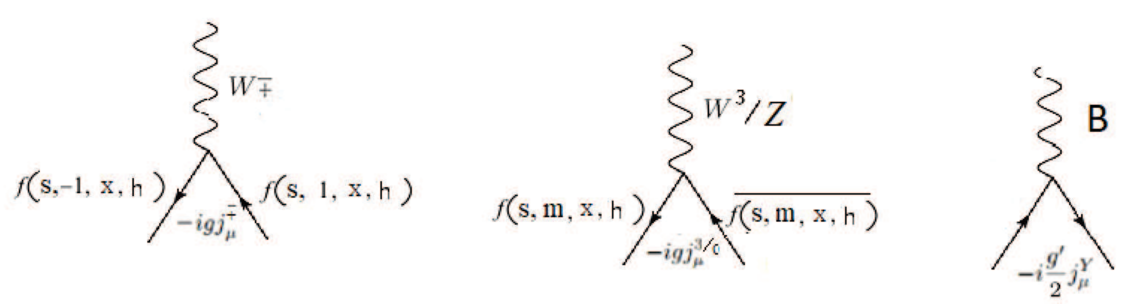
In this way, the $W^\div$ and Z^0 are transient conglomeration particles - not fundamental particles at all.

This may be generalized as follows:
Rewriting the above Weak force/interaction (vertex) interaction currents using the $f(x_1, x_2, x_3, x_4)$ notation:

$\nu_{e_R} = \nu(1) = f(0,\ 1,-1,-1)$	$\nu_\mu = \nu(2) = f(0,\ 1,-1,\ 0)$	$\nu_\tau = \nu(3) = f(0,\ 1,-1,\ 1)$
$e^- = e(1) = f(0,-1,\ 0,-1)$	$\mu^- = e(2) = f(0,-1,\ 0,\ 0)$	$\tau^- = e(3) = f(0,-1,\ 0,\ 1)$
$\nu_{e_L} = \nu(1) = f(0,\ 1,\ 1,-1)$	$\nu_\mu = \nu(2) = f(0,\ 1,\ 1,\ 0)$	$\nu_\tau = \nu(3) = f(0,\ 1,\ 1,\ 1)$
$u_R = u_1(1) = f(1,-1,-1,-1)$	$c_R = u_1(2) = f(1,-1,-1,\ 0)$	$t_R = u_1(3) = f(1,-1,-1,\ 1)$
$u_G = u_0(1) = f(1,-1,\ 0,-1)$	$c_G = u_0(2) = f(1,-1,\ 0,\ 0)$	$t_G = u_0(3) = f(1,-1,\ 0,\ 1)$
$u_B = u_{-1}(1) = f(1,-1,\ 1,-1)$	$c_B = u_{-1}(2) = f(1,-1,\ 1,\ 0)$	$t_B = u_{-1}(3) = f(1,-1,\ 1,\ 1)$
$d_R = d_1(1) = f(1,\ 1,-1,-1)$	$s_R = d_1(2) = f(1,\ 1,-1,\ 0)$	$b_R = d_1(3) = f(1,\ 1,-1,\ 1)$
$d_G = d_0(1) = f(1,\ 1,\ 0,-1)$	$s_G = d_0(2) = f(1,\ 1,\ 0,\ 0)$	$b_G = d_0(3) = f(1,\ 1,\ 0,\ 1)$
$d_B = d_{-1}(1) = f(1,\ 1,\ 1,-1)$	$s_B = d_{-1}(2) = f(1,\ 1,\ 1,\ 0)$	$b_B = d_{-1}(3) = f(1,\ 1,\ 1,\ 1)$

$x_1 = \begin{cases} -1: & \text{lepton} \\ 1: & \text{quark} \end{cases}$	$x_2 = \begin{cases} -1: & \text{up} \\ 1: & \text{down} \end{cases}$
$x_3 = \begin{cases} \underset{(x_1=1)}{color} = \begin{cases} -1: & \text{R} \\ 0: & \text{G} \\ 1: & \text{B} \end{cases} \\ \underset{(x_1=0)}{right/left} = \begin{cases} -1: & \text{Right} \\ 1: & \text{Left} \end{cases} \end{cases}$	$x_4 = generation = \begin{cases} -1: \\ 0: \\ 1: \end{cases}$

and noting the vertex diagrams for the quarks, neutrinos, and Z^0 :



$s \in \{0,1\} = \begin{cases} 0: & \text{lepton} \\ 1: & \text{quark} \end{cases}$

$m \in \{-1,1\} = \begin{cases} -1: & \text{up} \\ 1: & \text{down} \end{cases}$

$x \in \{-1,0,1\} = \begin{cases} -1: & \text{Right} \\ 1: & \text{Left} \end{cases}$

$h \in \{-1,0,1\} = \begin{cases} -1: \\ 0: & generation \\ 1: \end{cases}$

$$j^Y_{\mu(s,m,h)} = \left(-\overline{f(s,m,-1,h)}\gamma_\mu f(s,m,-1,h) - \overline{f(s,m,1,h)}\gamma_\mu f(s,m,1,h)\right) + \left(-\overline{f(s,m,-1,h)}\gamma_\mu f(s,m,-1,h) - \overline{f(s,-m,1,h)}\gamma_\mu f(s,-m,1,h)\right)$$

$$j^{W^3/Z}_{\mu(s,m,h)} = \left(\overline{f(s,-m,1,h)}\gamma_\mu f(s,-m,1,h) - \overline{f(s,m,1,h)}\gamma_\mu f(s,m,1,h)\right)$$

$$\begin{aligned} J^Y_{\mu(s,m,h)} &= \left(-\overline{f(s,m,-1,h)}\gamma_\mu f(s,m,-1,h) - \overline{f(s,m,\ 1,h)}\gamma_\mu f(s,m,\ 1,h)\right) + \\ &+ \left(-\overline{f(s,m,-1,h)}\gamma_\mu f(s,m,-1,h) - \overline{f(s,-m,\ 1,h)}\gamma_\mu f(s,-m,\ 1,h)\right) \\ J^{W3/Z}_{\mu(s,m,h)} &= \left(\overline{f(s,-m,\ 1,h)}\gamma_\mu f(s,-m,\ 1,h) - \overline{f(s,m,\ 1,h)}\gamma_\mu f(s,m,\ 1,h)\right) \end{aligned}$$

$\Downarrow$

$B_{\mu(s,m,h)} \equiv -\frac{i}{2}g_e\left[\left(-\overline{f(s,m,-1,h)}\gamma_\mu f(s,m,-1,h) - \overline{f(s,m,\ 1,h)}\gamma_\mu f(s,m,\ 1,h)\right) + \right.$
$\left. + \left(-\overline{f(s,m,-1,h)}\gamma_\mu f(s,m,-1,h) - \overline{f(s,-m,\ 1,h)}\gamma_\mu f(s,-m,\ 1,h)\right)\right]$
$W^3_{\mu(s,m,h)} \equiv -\frac{i}{2}g_e\left[\left(\overline{f(s,-m,\ 1,h)}\gamma_\mu f(s,-m,\ 1,h) - \overline{f(s,m,\ 1,h)}\gamma_\mu f(s,m,\ 1,h)\right)\right]$

Thus, again, define:

$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$	$\Leftrightarrow$	$A_\mu = B^\mu$	$W^3_\mu = Z^\mu$	$\Leftrightarrow$	$A^\mu_3 \equiv -A^\mu \sin\theta_W + Z^\mu \cos\theta_W$
$Z_\mu \equiv -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W$		$B_\mu = A^\mu$	$Z_\mu \equiv A^\mu_3$		$B^\mu = A^\mu \cos\theta_W + Z^\mu \sin\theta_W$

$(\sin^2\theta_W \approx 0.22)$

In this way, the $W^\pm$ and Z^0 are transient conglomeration particles - not fundamental particles at all.

***

Reference [2][8] evince the design of the I/O of the vertex diagrams, which may be classified as generation changing or not.

m_W/m_Z are invariant of generation change

Interactions of energy of at least the m_W/m_Z may produce m_W/m_Z resultant(s).

Since $|m_W|$ & $|m_Z|$ are always always the same these resultant weak interaction intermediate envelopes may be single wvelengths (additional interaction energies may be photons, Z's, or $W^+ W^-$pairs)

From measured minimum/maximum duration/extension $\Delta t_W/\Delta t_Z$ & $\Delta\ell_W/\Delta\ell_Z$ of m_W/m_Z may be determined using de Broglie Velocity Wavelength-Frequency:

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = hf \Rightarrow (m_0c)^2 = \left(\frac{hf}{c}\right)^2\left(1-\left(\frac{v}{c}\right)^2\right) \Rightarrow \left(\frac{m_0c^2}{hf}\right)^2 = 1-\left(\frac{v}{c}\right)^2$$

$$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{h}{\lambda} \Rightarrow (m_0 c)^2\left(\frac{v}{c}\right)^2 = \left(\frac{h}{\lambda}\right)^2\left(1-\left(\frac{v}{c}\right)^2\right)$$

$$\Rightarrow \frac{p}{E} = \left(\frac{v}{c^2}\right) = \frac{\left(\frac{h}{\lambda}\right)}{(hf)} \Rightarrow \frac{1}{f\lambda} = \frac{v}{c^2} \Rightarrow \left(\frac{v}{c}\right) = \frac{c}{f\lambda} \Rightarrow \frac{v}{c^2}\lambda = \frac{1}{f} \Rightarrow f\lambda = \frac{c^2}{v} \Rightarrow v = \frac{c^2}{f\lambda}$$

$$\Rightarrow \left(\frac{m_0 c^2}{h}\right)^2\left(\frac{1}{f}\right)^2 = 1-\left(\frac{v}{c}\right)^2 \Rightarrow \left(\frac{v}{c}\right)^2 = 1-\left(\frac{m_0 c^2}{h}\right)^2\left(\frac{1}{f}\right)^2$$

$$\Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{m_0 c^2}{h}\right)^2\left(\frac{1}{f}\right)^2}$$

$$\Rightarrow p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{h}{\lambda} = \frac{h}{\frac{c}{f_c}} = \left(\frac{h}{c}\right)f_c = \frac{h}{c}\frac{v}{\lambda_c} = \frac{\left(\frac{h}{c}v\right)}{\lambda_c} = \left(\frac{h}{c\lambda_c}\right)v$$

$$\Rightarrow \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \left(\frac{h}{c\lambda_c}\right) \Rightarrow \lambda_c = \left(\frac{h}{m_0 c}\right)\sqrt{1-\frac{v^2}{c^2}} = \frac{c}{f} \quad \checkmark$$

$$\left(\frac{h}{m_0}\right)\left(\frac{1}{v}\right)\sqrt{1-\left(\frac{v}{c}\right)^2} = \lambda = \frac{c}{f_c} \Rightarrow f_c = \left(\frac{\left(\frac{m_0 c}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)v$$

$\frac{v}{c} = \sqrt{1-\left(\frac{m_0 c^2}{h}\right)^2\left(\frac{1}{f}\right)^2}$	$f = \frac{\left(\frac{m_0 c^2}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$	$\lambda_c \equiv \frac{c}{f}$	$\lambda_c = \left(\frac{h}{m_0 c}\right)\sqrt{1-\frac{v^2}{c^2}}$
$\frac{v}{c} = \frac{1}{\sqrt{1+\left(\frac{m_0 c}{h}\right)^2\lambda^2}}$	$\lambda = \left(\frac{h}{m_0}\right)\left(\frac{1}{v}\right)\sqrt{1-\left(\frac{v}{c}\right)^2}$	$f_c \equiv \frac{c}{\lambda}$	$f_c = \left(\frac{\left(\frac{m_0 c}{h}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)v$
$E = hf$	$p = \frac{h}{\lambda}$	$p = \left(\frac{h}{c}\right)f_c$	$E = \frac{(hc)}{\lambda_c}$

$$\Delta t = \frac{1}{f} \;\&\; \Delta\ell = \lambda_c$$

thus, establish the energy of the intermediate envelope via the velocity.
(Clearly the minimum energies are ($v = 0$) $m_W c^2/m_Z c^2$ and wavelengths $\frac{h}{m_W c}/\frac{h}{m_Z c}$

The four parameters (s, m, x, h) of this weak theory may have an alternative relationship to the four parameters (m_1, m_2, m_3, m_0) :

$|m|^2 = m_1^2 + m_2^2 + m_3^2 + m_0^2 \Leftrightarrow m = (m_1, m_2, m_3, m_0)$

Since Left/Right & RGB do not affect fermion mass, consider:

$m_1 \propto s \in \{-1,1\}$: $\begin{cases} -1: & \text{lepton} \\ 1: & \text{quark} \end{cases}$	$m_2 \propto m \in \{-1,1\}$: $\begin{cases} -1: & \text{up} \\ 1: & \text{down} \end{cases}$	$m_3 \propto h \in \{-1,0,1\}$: $\begin{cases} -1: & \\ 0: & \textit{generation} \\ 1: & \end{cases}$

Now, for the pythagorean quadruples (m_1, m_2, m_3, m_0):

$$\frac{m_i}{|m_i|} = \pm 1, \quad (i \in \{1,2,3\})$$

so, without specifying the order of m_1, m_2, m_3 , let: $|m_1| < |m_2| < |m_3|$:

$$\frac{|m_1|-|m_2|}{||m_1|-|m_2||} = -1 \quad \& \quad \frac{|m_3|-|m_2|}{||m_3|-|m_2||} = +1$$

$$\Rightarrow \left(\frac{|m_1|-|m_2|}{||m_1|-|m_2||}, \frac{|m_3|-|m_2|}{||m_3|-|m_2||}\right) = \{-1,+1\} : \begin{cases} -1: & \text{Left} \\ +1: & \text{Right} \end{cases}$$

This is equivalent to the Right/Left specification.

and, for $(|m_1|,|m_2|,|m_3|) \propto (2,5,14) = 6\left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right); (4,10,28) = 6\left(\frac{2}{3},\frac{5}{3},\frac{14}{3}\right) = 12\left(\frac{1}{3},\frac{5}{6},\frac{7}{3}\right)$;etc.

$$\Rightarrow \left\{\begin{array}{|c|} \hline 0 > |m_1|-|m_2| \equiv (0 \bmod 3) \\ \hline 0 = |m_2|-|m_2| \equiv (0 \bmod 3) \\ \hline 0 < |m_3|-|m_2| \equiv (0 \bmod 3) \\ \hline \end{array}\right\}$$

and, of course, the average (right in the middle):

$$\frac{\left(\frac{|m_1|-|m_2|}{||m_1|-|m_2||} + \frac{|m_2|-|m_3|}{||m_2|-|m_3||}\right)}{2} = \frac{(-1)+(+1)}{2} = 0$$

$$\Rightarrow \left(\frac{|m_1|-|m_2|}{||m_1|-|m_2||}, \frac{\left(\frac{|m_1|-|m_2|}{||m_1|-|m_2||} + \frac{|m_2|-|m_3|}{||m_2|-|m_3||}\right)}{2}, \frac{|m_2|-|m_3|}{||m_2|-|m_3||}\right) = \{-1,0,1\} : \left\{ \underset{(x_1=1)}{color} = \begin{cases} -1: & \text{R} \\ 0: & \text{G} \\ 1: & \text{B} \end{cases} \right.$$

This is equivalent to the RGB specification!!!

And, finally, the Covariant Helmholtzian analysis:

Theorem II.1: For differentiable functions $\Phi, f^i, f_+^i, f_-^i, g_{ij}; \forall i,j \in \mathbb{N}$:

If: $\exists \mathbf{J}(x_3,x_2,x_1,x_0) \ni$

$$\mathbf{J}(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix} -D_{02} & D_{32}^{\Leftrightarrow} & -D_{22}^{\Leftrightarrow} & -D_{12} \\ -D_{32}^{\Leftrightarrow} & -D_{02} & D_{12}^{\Leftrightarrow} & -D_{22} \\ D_{22}^{\Leftrightarrow} & -D_{12}^{\Leftrightarrow} & -D_{02} & -D_{32} \\ -D_{12}^{\Updownarrow} & -D_{22}^{\Updownarrow} & -D_{32}^{\Updownarrow} & D_{02}^{\Updownarrow} \end{pmatrix} \begin{pmatrix} -D_{01}^{\Updownarrow} & -D_{31}^{\Leftrightarrow} & D_{21}^{\Leftrightarrow} & -D_{11} \\ D_{31}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & -D_{21} \\ -D_{21}^{\Leftrightarrow} & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & -D_{31} \\ -D_{11}^{\Updownarrow} & -D_{21}^{\Updownarrow} & -D_{31}^{\Updownarrow} & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

and

$$\exists \Phi(x_3,x_2,x_1,x_0) \equiv \begin{pmatrix} -D_{01}^{\Updownarrow} & -D_{31}^{\Leftrightarrow} & D_{21}^{\Leftrightarrow} & -D_{11} \\ D_{31}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & -D_{11}^{\Leftrightarrow} & -D_{21} \\ -D_{21}^{\Leftrightarrow} & D_{11}^{\Leftrightarrow} & -D_{01}^{\Updownarrow} & -D_{31} \\ -D_{11}^{\Updownarrow} & -D_{21}^{\Updownarrow} & -D_{31}^{\Updownarrow} & D_{01} \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

where:

$$D_{ij}^{+} \equiv (\partial_i + g_{ijh}^{\lambda}) \quad , \quad D_{ij}^{-} \equiv (\partial_i - g_{ijh}^{\lambda}) \qquad \text{[where: } D_{ik}^{\pm} f_{\sigma}^{m} = (\partial_i \pm g_{ijk}^{\lambda}) f_{\sigma}^{m} = \partial_i f_{\sigma}^{m} \pm g_{ijk}^{m} f_{\sigma}^{j} \text{]}$$

$$D_{ij} \equiv \begin{pmatrix} D_{ij}^{+} & 0 \\ 0 & D_{ij}^{-} \end{pmatrix} \; , \; D_{ij}^{\Updownarrow} \equiv \begin{pmatrix} D_{ij}^{-} & 0 \\ 0 & D_{ij}^{+} \end{pmatrix} ,$$

$$D_{ij}^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_{ij}^{-} \\ D_{ij}^{+} & 0 \end{pmatrix} \; , \; D_{ij}^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_{ij}^{+} \\ D_{ij}^{-} & 0 \end{pmatrix}$$

and:

$$J^i \equiv \begin{pmatrix} J_{+}^{i} \\ J_{-}^{i} \end{pmatrix} \; , \; \Phi^i \equiv \begin{pmatrix} \Phi_{+}^{i} \\ \Phi_{-}^{i} \end{pmatrix} \; , \; f^i \equiv \begin{pmatrix} f_{+}^{i} \\ f_{-}^{i} \end{pmatrix}$$

then:

$$\begin{pmatrix} -D_{02}\Phi^1 + D_{32}^{\Leftrightarrow}\Phi^2 - D_{22}^{\Leftrightarrow}\Phi^3 - D_{12}\Phi^0 \\ -D_{32}^{\Leftrightarrow}\Phi^1 - D_{02}\Phi^2 + D_{12}^{\Leftrightarrow}\Phi^3 - D_{22}\Phi^0 \\ D_{22}^{\Leftrightarrow}\Phi^1 - D_{12}^{\Leftrightarrow}\Phi^2 - D_{02}\Phi^3 - D_{32}\Phi^0 \\ -D_{12}^{\Updownarrow}\Phi^1 - D_{22}^{\Updownarrow}\Phi^2 - D_{32}^{\Updownarrow}\Phi^3 + D_{02}^{\Updownarrow}\Phi^0 \end{pmatrix} = \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} \quad \& \quad \begin{pmatrix} -D_{01}^{\Updownarrow}f^1 - D_{31}^{\Leftrightarrow}f^2 + D_{21}^{\Leftrightarrow}f^3 - D_{11}f^0 \\ D_{31}^{\Leftrightarrow}f^1 - D_{01}^{\Updownarrow}f^2 - D_{11}^{\Leftrightarrow}f^3 - D_{21}f^0 \\ -D_{21}^{\Leftrightarrow}f^1 + D_{11}^{\Leftrightarrow}f^2 - D_{01}^{\Updownarrow}f^3 - D_{31}f^0 \\ -D_{11}^{\Updownarrow}f^1 - D_{21}^{\Updownarrow}f^2 - D_{31}^{\Updownarrow}f^3 + D_{01}f^0 \end{pmatrix} \equiv \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \Phi^3 \\ \Phi^0 \end{pmatrix}$$

and:

$J_{+}^{1} = -\partial_0\Phi_{+}^{1} - \partial_1\Phi_{+}^{0} + \partial_3\Phi_{-}^{2} - \partial_2\Phi_{-}^{3} + (-g_{0j2}^{1} - g_{1j2}^{0})\Phi_{+}^{j} + (-g_{3j2}^{2} + g_{2j2}^{3})\Phi_{-}^{j}$	$-\partial_0 f_{+}^{1} - \partial_1 f_{+}^{0} - \partial_3 f_{-}^{2} + \partial_2 f_{-}^{3} + (+g_{0j1}^{1} - g_{1j1}^{0}) f_{+}^{j} + (+g_{3j1}^{2} - g_{2j1}^{3}) f_{-}^{j} = \Phi_{+}^{1}$
$J_{-}^{1} = -\partial_0\Phi_{-}^{1} - \partial_1\Phi_{-}^{0} + \partial_3\Phi_{+}^{2} - \partial_2\Phi_{+}^{3} + (+g_{3j2}^{2} - g_{2j2}^{3})\Phi_{+}^{j} + (+g_{0j2}^{1} + g_{1j2}^{0})\Phi_{-}^{j}$	$-\partial_0 f_{-}^{1} - \partial_1 f_{-}^{0} - \partial_3 f_{+}^{2} + \partial_2 f_{+}^{3} + (-g_{3j1}^{2} + g_{2j1}^{3}) f_{+}^{j} + (-g_{0j1}^{1} + g_{1j1}^{0}) f_{-}^{j} = \Phi_{-}^{1}$
$J_{+}^{2} = -\partial_0\Phi_{+}^{2} - \partial_2\Phi_{+}^{0} + \partial_3\Phi_{-}^{1} + \partial_1\Phi_{-}^{3} + (-g_{0j2}^{2} - g_{2j2}^{0})\Phi_{+}^{j} + (-g_{3j2}^{1} - g_{1j2}^{3})\Phi_{-}^{j}$	$-\partial_0 f_{+}^{2} - \partial_2 f_{+}^{0} + \partial_3 f_{-}^{1} - \partial_1 f_{-}^{3} + (-g_{2j1}^{0} + g_{0j1}^{2}) f_{+}^{j} + (-g_{3j1}^{1} + g_{1j1}^{3}) f_{-}^{j} = \Phi_{+}^{2}$
$J_{-}^{2} = -\partial_0\Phi_{-}^{2} - \partial_2\Phi_{-}^{0} + \partial_3\Phi_{+}^{1} + \partial_1\Phi_{+}^{3} + (+g_{3j2}^{1} + g_{1j2}^{3})\Phi_{+}^{j} + (+g_{0j2}^{2} + g_{2j2}^{0})\Phi_{-}^{j}$	$-\partial_0 f_{-}^{2} - \partial_2 f_{-}^{0} + \partial_3 f_{+}^{1} - \partial_1 f_{+}^{3} + (+g_{3j1}^{1} - g_{1j1}^{3}) f_{+}^{j} + (-g_{0j1}^{2} + g_{2j1}^{0}) f_{-}^{j} = \Phi_{-}^{2}$
$J_{+}^{3} = -\partial_0\Phi_{+}^{3} - \partial_3\Phi_{+}^{0} + \partial_2\Phi_{-}^{1} - \partial_1\Phi_{-}^{2} + (-g_{0j2}^{3} - g_{3j2}^{0})\Phi_{+}^{j} + (-g_{2j2}^{1} + g_{1j2}^{2})\Phi_{-}^{j}$	$-\partial_0 f_{+}^{3} - \partial_3 f_{+}^{0} - \partial_2 f_{-}^{1} + \partial_1 f_{-}^{2} + (-g_{3j1}^{0} + g_{0j1}^{3}) f_{+}^{j} + (+g_{2j1}^{1} - g_{1j1}^{2}) f_{-}^{j} = \Phi_{+}^{3}$
$J_{-}^{3} = -\partial_0\Phi_{-}^{3} - \partial_3\Phi_{-}^{0} + \partial_2\Phi_{+}^{1} - \partial_1\Phi_{+}^{2} + (+g_{2j2}^{1} - g_{1j2}^{2})\Phi_{+}^{j} + (+g_{0j2}^{3} + g_{3j2}^{0})\Phi_{-}^{j}$	$-\partial_0 f_{-}^{3} - \partial_3 f_{-}^{0} - \partial_2 f_{+}^{1} + \partial_1 f_{+}^{2} + (-g_{2j1}^{1} + g_{1j1}^{2}) f_{+}^{j} + (+g_{3j1}^{0} - g_{0j1}^{3}) f_{-}^{j} = \Phi_{-}^{3}$
$J_{+}^{0} = +\partial_0\Phi_{+}^{0} - \partial_1\Phi_{+}^{1} - \partial_2\Phi_{+}^{2} - \partial_3\Phi_{+}^{3} + (+g_{1j2}^{1} + g_{2j2}^{2} + g_{3j2}^{3} - g_{0j2}^{0})\Phi_{+}^{j}$	$+\partial_0 f_{+}^{0} - \partial_1 f_{+}^{1} - \partial_2 f_{+}^{2} - \partial_3 f_{+}^{3} + (+g_{0j1}^{0} + g_{1j1}^{1} + g_{2j1}^{2} + g_{3j1}^{3}) f_{+}^{j} = \Phi_{+}^{0}$
$J_{-}^{0} = +\partial_0\Phi_{-}^{0} - \partial_1\Phi_{-}^{1} - \partial_2\Phi_{-}^{2} - \partial_3\Phi_{-}^{3} + (-g_{1j2}^{1} - g_{2j2}^{2} - g_{3j2}^{3} + g_{0j2}^{0})\Phi_{-}^{j}$	$+\partial_0 f_{-}^{0} - \partial_1 f_{-}^{1} - \partial_2 f_{-}^{2} - \partial_3 f_{-}^{3} + (-g_{0j1}^{0} - g_{1j1}^{1} - g_{2j1}^{2} - g_{3j1}^{3}) f_{-}^{j} = \Phi_{-}^{0}$

or

$\mathbf{J}^1 = \begin{pmatrix} J_{+}^{1} \\ J_{-}^{1} \end{pmatrix} = -\partial_0\mathbf{\Phi}^1 - \partial_1\mathbf{\Phi}^0 + \partial_3\mathbf{\Phi}^2 - \partial_2\mathbf{\Phi}^3 + \left[(-g_{0j2}^{1} - g_{1j2}^{0})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g_{3j2}^{2} + g_{2j2}^{3})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right]\mathbf{\Phi}^j$
$\mathbf{J}^2 = \begin{pmatrix} J_{+}^{2} \\ J_{-}^{2} \end{pmatrix} = -\partial_0\mathbf{\Phi}^2 - \partial_2\mathbf{\Phi}^0 + \partial_3\mathbf{\Phi}^1 + \partial_1\mathbf{\Phi}^3 + \left[(-g_{0j2}^{2} - g_{2j2}^{0})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g_{3j2}^{1} - g_{1j2}^{3})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right]\mathbf{\Phi}^j$
$\mathbf{J}^3 = \begin{pmatrix} J_{+}^{3} \\ J_{-}^{3} \end{pmatrix} = -\partial_0\mathbf{\Phi}^3 - \partial_3\mathbf{\Phi}^0 + \partial_2\mathbf{\Phi}^1 - \partial_1\mathbf{\Phi}^2 + \left[(-g_{0j2}^{3} - g_{3j2}^{0})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g_{2j2}^{1} + g_{1j2}^{2})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right]\mathbf{\Phi}^j$
$\mathbf{J}^0 = \begin{pmatrix} J_{+}^{0} \\ J_{-}^{0} \end{pmatrix} = +\partial_0\mathbf{\Phi}^0 - \partial_1\mathbf{\Phi}^1 - \partial_2\mathbf{\Phi}^2 - \partial_3\mathbf{\Phi}^3 + (+g_{1j2}^{1} + g_{2j2}^{2} + g_{3j2}^{3} - g_{0j2}^{0})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\mathbf{\Phi}^j$

$-\partial_0\mathbf{f}^1 - \partial_1\mathbf{f}^0 - \partial_3\mathbf{f}^2 + \partial_2\mathbf{f}^3 + \left[(+g_{0j1}^{1} - g_{1j1}^{0})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (+g_{3j1}^{2} - g_{2j1}^{3})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right]\mathbf{f}^j = \begin{pmatrix} \Phi_{+}^{1} \\ \Phi_{-}^{1} \end{pmatrix} = \mathbf{\Phi}^1$
$-\partial_0\mathbf{f}^2 - \partial_2\mathbf{f}^0 + \partial_3\mathbf{f}^1 - \partial_1\mathbf{f}^3 + \left[(-g_{2j1}^{0} + g_{0j1}^{2})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (-g_{3j1}^{1} + g_{1j1}^{3})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right]\mathbf{f}^j = \begin{pmatrix} \Phi_{+}^{2} \\ \Phi_{-}^{2} \end{pmatrix} = \mathbf{\Phi}^2$
$-\partial_0\mathbf{f}^3 - \partial_3\mathbf{f}^0 - \partial_2\mathbf{f}_{-}^{1} + \partial_1\mathbf{f}_{-}^{2} + \left[(-g_{3j1}^{0} + g_{0j1}^{3})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + (+g_{2j1}^{1} - g_{1j1}^{2})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right]\mathbf{f}^j = \begin{pmatrix} \Phi_{+}^{3} \\ \Phi_{-}^{3} \end{pmatrix} = \mathbf{\Phi}^3$
$-\partial_1\mathbf{f}^1 - \partial_2\mathbf{f}^2 - \partial_3\mathbf{f}^3 + \partial_0\mathbf{f}^0 + \left[(+g_{0j1}^{0} + g_{1j1}^{1} + g_{2j1}^{2} + g_{3j1}^{3})\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]\mathbf{f}^j = \begin{pmatrix} \Phi_{+}^{0} \\ \Phi_{-}^{0} \end{pmatrix} = \mathbf{\Phi}^0$

and:

$$\mathbf{J} = \left(\begin{array}{c} \left(\begin{array}{l} (\partial_0^2 + \partial_3^2 + \partial_2^2 + \partial_1^2)f_+^1 + \\ +\partial_0([+g_{11h}^0 - g_{01h}^1]f_+^h + [-g_{31h}^2 + g_{21h}^3]f_-^h) + \\ +\partial_1([-g_{01h}^0 - g_{11h}^1 - g_{21h}^2 - g_{31h}^3]f_+^h) + \\ +\partial_2([+g_{21h}^1 - g_{11h}^2]f_+^h + [-g_{31h}^0 + g_{01h}^3]f_-^h) + \\ +\partial_3([-g_{11h}^3 + g_{31h}^1]f_+^h + [-g_{01h}^2 + g_{21h}^0]f_-^h) + \\ +(-g_{12h}^0 + g_{02h}^1)\partial_0 f_+^h + (+g_{32h}^2 - g_{22h}^3)\partial_0 f_-^h \\ +(+g_{02h}^0 + g_{12h}^1 + g_{22h}^2 + g_{32h}^3)\partial_1 f_+^h + \\ +(-g_{22h}^1 + g_{12h}^2)\partial_2 f_+^h + (+g_{32h}^0 - g_{02h}^3)\partial_2 f_-^h + \\ +(-g_{32h}^1 + g_{12h}^3)\partial_3 f_+^h + (-g_{22h}^0 + g_{02h}^2)\partial_3 f_-^h + \\ +([-g_{12k}^0 - g_{02k}^1]g_{01h}^k + [+g_{02k}^0 - g_{12k}^1 + g_{22k}^2 + g_{32k}^3]g_{11h}^k + [-g_{22k}^1 - g_{12k}^2]g_{21h}^k + [-g_{32k}^1 - g_{12k}^3]g_{31h}^k)f_+^h + \\ +([+g_{32k}^2 - g_{22k}^3]g_{01h}^k + [-g_{32k}^0 + g_{02k}^3]g_{21h}^k + [+g_{22k}^0 - g_{02k}^2]g_{31h}^k)f_-^h \end{array} \right) \\ \vdots \end{array} \right)$$

Thus, for suitable g_{mjn}^h (Lie group G, G×G→G $\Rightarrow W_{\mu\nu}^h \in$G $\Rightarrow b_{\mu\nu}^m W_{\mu\nu}^j W_{\mu\nu}^h = W_{\mu\nu}^m \in$G):

$$([-g_{12k}^0 - g_{02k}^1]g_{01h}^k + [+g_{02k}^0 - g_{12k}^1 + g_{22k}^2 + g_{32k}^3]g_{11h}^k + [-g_{22k}^1 - g_{12k}^2]g_{21h}^k + [-g_{32k}^1 - g_{12k}^3]g_{31h}^k)f_+^h =$$
$$= b_{\mu\nu}^{h_1} f_+^\mu f_+^\nu + b_{\mu\nu}^{h_2} f_+^\mu f_-^\nu + b_{\mu\nu}^{h_3} f_-^\mu f_-^\nu$$

etc.

So, quantum field theory flows from the covariant Helmholtzian, including the weak interaction.

And, the next part of the covariant Helmholtzian theorem II.1 and corollaries, generalizing the ordinary Helmholtzian factorization, shows how it yields gravitational field equations in curved spacetime.

And, so, the Helmholtzian approach yields unification and more without tricks or obfuscations.

Clearly, the (Covariant) Helmholtzian factorization approach is superior to the gauge theory/combined symmetry group/symmetry breaking/Higgs mechanism patchwork quilt approach (which doesn't even include gravitation).

THAT CAN BE DESIGNED BY A SUFFICIENTLY INTELLIGENT MIND

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