Brook Taylor

by

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Abstract

Brook Taylor was an English mathematician, invented integration by parts, and discovered the celebrated formula known as Taylor's expansion.

Introduction

Brook Taylor (1685-1731), British mathematician, who pioneered the infinitesimal calculus and wrote two works on perspective . Because he did not publish his results, some were claimed by Johann Bernoulli, and the importance of Taylor's theorem was only recognized some 60 years later, by Lagrange. He became a fellow of the Royal Society, and sat on the committee that adjudicated between the claims of Newton and Leibniz to have first invented infinitesimal calculus.

This note is dedicated to Brook Taylor

Taylor's theorem

Taylor's theorem (and its variants) is widely used in several areas of mathematical analysis, functional analysis, and partial differential equations.

Theorem 1. (Fundamental theorem of calculus for the Lebesgue integral) A function $f:[a,b]\to\mathbb{R}$ is absolutely continuous if and only if it is differentiable almost everywhere, its derivative $f'\in L^1[a,b]$ and, for each $x\in[a,b]$,

$$f(x)=f(a)+\int_a^x f'(t)\,dt$$

Theorem 2. (Taylor's theorem) Given a function $f:[a,b]\to\mathbb{R}$ such that the Nth derivative $f^{(N)}$ is absolutely continuous on $[a,b]$, them for all $x\in[a,b]$

$$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}\ f''(a)+\cdots+\frac{(x-a)^N}{N!}\ f^{(N)}(a)+R_N(x)$$

where $R_N(x)$ satisfies the inequality

$$|R_N(x)|\le\sup_{c\in[a,x]}\left|f^{(N+1)}(c)\right|\frac{(x-a)^{N+1}}{(N+1)!}\ ,$$

and $R_N(x)$ is given exactly by the expression

$$R_N(x)=\int_a^x\frac{(x-t)^N}{N!}\ f^{(N+1)}(t)\,dt$$

Taylor series for $\arctan(x)=tan^{-1}(x)$

The Taylor series of an infinitely differentiable function f is given by:

$$f(x)=\sum_{n=0}^{\infty}\frac{(x-a)^n f^{(n)}(a)}{n!}\ ,$$

where $f^{(n)}(a)$ denotes the nth derivative of f . If we put $a=0$, we get a series named after Colin Maclaurin (1698-1746), who published it in his A Treatise of Fluxions.

If $f(x)=\tan^{-1}x$, we have

$$\tan^{-1}x=\tan^{-1}a+\sum_{n=1}^{\infty}\frac{(x-a)^n\,p_n(a)}{n!\ (1+a^2)^n}$$

where

$$p_{n+1}(a)=(1+a^2)p_n'(a)-2\,n\,a\,p_n(a)\ ,p_1(a)=1\ \ ,n=1,2,3,\ldots$$

If $a=0$, we get

$$\tan^{-1}x=\sum_{n=0}^{\infty}\frac{(-1)^n\,x^{2n+1}}{2n+1}\quad,\qquad |x|<1$$

recall that

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots$$

Some formulas for Pi

Entry 1. For $0<x\le\frac{\pi}{2}$, we have

$$x=\sin x+\sum_{n=1}^{\infty}2^n\left(1-\cos\left(\frac{x}{2^n}\right)\right)\sin\left(\frac{x}{2^n}\right)$$

Examples:

$$\pi = 2\sqrt{2} + \sum_{n=1}^{\infty} 2^{n+2}\left(1-\cos\left(\frac{\pi}{2^{n+2}}\right)\right)\sin\left(\frac{\pi}{2^{n+2}}\right)$$

$$\pi = 4\sqrt{2-\sqrt{2}} + \sum_{n=1}^{\infty} 2^{n+3}\left(1-\cos\left(\frac{\pi}{2^{n+3}}\right)\right)\sin\left(\frac{\pi}{2^{n+3}}\right)$$

Entry 2.

$$\pi = 4 - \sum_{n=1}^{\infty} 2^{n+2+2^{-n-1}}\left(2^{2^{-n-1}}\cos\left(\frac{\pi}{2^{n+2}}\right)-1\right)\sin\left(\frac{\pi}{2^{n+2}}\right)$$

Entry 3.

$$\pi = 4 + 2i\ln 2 + 4i\sum_{n=1}^{\infty} 2^{n-1}\left(F(-2^{-n},1,1,-i)-1\right)^2$$

$$\pi = 4 + 2i\ln 2 + 4i\sum_{n=1}^{\infty} 2^{n-1}\left(\left(\frac{1-i}{2}\right)F\left(-2^{-n},1,1,\frac{1+i}{2}\right)-1\right)^2$$

Remark: $i=\sqrt{-1}$ and F is the Gauss hypergeometric function.

Entry 4.

$$\pi^2 = 8 + \sum_{n=1}^{\infty} 2^{2n+1}\left(3+\cos\left(\frac{\pi}{2^n}\right)-4\sin\left(\frac{\pi}{2^{n+1}}\right)\right)$$

Entry 5. For $a>1$, we have

$$\pi = 4\sum_{n=1}^{\infty}\frac{a^{-n}}{n}\sum_{k=0}^{\left[\frac{n-1}{2}\right]}(-1)^k(a-1)^{n-2k-1}\binom{n}{2k+1}$$

Entry 6. For $a>0$, we have

$$\pi = \frac{4}{1+a^2} + 4\sum_{n=1}^{\infty}(1+a^2)^{-n-1}\sum_{k=0}^{n}\binom{n}{k}\frac{(-1)^k a^{2n-2k}}{2k+1}$$

Entry 7. For $a>0$, we have

$$\pi = 4\tan^{-1}a + 4\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(\frac{1-a}{1+a^2}\right)^n\sum_{k=0}^{\left[\frac{n-1}{2}\right]}(-1)^k a^{n-2k-1}\binom{n}{2k+1}$$

Entry 8.

$$\frac{9-4\sqrt{3}}{9}\,\pi = \sum_{n=1}^{\infty}\frac{2^{-n}}{2n+1}F\left(1,n+1,n+\frac{3}{2},\frac{3}{4}\right)$$

$$\frac{2\sqrt{3}-3}{12}\,\pi = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n+1}\left(\frac{2}{3}\right)^{n} F\left(n+1,n+\frac{1}{2},n+\frac{3}{2},-\frac{1}{3}\right)$$

Remark: $F(a,b,c,x)$ is the Gauss hypergeometric function.

Entry 9.

$$\pi = 4\sqrt{3}\sum_{n=0}^{\infty}\left(\frac{3-\sqrt{3}}{4}\right)^{n+1}\sum_{k=0}^{[n/2]}\binom{n}{n-2k}\frac{(-3)^{-k}}{2k+1}$$

$$\pi = 24\sum_{n=0}^{\infty}\left(\frac{\sqrt{6}+2\sqrt{3}-\sqrt{2}-4}{4}\right)^{n+1}\sum_{k=0}^{[n/2]}\binom{n}{n-2k}\frac{(-1)^{k}\left(\sqrt{2}-1\right)^{n-2k}}{2k+1}$$

Entry 10.

$$\sum_{n=0}^{\infty}\frac{6^{-n}}{2n+1}\sum_{k=[n/3]}^{[n/2]}2^{3k}\binom{k}{n-2k} = \frac{\pi}{2\sqrt{3}} + \sum_{n=0}^{\infty}\frac{3^{-n}}{2n+1}\sum_{k=0}^{[(n-1)/2]}\binom{n-k-1}{k}$$

Entry 11.

$$\pi^{3} = 8 + \sum_{n=0}^{\infty}2^{-3n}\left(h_{n+1}\right)^{3} + 12\sum_{n=0}^{\infty}2^{-n}\,h_{n+1}g_{n}g_{n+1}$$

where

$$h_{n} = \sum_{k=0}^{n}(-1)^{k}\binom{n}{k}(2k+1)^{-1}$$

$$g_{n} = \sum_{k=0}^{n}2^{-k}\,h_{k}$$

Entry 12.

$$\pi^{3} = 8 + 24\sum_{n=0}^{\infty}\frac{2^{n+1}\,h_{n}\,h_{n+1}}{\binom{2n+2}{n+1}(2n+3)} + 8\sum_{n=0}^{\infty}\frac{2^{3n+3}}{\left(\binom{2n+2}{n+1}(2n+3)\right)^{3}}$$

where

$$h_{n} = \sum_{k=0}^{n}\frac{2^{k}}{\binom{2k}{k}(2k+1)}$$

Entry 13. For $0 < a < x$, we have

$$\tan^{-1} x = \tan^{-1} a + \frac{2}{x^2+a^2} \sum_{n=0}^{\infty} \left(\frac{x^2+a^2}{2+x^2+a^2}\right)^{n+1} \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{2k+1} \left(x \left(\frac{x^2}{x^2+a^2}\right)^k - a \left(\frac{a^2}{x^2+a^2}\right)^k \right)$$

Examples:

$$\pi = 18 \sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^{n+1} \sum_{k=0}^{n} \binom{n}{k} \frac{(-2)^k}{2k+1} \left(\left(\frac{3}{4}\right)^k - \frac{1}{\sqrt{3}} \left(\frac{1}{4}\right)^k \right)$$

$$\pi = \frac{32}{37} \sum_{n=0}^{\infty} \left(\frac{37}{45}\right)^{n+1} \sum_{k=0}^{n} \binom{n}{k} \frac{(-2)^k}{2k+1} \left(3 \left(\frac{36}{37}\right)^k - \frac{1}{2} \left(\frac{1}{37}\right)^k \right)$$

Entry 14. For $x > 0$, we have

$$\tan^{-1} x = x\, F\left(1, \frac{1}{6}, \frac{7}{6}, -x^6\right) - \frac{x^3}{3}\, F\left(1, \frac{1}{2}, \frac{3}{2}, -x^6\right) + \frac{x^5}{5}\, F\left(1, \frac{5}{6}, \frac{11}{6}, -x^6\right)$$

$$\tan^{-1} x = \frac{x}{1+x^6}\, F\left(1,1, \frac{7}{6}, \frac{x^6}{1+x^6}\right) - \frac{x^3}{3(1+x^6)}\, F\left(1,1, \frac{3}{2}, \frac{x^6}{1+x^6}\right) + \frac{x^5}{5(1+x^6)}\, F\left(1,1, \frac{11}{6}, \frac{x^6}{1+x^6}\right)$$

$$\tan^{-1} x = \frac{x}{(1+x^6)^{1/6}}\, F\left(\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{x^6}{1+x^6}\right) - \frac{x^3}{3(1+x^6)^{1/2}}\, F\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{x^6}{1+x^6}\right) + \frac{x^5}{5(1+x^6)^{5/6}}\, F\left(\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{x^6}{1+x^6}\right)$$

Example:

$$\pi = 2\, F\left(1,1, \frac{7}{6}, \frac{1}{2}\right) - \frac{2}{3}\, F\left(1,1, \frac{3}{2}, \frac{1}{2}\right) + \frac{2}{5}\, F\left(1,1, \frac{11}{6}, \frac{1}{2}\right)$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.

Entry 15. For $|x\, y| < 1, |y| < 1$, we have

$$\tan^{-1}(x\, y) = (1+y^2) \sum_{n=0}^{\infty} (-1)^n\, y^{2n+1} \sum_{k=0}^{n} \frac{x^{2k+1}}{2k+1}$$

Example:

$$\pi = \frac{20}{3\sqrt{3}} \sum_{n=0}^{\infty} (-1)^n\, 3^{-2n} \sum_{k=0}^{n} \frac{3^k}{2k+1}$$

Entry 16. For $|x\,y| < 1, |y| < 1$, we have

$$\tan^{-1}(x\,y) = (1-y^2)\sum_{n=0}^{\infty} y^{2n+1} \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{2k+1}$$

Example:

$$\pi = \frac{16}{3\sqrt{3}} \sum_{n=0}^{\infty} 3^{-2n} \sum_{k=0}^{n} \frac{(-3)^k}{2k+1}$$

Entry 17. For $|x| < 1$, we have

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n\,(n+1) x^{2n+1} \left(\frac{1}{2n+1} + \frac{x^2}{2n+3}\right)$$

Entry 18. For $|x| < 1$, we have

$$\tan^{-1} x = (1-x^2)\sum_{n=0}^{\infty} x^{2n+1} \sum_{k=0}^{n} \frac{(-1)^k}{2k+1}$$

$$\tan^{-1} x = (1+x^2)\sum_{n=0}^{\infty} (-1)^n\, x^{2n+1} \sum_{k=0}^{n} \frac{1}{2k+1}$$

Examples:

$$\pi = \frac{4}{\sqrt{3}} \sum_{n=0}^{\infty} 3^{-n} \sum_{k=0}^{n} \frac{(-1)^k}{2k+1}$$

$$\pi = \frac{8}{\sqrt{3}} \sum_{n=0}^{\infty} (-3)^{-n} \sum_{k=0}^{n} \frac{1}{2k+1}$$

Entry 19. For $|x| < 1$, we have

$$\tan^{-1} x = \frac{x}{1+x^2} + \frac{2x^3}{3}\ F(2, \frac{3}{2}, \frac{5}{2}, -x^2)$$

Example:

$$\pi = \frac{3\sqrt{3}}{2} + \frac{4}{3\sqrt{3}}\ F(2, \frac{3}{2}, \frac{5}{2}, -\frac{1}{3})$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.

Entry 20. For $|x| < 1$, we have

$$\tan^{-1} x = \frac{x}{1+x^2} + \frac{2x^3}{(1+x^2)^2} - \frac{4x^3}{3}\ F\left(3, \frac{3}{2}, \frac{5}{2}, -x^2\right) + \frac{4x^5}{5}\ F\left(3, \frac{5}{2}, \frac{7}{2}, -x^2\right)$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.

Entry 21. For $0 \leq x < 1, k = 2,3,4,5, \dots$, we have

$$\tan^{-1} x = \sum_{n=1}^{2^{k-1}} \frac{(-1)^{n-1}\, x^{2n-1}}{2n-1}\ F(1, \frac{2n-1}{2^k}, \frac{2n-1+2^k}{2^k}, x^{2^k})$$

Example: $k = 2,3,4,5, \dots$, we have

$$\pi = 6\sqrt{3} \sum_{n=1}^{2^{k-1}} \frac{(-1)^{n-1}\, 3^{-n}}{2n-1}\ F(1, \frac{2n-1}{2^k}, \frac{2n-1+2^k}{2^k}, 3^{-2^{k-1}})$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.

Entry 22. For $n = 0,1,2,3, \dots$, we have

$$\tan^{-1} x = \sum_{k=0}^{n} \frac{x^{2k+1}}{2k+1}\ F\left(k+2, k+\frac{1}{2}, k+\frac{3}{2}, -x^2\right) + \frac{x^{2n+3}}{2n+3}\ F\left(n+2, n+\frac{3}{2}, n+\frac{5}{2}, -x^2\right)$$

Example: $n = 0,1,2,3, \dots$, we have

$$\pi = 2\sqrt{3} \sum_{k=0}^{n} \frac{3^{-k}}{2k+1}\ F\left(k+2, k+\frac{1}{2}, k+\frac{3}{2}, -\frac{1}{3}\right) + \frac{2\sqrt{3}\, 3^{-n-1}}{2n+3}\ F\left(n+2, n+\frac{3}{2}, n+\frac{5}{2}, -\frac{1}{3}\right)$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.

Entry 23. For $|x| < 1, i = \sqrt{-1}$, we have

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \binom{2n}{n} \frac{2^{-2n}}{n+1}\ Im\left((x\, i)^{n+1} F\left(\frac{1}{2}, n+1, n+2, -x\, i\right)\right)$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.
Remark: $Im(z)$ is the complex part of z .

Entry 24. For $-1 < x < 3, i = \sqrt{-1}$, we have

$$\tan^{-1} x = \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} \binom{2n}{n} 2^{-3n} \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{k+1}\ Im\left((x\, i)^{k+1} F\left(\frac{1}{2}, k+1, k+2, -x\, i\right)\right)$$

Remark: $F(a, b, c, x)$ is the Gauss hypergeometric function.
Remark: $Im(z)$ is the complex part of z .

Entry 25. For $-1 < x < 1$, we have

$$\tan^{-1} x = \sum_{n=1}^{\infty} (-1)^{n-1}\, x^{2n-1} H_n \left(\frac{(1+x^2)n(2n+1) - x^2}{4n^2-1} \right)$$

where

$$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad , n = 1,2,3, \ldots$$

Example:

$$\pi = 2\sqrt{3} \sum_{n=1}^{\infty} (-1)^{n-1}\, 3^{-n} H_n \left(\frac{8n^2+4n-1}{4n^2-1} \right)$$

Entry 26. For $-1 < x < 1$, we have

$$\tan^{-1} x = \sum_{n=1}^{\infty} x^{2n-1}\, h_n \left(\frac{(1-x^2)n(2n+1) + x^2}{4n^2-1} \right)$$

where

$$h_n = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k} \quad , n = 1,2,3, \ldots$$

Example:

$$\pi = 2\sqrt{3} \sum_{n=1}^{\infty} 3^{-n}\, h_n \left(\frac{4n^2+2n+1}{4n^2-1} \right)$$

Entry 27. For $-1 < x < 1$, we have

$$\tan^{-1} x = \sum_{n=1}^{\infty} x^{n-1} \left(\frac{(1-x)n(2n+1) + x}{4n^2-1} \right) \sum_{k=1}^{n} \frac{(-1)^{k-1} x^k}{k}$$

$$\tan^{-1} x = \sum_{n=1}^{\infty} (-1)^{n-1} x^{n-1} \left(\frac{(1+x)n(2n+1) - x}{4n^2-1} \right) \sum_{k=1}^{n} \frac{x^k}{k}$$

Entry 28. For $0 < x < y < 1$, we have

$$y\ \tan^{-1} x = x\ \tan^{-1} y + x\, y \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+3} (y^{2n+2} - x^{2n+2})$$

Entry 29. For $-1 < x < 1$, we have

$$\tan^{-1} x = \sum_{n=1}^{\infty} (-1)^{n-1} \left(H_{2n-1} x^{2n-1} + H_{2n} x^{2n+1} \right)$$

where

$$H_n = \sum_{k=1}^{n} \frac{1}{k} \quad , n = 1,2,3, \dots$$

Example:

$$\pi = 2\sqrt{3} \sum_{n=1}^{\infty} (-1)^{n-1} 3^{-n} (3H_{2n-1} + H_{2n})$$

Entry 30. For $-1 < x < 1$, we have

$$\tan^{-1} x = \sum_{n=1}^{\infty} (-1)^{n-1} (h_{2n-1} x^{2n-1} + h_{2n} x^{2n+1})$$

where

$$h_n = \sum_{k=1}^{n} \frac{(-1)^{k-1}}{k} \quad , n = 1,2,3, \dots$$

Example:

$$\pi = 2\sqrt{3} \sum_{n=1}^{\infty} (-1)^{n-1} 3^{-n} (3h_{2n-1} + h_{2n})$$

Entry 31. For $-\frac{1}{2} < x < 1$, we have

$$\tan^{-1} x = \sum_{n=0}^{\infty} \left(\frac{x}{1+x}\right)^{n+1} \sum_{k=0}^{[n/2]} \frac{(-1)^k}{2k+1} \binom{n}{n-2k}$$

Entry 32. For $0 < x < 1$, we have

$$\tan^{-1} x = \frac{2x}{1+x} \sum_{n=0}^{\infty} (-1)^n \left(\frac{1-x}{1+x}\right)^n \sum_{k=0}^{[n/2]} \frac{(-1)^k}{2k+1} \binom{n}{n-2k} \left(\frac{2x}{1-x}\right)^{2k}$$

Entry 33. For $0 < x < 1$, we have

$$\tan^{-1} x = \sum_{n=0}^{\infty} x^{n+1} \sum_{k=0}^{[n/2]} \frac{(-1)^k}{2k+1} \binom{n}{n-2k} (1-x)^{2k+1}$$

Entry 34. For $-1 \le x \le 1$, we have

$$\tan^{-1} x = -\frac{1}{x} \sum_{n=1}^{\infty} 2^{-n} Li_n(-x^2) = \frac{\ln(1+x^2)}{2x} - \frac{1}{x} \sum_{n=2}^{\infty} 2^{-n} Li_n(-x^2)$$

where

$$Li_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n} \quad , |x| < 1, n = 1,2,3, \dots$$

Example:

$$\pi = -6\sqrt{3}\sum_{n=1}^{\infty} 2^{-n}\, Li_n\left(-\frac{1}{3}\right) = 3\sqrt{3}\,\ln\left(\frac{4}{3}\right) - 6\sqrt{3}\sum_{n=2}^{\infty} 2^{-n}\, Li_n\left(-\frac{1}{3}\right)$$

Future research

Entry 35.

$$\frac{\pi + \ln\left(3-2\sqrt{2}\right)}{4\sqrt{2}} = -\int_0^{1/2} \ln\left(\sqrt[3]{\frac{x}{1-x\sqrt[3]{\frac{x}{1-x\sqrt[3]{\frac{x}{1-\cdots}}}}}}\right) dx$$

$$\frac{\pi + \ln\left(3-2\sqrt{2}\right)}{4\sqrt{2}} = -\int_0^{1/2} \ln\left(\left(x + x\left(x + x(x+\cdots)^{4/3}\right)^{4/3}\right)^{1/3}\right) dx$$

Entry 36.

$$\frac{\pi + \ln\left(3-2\sqrt{2}\right)}{4\sqrt{2}} = -\int_0^{1/2} \ln\left(x^{1/3} + \frac{x^{5/3}}{3} + \frac{x^3}{3} + \frac{35\,x^{13/3}}{81} + \frac{154\,x^{17/3}}{243} + \frac{x^7}{1} + \cdots\right) dx$$

Entry 37.

$$\frac{\pi + \ln\left(3-2\sqrt{2}\right)}{4\sqrt{2}} = -\int_0^{1/2} \ln(f(x))\,dx$$

where

$$f(x) = \frac{1}{4x} + \frac{1}{2}\sqrt{\frac{1}{4x^2} + g(x)} - \frac{1}{2}\sqrt{\frac{1}{2x^2} - g(x) + \frac{1}{4x^3\sqrt{\frac{1}{4x^2} + g(x)}}}$$

$$g(x) = 4\left(\frac{2}{3}\right)^{1/3} x\,\left(9x - \sqrt{3}\sqrt{27x^2 - 256x^6}\right)^{-1/3} + \frac{\left(9x - \sqrt{3}\sqrt{27x^2 - 256x^6}\right)^{1/3}}{2^{1/3}\,3^{2/3}\,x}$$

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