An Information Based Theory of Stationary Action

David Trost
david_trost@icloud.com

21-March-2024[*]

The quantization of energy proposed by Planck to account for the observed spectrum of black body radiation has associated with it a quantization of entropy. This in turn implies a quantization of observable information, directly implying observational uncertainty on the order of Planck's constant. The effect of that uncertainty is analyzed. In order to adhere strictly to the use of observable quantities, a probability measure is employed based on the distinguishability of statistical samples. This leads directly to the description of probability in terms of the absolute square of a complex amplitude. The Feynman rules may then be applied naturally for indistinguishable events without contradiction to the conventional rules for distinguishable events. This enables the straightforward calculation of the probability that a particle moves from one arbitrary point to another. The Feynman formulation of quantum phenomena and the principle of stationary action results when it is assumed that the classical action represents the measure of distinguishability. Parallel analysis on a Lorentz manifold yields the geodesic principle.

Introduction

Early efforts to understand black body radiation within the confines of classical physics focused on the entropy of electromagnetic radiation. In 1884 Boltzmann studied black body radiation in a perfectly reflecting enclosure. Treating the radiation pressure as that of a continuum gas he was able to define its entropy [1]. From that followed a theoretical basis for Stefan's empirically determined dependence of total radiated power on the fourth power of temperature.

Several years earlier Boltzmann had shown a correspondence between classical entropy and the discrete quantity that was later termed the *statistical multiplicity* [2] of a molecular gas, though he made no use of this in his black body work. It was not until the mid-twentieth century introduction of information theory [3] that entropy could be understood as information lost due to the statistical treatment of trajectories in lieu of a more complete microscopic model of molecular states [4].

By 1900, Planck had developed a classical model of the entropy of a black body at temperature T in which electromagnetic dipole resonators operated in equilibrium with the radiant energy in Boltzmann's reflective enclosure. Comparing the latest empirical data to his model, he found it necessary to introduce the constant h limiting radiation energy to discrete multiples of $h\nu$ [5]. This quantization of energy has the effect of limiting the entropy to increments of $h\nu/T$ when expressed in the prevailing units, those of Boltzmann's constant k.

The appearance of entropy in discrete increments is consistent with the situation in statistical mechanics. In that case discrete values replace the continuum model since entropy is now based on the discrete statistical multiplicity of macro-states. As in the case of the continuum gas model, the entropy in the black body model will be interpreted as a paucity of accurate information, in this case due to some inherent natural limit.

Consider an ideal gas consisting of a single molecule. The Planck entropy implies an inability of the classical model to describe its trajectory in phase space with uncertainty less than h. It may be that a more accurate description of the trajectory is not possible. Alternately, the trajectory may be fully deterministic but some inherent limit on observational accuracy produces the uncertainty, even with perfectly accurate measuring equipment.

[*]Initial text posted 25-March-2023

In either case the description of observable quantities in the classical model, like that of a continuum gas is only approximate, with the action of observed natural phenomena differing from the classical description. The result is an observed stochastic component of order h in the molecular trajectory. In this situation, Planck's constant h is a more convenient unit of entropy.

Let η be a system dependent parameter, then let

$$I_1 = \eta h \tag{1}$$

represent the information in a hypothetical, more accurate and possibly fully deterministic model that supersedes the inaccurate part of the information in the classical description. Let H_1 represent the entropy of the more accurate model and H_0 the entropy of the classical model. Then [6]

$$H_1 = H_0 - I_1 \tag{2}$$

If the more accurate model is both fully deterministic and completely accurate

$$H_1 = 0 \tag{3}$$

Then

$$H_0 = I_1 = \eta h \tag{4}$$

The analysis that follows explicitly acknowledges statistical uncertainties in observations of physical systems. Following classical practice, we assume no explicit limit on the accuracy of measuring instruments. Also, as in the classical model, the explicit effect of a measurement is assumed in advance to significantly affect neither its own result, nor the results of future measurements.

The existence of uncertainty requires that the inherently stochastic nature of the result of observation be incorporated in the analysis. The choice of probability measure can be of profound importance [7]. Following modern practice, careful attention is paid to ensuring our analysis is based strictly on what can be observed. To that end we employ a probability measure based on stochastic outcomes that are equal in statistical distinguishability from one another.

Distinguishing one experimental outcome from another is necessarily a matter of distinguishing between their probability distributions. The distinguishability of probability distributions has been quantified by Fisher [8]. Wootters has employed the term *statistical distance* for that quantity and studied its properties along contiguous paths. He has found an association between statistical distance and quantum phase [9].

Consider two N sided loaded dice with different loadings, where the differences in the probabilities of corresponding faces are $\delta p_1 \cdots \delta p_N$. The dice are said to be distinguishable in n trials if

$$\frac{\sqrt{n}}{2}\left[\sum_{i=1}^{N}\frac{(\delta p_i)^2}{p_i}\right] > 1 \tag{5}$$

Then the statistical distance $\mathcal{S}$ between these dice on the appropriate probability space is defined by [9]

$$\mathcal{S} = \lim_{n\to\infty}\frac{1}{\sqrt{n}}\times\begin{bmatrix}\text{the maximum number of intermediate outcomes each of}\\ \text{which is distinguishable (in } n \text{ trials) from its neighbors}\end{bmatrix} \tag{6}$$

Along with the stochastic analysis of what can be observed, the question remains: are these stochastic processes consistent with more deterministic, or even fully deterministic underlying natural processes, even if some of the variables necessary to make use of a more deterministic model are inherently hidden due to some natural limitation on their observability.

Analysis

Let $x = (x_0, x_1, x_2, x_3)$ represent the ordinary space and time of classical physics where x_0 represents time and x_1, x_2 and x_3 represent three-dimensional Euclidean space. Let us consider a particle that moves from start point A to end point B by an unknown trajectory through this space and time. Let $x(t)$ be an arbitrary trajectory with the same end points, where t is a time like parameter. We stipulate, in view of uncertainty, that for any value of the parameter t assigning a definite time and place on $x(t)$ there may be a nonzero

probability that the particle can be observed at any time and place x. This defines the probability distribution $p(x,t)$. The set of all possible probability distributions constitutes a probability space.
Let $p(x,t)$ be piecewise differentiable with respect to t. The statistical distance between points on $x(t)$ in the physical space may then be expressed as the statistical distance $\mathcal{S}$ between corresponding points in the associated probability space [9].

$$d\mathcal{S}(t) = \frac{1}{2}\left\{\int_{-\infty}^{\infty} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \frac{1}{p(x,t)}\left[\frac{dp(x,t)}{dt}\right]^2\right\}^{1/2} dt \tag{7}$$

This expression may be simplified by the substitution $\zeta(x,t) = p^{1/2}(x,t)$. Then [9]

$$d\mathcal{S}(t) = \left\{\int_{-\infty}^{\infty} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \left[\frac{d\zeta(x,t)}{dt}\right]^2\right\}^{1/2} dt \tag{8}$$

This new expression defines an element of length $d\mathcal{S}$ in an infinite dimensional Euclidean ζ space described in rectilinear coordinates [9].

Since $\zeta^2(x,t)$ is a probability, it's integral over all of space and time is unity for any value of the parameter t. Thus $d\mathcal{S}$ lies on the surface of an infinite dimensional unit hypersphere. This surface constitutes the probability space, with each point in this space representing a probability distribution. Then statistical distance $\mathcal{S}$ between points on $x(t)$ is measured by the length of the arc traced on the surface of the hypersphere as the progress of parameter t traces out the trajectory in space and time between them [9].

Now let us divide the integral on the right-hand-side of (8) in two. Let $[da(t)/dt]^2$ represent the portion of the integral for which $x_0 < t$ and $[db(t)/dt]^2$ the portion for which $x_0 > t$.

$$\left[\frac{da(t)}{dt}\right]^2 = \int_{-\infty}^{t} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \left[\frac{d\zeta(x,t)}{dt}\right]^2 \tag{9}$$

$$\left[\frac{db(t)}{dt}\right]^2 = \int_{t}^{\infty} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \left[\frac{d\zeta(x,t)}{dt}\right]^2 \tag{10}$$

Then

$$d\mathcal{S}(t) = \left\{\left[\frac{d(a)}{dt}\right]^2 + \left[\frac{d(b)}{dt}\right]^2\right\}^{1/2} \tag{11}$$

Since the probabilities must still sum to unity, $a^2 + b^2 = 1$. Analogous to (8) the unit hypersphere is collapsed into the unit circle, its two dimensions representing past and future. Now the statistical distance $\mathcal{S}$ between points on $x(t)$ is measured by the length of the arc traced on the new *circular probability space* as the progress of parameter t traces out the trajectory in space and time. Since an element of that arc length $d\theta(t)$ on the unit circle is described by the right-hand side of (11) we now have

$$d\mathcal{S}(t) = d\theta(t) \tag{12}$$

Let us form the complex quantity $a(t) + b(t)i = e^{i\theta(t)}$ representing the same unit circle on the Argand plane. Now in order to express the probability $p[x(t)]$ of our particle being found at a point on the trajectory $x(t)$ as an explicit function of the statistical distance $\theta(t)$ we may define the probability amplitude

$$\varphi[x(t)] \equiv \zeta[x(t)]e^{i\theta(t)} \tag{13}$$

Then

$$p[x(t)] = |\varphi[x(t)]|^2 = \langle\varphi^*|\varphi\rangle \tag{14}$$

Representation of the probability in terms of a Hilbert space has the effect of including the real valued measure θ along with the real valued magnitude $|\varphi|$ in the statement of the probability expressed by the single complex variable φ [7].

The Feynman Rules

The identification of a probability with the absolute square of a complex quantity constitutes the Feynman *amplitude-probability rule* [10, 11].

While the Feynman rules are explicitly intended for the purpose of quantum mechanical calculation, the conditions assumed here in developing the probability amplitude consist of no more than a random variable with probability described by a function piecewise differentiable with respect to some parameter. This allows the Feynman rules to be treated as a feature of probability theory under this circumstance, when a measure based on statistical distinguishability is employed. Indeed, the peculiarities of quantum theory, depending upon what can and cannot be measured, may be regarded as depending upon distinguishability.

The conventional Laplace rule for the probability p_L of an event that may occur by any of multiple alternative means is

$$p_L = \sum p_i = \sum |\varphi_i|^2 \tag{15}$$

where p_i represents the probability of each of the alternatives. The Laplace rules are empirical in nature, verified by counting occurrences of the various constituent events [12]. We know from a century of experience with quantum phenomena that when the alternatives are *indistinguishable* the probability p_F is [10].

$$p_F = |\varphi_F|^2 = \left|\sum \varphi_i\right|^2 \tag{16}$$

This constitutes the *Feynman amplitude sum rule* [11, 13].

The conventional Laplace rule for the probability p_L that multiple events all occur is

$$p_L = \prod p_i = \prod |\varphi_i|^2 \tag{17}$$

where p_i represents the probability of each of the multiple event. Experience with quantum phenomena informs us that when the multiple events are indistinguishable the probability p_F is [10].

$$p_F = |\varphi_F|^2 = \left|\prod \varphi_i\right|^2 \tag{18}$$

This constitutes the *Feynman amplitude product rule* [11,13]. While the two formulae yield identical probabilities, the latter establishes the phase $\theta_F = \sum \theta_i$ of the probability amplitude for the composite event.

Goyal, Knuth and Skilling have shown that the Feynman rules are a necessary result of any probability represented by a pair of real numbers [14]. Goyal and Knuth have further shown that the Feynman rules can coexist free of conflict with conventional probability [11]. Earlier Sykora emphasized that while probabilities are routinely described in terms of a single number, a measure is also necessary. Though frequently it is not explicitly stated, the need for this second real number is never-the-less implied. He further noted that clear statement of the measure has the salutary effect of eliminating ambiguities in statistical evaluation of observed result. [7].

Given that two real numbers are the minimum necessary to fully express a probability; given also that the Feynman rules are universally observed in nature when dealing with indistinguishable events; and given as well that the Feynman rules are the only two-component alternative to the Laplace rules that are compatible with them, we elect to treat Feynman's rules as a natural part of probability theory.

In view of the definition of the probability amplitude (13), $\theta(t) = \tan^{-1} b(t)/a\,(t)$. This implies the relationships $a(t) = \cos\theta(t)$, $b(t) = \sin\theta(t)$. Because (8) through (12) are formulated entirely in terms of time derivatives of $p(t)$, $\zeta(t)$, $a(t)$ and $b(t)$ we know that $\theta(t)$ must include an arbitrary additive constant θ_A. This indicates that the initial assignment of past and future contributions to the probability is arbitrary, while as time progresses these contributions vary sinusoidally between the two.

With the origin of quantum phase apparently tied to the use of statistical distance as a natural probability measure, the presence of an arbitrary additive constant may be understood by noting that the notion of an absolute statistical position has no apparent meaning.

Feynman [13,15], citing von Neumann [16] has shown that the introduction of a means of observation within a trajectory resets the arbitrary additive phase constant yielding a random shift in phase for future unobservable events. Let $p_F = |\varphi_1 + \varphi_2|^2$ be the probability of an event with two indistinguishable alternative ways of occurring, signified by the two subscripts. Suppose now that a means of observing the alternatives is provided. The presence of the measuring equipment introduces arbitrary additive constants $\delta\theta_1$ and $\delta\theta_2$ to the phases of the probability amplitudes. Now $p_F = \left|\varphi_1 e^{i\delta\theta_1} + \varphi_2 e^{i\delta\theta_2}\right|^2$. The multiple observations required to establish a frequentist probability p_F require that the phases of the alternatives be averaged over all angles. This results in reversion to the Laplace rule $p_L = |\varphi_1|^2 + |\varphi_2|^2$ as the sample size approaches infinity. Multiple observations have the simultaneous effect of averaging over all initial assignments of past and future contributions to the probability.

Von Neumann's phase reset was determined based on the properties of the Dirac-von Neumann model of quantum mechanics. Given the undeniable long-term success of that model, reset of phase upon the possibility of measurement is treated here as an empirically based supplement to the Feynman rules.

In view of this one may argue that these supplemented Feynman rules be treated as the more fundamental empirical rule of probability applying to any inherently indistinguishable alternatives lying on differentiable probability continua. The conventional Laplace rules then become derivative of them.

Until the possibility of an observation is present, an amplitude exists without a corresponding frequentist probability. Where observation is possible, a frequency may be observed and a frequentist probability emerges. The real valued probability is independent of phase, and the phase of the probability amplitude is lost with the emergence of a frequentist probability. It appears then that the supplemented Feynman rules are a natural replacement for the Laplace rules which follow from Feynman's when observation is possible.

To make proper use of the Feynman rules it becomes necessary to define all cases where events are *inherently* indistinguishable [17]. Clearly that is the case when no means of measurement is present. In addition, from (5), measurement results are indistinguishable when the statistical distance between them is less than or equal to unity. Kok [18] notes that this is a consequence of the Kramer-Rao bound.

For indistinguishability to be inherent there must be some mechanism, discussed further on, whereby any variables describing underlying physical processes are either inherently random, or inherently hidden from direct observation.

The Path Integral

Let $\Delta\theta = \int_{x(t)} [d\theta(t)/dt]\, dt$. This is the statistical distance traced by a particle as it traverses the path $x(t)$. Let m be an arbitrary integer and let $\delta\theta = \Delta\theta/m$ so that $x[\theta(t)]$ is divided into m equally distinguishable segments.

Let $p_j^{(m)}$, dependent on the value of m, be the probability that the particle is found in the j^{th} segment when $x(t)$ calls for it to be there and measurement is possible. Let $\zeta_j^{(m)} \equiv \left[p_j^{(m)}\right]^{1/2}$. Then the probability amplitude for the j^{th} segment is $\varphi_j = \zeta_j^{(m)} e^{i\delta\theta}$.

From the amplitude product rule, the probability amplitude $\varphi_{x(t)}(BA)$ for a test particle following an arbitrary path $x(t)$ between A and B when the individual points cannot be observed, is the product of the probability amplitudes that the particle traverses every point on $x[\theta(t)]$

$$\varphi_{x(t)}(BA) = \lim_{m\to\infty} \prod_{j=1}^{m} \left[\zeta_j^{(m)} e^{i\delta\theta}\right] = \exp\left\{ \int_A^B \frac{d \ln \zeta[\mathrm{x}(\theta)]}{d\theta} d\theta + i\Delta\theta \right\} = \zeta_{x(t)} e^{i\Delta\theta} \tag{19}$$

where ${\zeta_{x(t)}}^2 = \left\{\exp \int_A^B \frac{d \ln \zeta[\mathrm{x}(\theta)]}{d\theta} d\theta\right\}^2 = \exp \int_A^B \frac{d \ln \zeta^2[\mathrm{x}(\theta)]}{d\theta} d\theta$ is the probability that the path $x(t)$ is followed when observation of the path is possible.

We, know based on many decades of empirical experience verifying the Feynman formulation of quantum mechanics, that the proper expression for $\varphi_{x(t)}(BA)$ is [10]

$$\varphi_{x(t)}(BA) = \text{Const}\ e^{iS/\hbar} \tag{20}$$

where S is the classical action. In parallel with Feynman let us assume that the magnitude of the probability amplitude varies much more slowly than the phase. Then for trajectories of stationary phase $\zeta_{x(t)}$ will be stationary as well and the substitution

$$S/\hbar = \Delta\theta \tag{21}$$

yields the Feynman result. The classical action S corresponds to the statistical distance $\mathcal{S}$ traced by a particle as it traverses $x(t)$ while the classical Lagrangian corresponds to the rate of change of statistical distance with time.

The probability that a test particle goes from start point A to end point B by any path is then computed according to the Feynman amplitude sum rule with the path integral replacing the summation in (16).

$$p(BA) = \left| \int \varphi_{x(t)}(BA)\, \mathcal{D}x(t) \right|^2 \tag{22}$$

Quantum Gravity

Our derivation of the Feynman formalism for ordinary quantum mechanics does not rely on the Euclidean nature of the physical space to which it is applied. A parallel derivation can be applied to a particle trajectory on a Lorentz manifold when uncertainty is postulated in the observed *four-space location* of a particle on the manifold. This leads directly to the geodesic principle in lieu of Hamilton's principle. The equivalence of these two principles in the flat space limit [19] highlights a correspondence between the two forms of uncertainty previously established in gravitational theory [20].

Let $x^* = (x_0^*, x_1^*, x_2^*, x_3^*)$ represent the four-dimensional Lorentz manifold of general relativity, where x_0^* represents time and x_1^*, x_2^* and x_3^* represent the three spatial dimensions. Substituting x^* for x in (7) through (12) and more carefully specifying t as an affine parameter, we find the equivalent definition of statistical distance. Continuing in the same vein through (19), we find the equivalent expression for the probability amplitude of an arbitrary path in terms of the statistical length of the path $\Delta\theta^*$.

The Einstein Hilbert action for empty space is [21]

$$S_g = \kappa \int \sqrt{-g}\, R\, d^4x \tag{23}$$

Where κ is the Einstein gravitational constant, R is the Ricci scalar and g is the negative valued determinant of the metric tensor.

The scalar invariant integral carries the units of length squared. This allows us to define a scalar invariant quantum unit of gravitational action proportional to ${s_c}^2$ where s_c is a small interval on the Lorentz manifold characterizing uncertainty in spacetime location. In the flat space limit $s_c = \sqrt{32}\,\pi\, l_P \approx 17.77\ l_P$ where l_P is the Planck length [20].

We may now omit (21) which identifies statistical distance with the classical action, and substitute s/s_c for $\Delta\theta$ where $s = \int_{x^*(t)} [dx^*(t)/dt]\, dt$ is the length of $x^*(t)$. Then the probability amplitude for a particle to follow an arbitrary path $x^*(t)$ from A to B is

$$\varphi^*_{x^*(t)}[BA] = \zeta^*_{x^*(t)} e^{is/s_c} \tag{24}$$

As the path integral (22) yields Hamilton's principle, the new path integral

$$p^*(BA) = \left| \iint \varphi^*_{x^*(t)}(BA)\, \mathcal{D}x^*(t) \right|^2 \tag{25}$$

yields the geodesic principle. The foundational principle of general relativity is recovered directly. The principle of stationary action follows in the flat space limit [19]. The result is the relativistic equivalent to the Feynman space time formulation of non-relativistic quantum mechanics [15] providing a plausible and equally general model of quantum gravity subject to empirical trial.

The association of uncertainty with the geometric quantity spacetime-location provides some additional clarity. The yet to be defined source of uncertainty in spacetime location provides a manifest source of the limitations of classical physics.

The Black Body Spectrum Revisited

When viewed in the laboratory frame, uncertainty of location in spacetime must appear as a small apparently random motion. This in turn results in a small zero-point energy. As early as 1913 employing purely classical analysis, Einstein and Stern showed that the assumption of zero-point energy $h\upsilon$ in Planck's dipole oscillators led to the Planck spectrum *without the independent assumption of energy quantization* [22].

Milonni has extended that analysis [22] noting each Planck oscillator is in equilibrium with an associated field mode of Planck's cavity. Employing the same zero-point energy $h\upsilon$, the equipartition theorem requires the oscillator and field each have energy $h\upsilon/2$.

Let us now momentarily assume that deterministic physical laws resembling those of classical physics continue to govern even at the microscopic level, subject to some undefined source of uncertainty in observed spacetime location. Since the apparently, or actually random motion of the electron and the field at the same location are identical, we would expect no coupling between the zero-point motion of the dipole oscillator and the zero-point component of the field under this assumption.

Returning to Milonni, still employing purely classical analysis he demonstrates that when there is no interaction between the random components of the oscillator and the field, black body spectral density is

$$\rho(\upsilon) = \frac{8\pi h\upsilon^3/c^3}{e^{h\upsilon/kT} - 1} + 4\pi h\upsilon^3/c^3 \tag{26}$$

in agreement with quantum electrodynamic theory. Though short of proof, this suggests that even at the unobservable level the concept of spacetime location remains valid and fully deterministic laws resembling the classical ones may prevail. Milonni's result further indicates that in the unobservable domain treatment of the electromagnetic field in the classical way can be a valid approach. It allows for uncertainty to be induced either by random fluctuations of the spacetime metric, or by random fluctuations in the locations of material particles. Both possibilities are discussed below.

Discussion

The present analysis leads naturally to the Feynman formulation of quantum mechanics under the assumption that our ability to know the state of physical phenomena is inherently imperfect. It relies on a revised form of probability theory that employs the Feynman rules as the empirically determined rules of probability for indistinguishable events, reducing to the Laplace rules when events can be distinguished.

A similar point of view can be applied to the Dirac-von Neumann formalism. Consistent with the "ψ-epistemic" view [23], the probability amplitude does not represent the state of the system. Instead, it represents the state of information available about the system. When the amplitude is understood in this way, its collapse no longer represents a change in the physical system. Instead, it indicates the state of available information about the system has changed as the result of the possibility of measurement. A frequentist probability has emerged, while simultaneously the phase of the probability amplitude has vanished.

The probability amplitude may be regarded as representing a conditional probability [11] based on the state of information prior to measurement. The amplitude after measurement represents a new conditional probability based on the new state of information generated by the measurement.

Goyal's analysis has shown [24] that the logic of the Feynman formalism is equivalent to that of Dirac-von Neumann when it is supplemented with a *no-disturbance postulate*. This posits that there exists a class of trivial measurements which have no effect on the probability amplitude. Trivial measurements are defined by the property that they yield no new information about the system being measured. This principle is a natural consequence of the epistemic interpretation of the probability amplitude. With no change in information the conditional probabilities of subsequent outcomes are unchanged.

The no-disturbance postulate resides uncomfortably alongside the notion that uncertainty is caused by the process of measurement. The apparent conflict is eliminated with the adoption of the ψ-epistemic viewpoint coupled with the proposed entropic origin of uncertainty. The fact that the no disturbance principle requires a change in available information to change the probability amplitude is a strong indicator of the amplitude's epistemic nature.

The entropy and information considerations that motivate the present analysis allow for, but do not require fully deterministic underlying spacetime locations of both particles and waves that are beyond our ability to accurately observe. The ψ-epistemic viewpoint allows for the same latitude in [23].

The Wave Particle Duality

The historically empirical Laplace rules of probability among distinguishable alternatives have been shown here to lead naturally to the description of particle probabilities in terms of a complex probability amplitude with wavelike characteristics. With the adoption of the more general Feynman rules, empirically derived more recently, these probability amplitudes add as do the amplitudes of physical wave phenomena when observations of alternative events are not possible. This imparts wavelike properties to classical particles resulting strictly from the novel rules of probability.

Planck's derivation of the black body spectrum was based almost entirely on a combination of classical mechanics and electromagnetic theory. It deviated only with an unexplained quantization of electromagnetic field energy. Planck demonstrated that the observed black body spectrum was consistent with the effect of this energy quantization on the entropy of radiation.

Particle like behavior of classical electromagnetic waves with entropy empirically imposed by the observed black body spectrum was famously explored as early as 1905 [25, 26]. Einstein's classical analysis of the empirically determined high frequency portion of the black body spectrum found that the entropy of any narrow band about frequency ν matched that of an ideal molecular gas with particle energy concentrated in a narrow band around the value $h\nu$. Heuristic particle like behavior in classical wave phenomena thus explained the photoelectric effect twenty years before the advent of modern quantum theory.

With the benefit of information theoretic insights not available for another half century [6], we have adopted a ψ-epistemic interpretation of that result. In this view the quantization of entropy leads to the quantization of *information about the energy* rather than the energy proper. The present analysis allows us to associate the entropy of the black body spectrum with our inability to account for the apparently, or actually random common spacetime location of the resonator and field. Due to the resulting uncertainty, the quantization of the field entropy of inherently wavelike phenomena mimics the entropy of a particulate gas. This accounts for the appearance of photons.

While the present analysis takes no exception to the notion that light consists of photons in all observable phenomena, it also allows for the existence of unobservable fully deterministic underlying behavior in which electromagnetic effects are not quantized, as in the Einstein-Stern-Milonni black body analysis. Though this is an isolated result, it suggests that deterministic physical processes may continue to operate in the quantum regime even though available information about the location of events in spacetime may suffer inherent limitations. In this underlying model there need be no intrinsic particle like properties in the nature of waves.

In this view, we see photons as physical particles imperfectly substituting for quanta of information about the properties of electromagnetic fields in the face of uncertainty in observable spacetime location of order s_c. At locations separated by spacetime distances greater than s_c.the individual regions of uncertainty have minimal overlap. At these same scales the Feynman QED model has demonstrated remarkable fidelity to nature. For locations separated by less than s_c regions of uncertainty are highly overlapped. Substitution of separate energetic particles for these overlapped regions becomes questionable since much of the uncertain region each of these separate point particles represents may be common to multiple overlapped regions of uncertainty.

In fact, attempts to compute probabilities with the Feynman QED model that include events at separations much finer than s_c yield total probabilities greater than unity [27, 28]. This provides clear motivation for limiting the inclusion of events in that model to those separated by more than s_c or whatever other interval is the lower limit yielding unit total probability.

Fixing a minimum scale in the Feynman QED model has the salutary effect of eliminating the need for renormalization, while simultaneously fixing values for the mass and charge of the naked electron [27]. Ultimately though, a more inclusive theory is required to properly model the situation [28]. The present analysis in which more deterministic behavior underlies what can be observed in the presence of observational uncertainty may provide a useful framework.

<u>Emergence of the Geodesic Principle and Stationary Action</u>
Regardless of the source of uncertainty, we have shown that the principle of stationary action as well as the more general geodesic principle follow naturally from a small set of assumptions:

1. Natural phenomena are described in a Lorentz four-space model.
2. Observers of these phenomena experience some level of inherent uncertainty in their knowledge of location in this four-space.
3. The Feynman rules, supplemented with random phase shift upon the possibility of observation, apply to the probability of physical phenomena that are inherently indistinguishable.
4. The probability amplitudes for particles being found at any location in four-space are piecewise differentiable with respect to the time coordinate.

We have argued that the Feynman rules constitute a more plausible empirically justified axiomatic basis for probability theory than the Laplace rules in that, at least when the probability is piecewise differentiable with respect to a time like parameter, they cover a broader range of phenomena including both indistinguishable and distinguishable events. In the face of uncertainty at the observable level, these rules provide a mechanism for the deterministic laws of mechanics to emerge.

Our analysis has placed no restrictions on the probability distribution underlying uncertainty. We continue to assume that the magnitude of the probability amplitude varies in spacetime much more slowly than the phase. Then the path integral assures that the probability amplitude of indistinguishable particles that just happen to follow paths of near stationary length will be coherently enhanced, while all others will be coherently suppressed. Thus, both the principle of stationary action, and the geodesic principle emerge in the observable universe on the strength of the Feynman rules.

<u>The Origin of Uncertainty</u>
We have characterized uncertainty as either "apparently random" or "actually random". The former refers to a model of nature in which indistinguishable physical phenomena are fully deterministic but appear to

the observer to contain a random component due to inherent limitations on observability from an unspecified source of error in the detection of spacetime location. The latter refers to a model in which the source of the randomness is the presence of actually random physical phenomena at scales $\leq s_c$. The following examples range from highly deterministic to highly random.

The general relativistic model contains within it a mechanism whereby an observer with only local information will observe a small apparently random zero-point energy. Correspondence between Newtonian mechanics and general relativity occurs when energies of objects under observation are suitably small, and there are no variations in the spacetime metric due to events outside the range of observation [29]. The latter of these conditions precludes from consideration a background level of broadband gravitational radiation.

If such background exists then, it can be expected to impose on the classical picture a small apparently random source of zero-point energy. Even in the full general relativistic model, background gravitational radiation that appears stochastic to an observer with only local knowledge must add an apparently random component to the predictable trajectories of ponderable masses.

The full nature of such a stochastic background of gravitational radiation is an open question [30]. Its spectrum is unknown. That this is the source of uncertainty is of course speculation. If this is the case, the general relativistic model is the fully deterministic but not-fully-predictable model underlying observable phenomena. The lack of predictability stems from our inability to know the background gravitational radiation in anything but stochastic terms. This model is characterized by fully deterministic location in fully deterministic spacetime with fully deterministic laws of physics impaired by an inherent limit on observability of deterministic spacetime locations. The observer, with inherently incomplete information, experiences uncertainty where none exists in the underlying behavior.

The logic of this model is that the empirical realities of uncertainty and the supplemented Feynman rules lead directly to the geodesic principle. Despite arising from uncertainty in observation, the geodesic principle is fully deterministic. The deterministic relativistic model then exhibits the source of the uncertainty in the form of the *apparent* randomness that gravitational radiation imparts to the observer within the system.

We have also proposed the possibility of *inherent* randomness in the behavior that underlies observation. The source of uncertainty may be a truly random component of the spacetime metric outside of the deterministic confines of relativity theory, not just apparent randomness as determined by a local observer. Alternatively, the source may be a random component in the motion of material particles outside the confines of the Einstein equation.

In the first alternative, distinguishing between apparent and actual randomness in the already unobservable spacetime metric entails distinguishing between two alternative unobservable conditions that produce the same observable outcome. Given that there is no apparent way for an observer to make this distinction, it is not clear that there is a meaningful difference between random and apparently random in this situation. The two alternatives may indeed be one and the same.

In the second alternative, involving random motion of particles against a deterministic spacetime metric, we note the unconstrained probability distribution underlying uncertainty. Our assumption that the probability distribution for a particle at a given location in spacetime be piecewise differentiable is satisfied by a uniform distribution that makes any location as likely as any other. This posits a maximally random universe.

Even under this most extreme assumption, the path integral still assures that indistinguishable particles with piecewise differentiable probability amplitudes that just happen to follow paths of near stationary length will be coherently enhanced, while all others will be coherently suppressed. The principle of stationary action, and the geodesic principle still emerge in the observable universe on the strength of the Feynman rules, even in the face of extreme underlying randomness in the underlying universe. Remnants of the imperfectly coherently suppressed random background may be the source of observed uncertainty.

The logic of this model is that natural particle behavior is maximally random. The supplemented Feynman rules produce the fully deterministic geodesic principle out of this chaos. Some residual uncertainty is imposed on this determinism by the necessary imperfection of the coherent cancellation of the underlying random behavior.

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