

# Net-Proton Number Fluctuations in the SST Nuclear Plasma

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**Abstract:** Here we show that the kurtosis multiplied by the squared standard deviation for net-proton distributions in nuclear collisions validate the nuclear-plasma structure described in the Scale-Symmetric Theory (SST). We also described the SST nuclear phase diagram.

## 1. Introduction

One of the fundamental goals in physics is to understand the properties of the nuclear plasma and hadronic phase of matter when we change the collision energy of ions,  $\sqrt{s_{NN}}$ .

The complexity of the system formed in heavy-ion collisions forced scientists to use advanced statistical methods. They use the moments to describe the characteristic of a distribution.

If a function is a probability distribution, then the first moment is the expectation value (the arithmetic mean of a large number of independently selected outcomes of a random variable), the second moment is the variance (a measure of how far a set of the same physical quantity is spread out from their average value), the third standardized moment (normalized by a power of the standard deviation  $\sigma$ ) is the skewness (positive skew indicates that the longer tail is on the right side of the distribution), **the fourth standardized moment is the kurtosis (it describes how much of a probability distribution falls in the tails instead of its center)**, and so on.

The most important information about the nuclear structure of plasma is observed in the energy dependence of  $\kappa\sigma^2$ , where  $\kappa$  defines kurtosis, and  $\sigma$  is the standard deviation, of net proton distribution (i.e. number of protons minus number of antiprotons) from the 0 – 5% most central collisions because such dependence is non-monotonic.

But the Relativistic Heavy-Ion Collider (RHIC) data are incomplete and they sometimes differ from the theoretical predictions, so a new model is needed to explain discrepancies, deficiencies in description, and to predict results for missing energies of the central ion collisions.

Here we use the Scale-Symmetric Theory (SST) [1] to describe structure of the SST nuclear plasma, to describe the phase transitions in the SST plasma core and plasma corona, and to show how the function  $\kappa\sigma^2 = f(\sqrt{s_{NN}})$  should look, so we can verify our model.

## 2. The SST nomenclature and calculations

### 2.1. Reconstruction of proton, nuclear ionization and **the SST nuclear-plasma core**

In SST [1], nucleons (p and n) consist of the neutral core  $H^0 = 724.776 \text{ MeV}$  or positively charged core  $H^+ = 727.439 \text{ MeV}$ , and of a relativistic pion (neutral  $W_{(o),d=1} =$

208.643 MeV or charged  $W_{(+\rightarrow),d=1} = 215.761 \text{ MeV}$ ) in distance  $R_{d=1} = A + B = 1.199278 \text{ fm}$ , where  $A = 0.6974425 \text{ fm}$  is the equatorial radius of the baryonic core.

Reconstruction of the proton is a capture of the relativistic pion by the core of baryon.

Nuclear ionization is the emission of the relativistic pion by proton or neutron.

The charged core of baryons is most stable, so the SST nuclear-plasma core consists of the baryonic cores,  $H^+$ , packed to maximum.

To reconstruct a proton, first of all, we must reconstruct the Titius-Bode (TB) orbits for the nuclear strong interactions. According to SST, it is done by quanta with a mass  $M_{TB} = 750.296 \text{ MeV}$  that are quadrupoles, so it can be also  $M_{TB}/4$ ,  $M_{TB}/2$ ,  $2M_{TB}$ ,  $4M_{TB}$ , and so on.

## 2.2. Torus/electric-charge of proton and the SST nuclear-plasma corona

Torus/electric-charge of proton is localized in the core of baryons and its mass is  $X^\pm = 318.2955 \text{ MeV}$ . By an analogy to the electron:  $e^\pm_{\text{bare,real}} + (e^+e^-)_{\text{virtual}}$ , where  $e^\pm_{\text{bare,real}}$  is the real mass of bare electron, we have  $X^\pm_{\text{real}} + (X^+X^-)_{\text{virtual}}$ .

The  $X^+X^-$  pairs are the components of the SST nuclear-plasma corona.

## 2.3. Net-proton number

Net-proton number is the number of reconstructed protons minus number of reconstructed antiprotons. For the nuclear-plasma corona, such number distance,  $\Delta N$ , is zero. Just reconstruction of nucleons from the  $X^+X^-$  pairs leads to the same number of protons and antiprotons.

Each baryonic core  $H^+$  can lead to one proton so we can normalize the function  $\kappa\sigma^2 = f(\sqrt{S_{NN}})$  for the nuclear-plasma core at very high energies as  $\Delta N_{\text{high}} = 1$ . But at lower collision energies, due to the reconstruction of nucleons, there are created locally the alpha particles and the  $1p1n$  pairs, so  $\Delta N_{\text{lower}} = 1.5$ . At lowest collision energies, in the plasma core there can be created locally the cuboids  $4p4n$  or  $3p5n$  (it dominates is heavier nuclei), so  $\Delta N_{\text{lowest}} = 3.5$ .

## 2.4. The nuclear-strong constant

It is an analog of the gravitational constant,  $G$ , for the nuclear strong interactions:

$$G_S = 5.45651 \cdot 10^{29} \text{ m}^3/(\text{kg s}^2).$$

## 2.5. The characteristic baryonic chemical potential $\mu_{B,\text{ch}}$ at $T = 0$

It is defined as energy needed to emit the relativistic pion in the  $d = 1$  state (the nuclear ionization)

$$\mu_{B,\text{ch}} = G_S H^{+0} W_{(+\rightarrow),d=1} / R_{d=1}. \quad (1)$$

The upper and lower limits are

$$\mu_{B,\text{ch,upper}} = 1417 \text{ MeV}. \quad (2)$$

$$\mu_{B,\text{ch,lower}} = 1365 \text{ MeV}. \quad (3)$$

Such energies are needed to “ionize” the nucleons, so at  $T = 0$  for  $\mu_B = 1417 \text{ MeV}$  there appears the pure SST nuclear plasma (see Fig.1).

## 2.6 Temperature of the SST nuclear matter and the temperature of phase transition

Temperature  $T$  we define as follows

$$T \rightarrow T k_B [\text{MeV}], \quad (4)$$

where  $k_B = 1.380648 \cdot 10^{-23} \text{ J/K} = 8.6173 \cdot 10^{-11} \text{ MeV/K}$  is the Boltzmann constant. The  $T$  we calculate from the Wien’s displacement law – it states that the black-body-radiation curve, for different temperatures,  $T$ , peaks at different wavelengths,  $\lambda_{\text{Peak}}$ , and  $\lambda_{\text{Peak}}$  is inversely proportional to  $T$

$$\lambda_{\text{Peak}} \sim 1 / T, \quad (5)$$

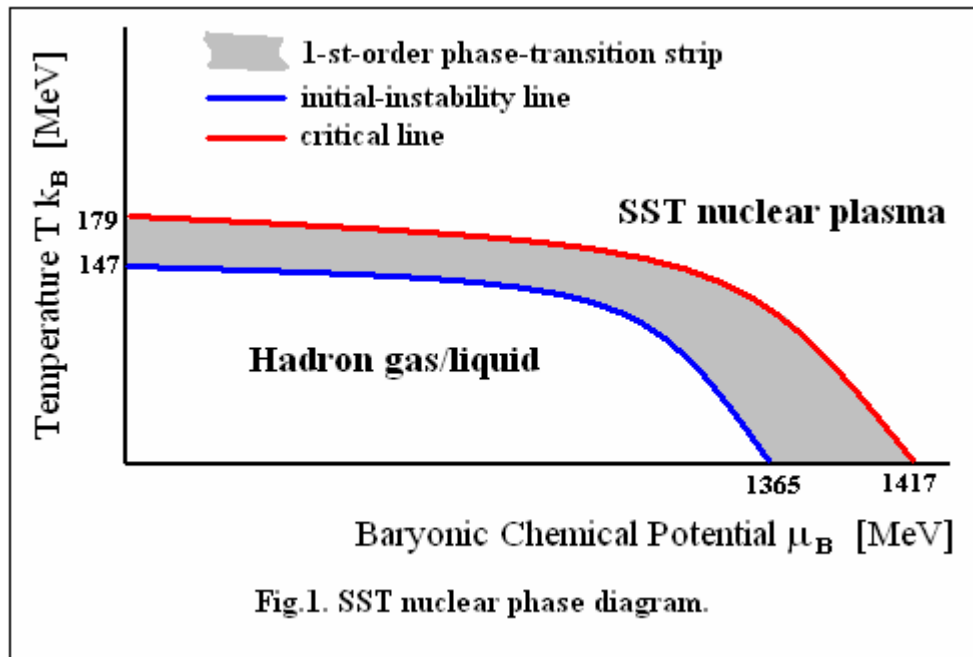
$$T \lambda_{\text{Peak}} = 2.8978 \cdot 10^{-3} [\text{K m}]. \quad (6)$$

To create the SST nuclear-plasma core, we must pack the baryonic cores to maximum, i.e. their distance must be  $\lambda_{\text{Peak}} = 2A = 1.395 \text{ fm}$ . From (6) and (4) we obtain

$$T_{\text{transition}} = 179 [\text{MeV}]. \quad (7)$$

But instability of the nuclear matter starts for  $\lambda_{\text{Peak}} = R_{d=2} = (A + 2B) = 1.7011 \text{ fm}$  (it is the ground state for the nuclear strong interactions above the Schwarzschild surface for the nuclear strong interactions), so we have

$$T_{\text{instability}} = 147 [\text{MeV}]. \quad (8)$$



From SST follows that when distances between the components of the SST nuclear plasma are smaller (i.e. temperature  $T$  is higher) then the nuclear-ionization energy (i.e. the baryonic

chemical potential  $\mu_B$ ) is lower, i.e. the energy needed to emit the relativistic pions by baryons is lower.

There is no critical point in the SST nuclear phase diagram, there is the critical line.

SST shows that the baryonic chemical potential (the energy of nuclear ionization) at  $T = 0$  of the relativistic charged pion in the last  $d = 4$  TB orbit is **1064 MeV**.

## 2.7. Relationship between the collision energy $\sqrt{s_{NN}}$ and the characteristic masses created in the SST nuclear matter

New fermions created in matter must satisfy following formula

$$M_{\text{bare}} c = M v, \quad (9)$$

where  $M_{\text{bare}}$  is the bare mass of the fermion mass  $M$ .

For the electron or the torus/electric-charge of proton ( $X^\pm$ ) or a particle containing such objects, we have [1]

$$v / c = M_{\text{bare}} / M = 1 / 1.0011596522. \quad (10)$$

Then the ratio of relativistic mass and rest mass is

$$F = m_{\text{rel}} / m_0 = (1 - v^2 / c^2)^{-1/2} = \sqrt{s_{NN}} / M = 20.78. \quad (11)$$

When energies of the exchanged particles have mass equal to  $M_i = M_{\text{TB}}/4, M_{\text{TB}}/2, M_{\text{TB}}, 2M_{\text{TB}}, 4M_{\text{TB}}, 8M_{\text{TB}}$ , and so on, then the TB orbits of some  $H^\pm$  cores are restored, so number of protons increase. It causes that the net-proton distribution increases as well, i.e. the plasma contains regions that are not nuclear plasma. From (11) we obtain that the masses  $M_i$  relate to

$$(\sqrt{s_{NN}})_{\text{Maxima}} = 3.9, 7.8, 15.6, 31.2, 62.4, 124.7 \text{ GeV}. \quad (12)$$

We can see that the SST nuclear plasma starts at energy  $\sim 3.9 \text{ GeV}$ .

For such collision energies, we should observe some maxima in the function  $\kappa\sigma^2 = f(\sqrt{s_{NN}})$  because they partially restore the structure of nucleons.

On the other hand, we should observe some minima for following energies (then the SST nuclear-plasma corona dominates)

$$E_d = d (X^+ + X^-), \quad (13)$$

where  $d = 1, 2, 4$ , are the TB numbers [1].

From (11) we obtain that the masses  $E_d$  relate to

$$(\sqrt{s_{NN}})_{\text{Minima}} = 13.2, 26.5, 52.9 \text{ GeV}. \quad (14)$$

But from experimental data follows that in the nuclear plasma are additionally produced, first of all, the pions and kaons, so for the collision energies related to  $\pi^+\pi^-$  and  $K^+K^-$  pairs we also should observe deep minima

$$(\sqrt{s_{NN}})_{\text{Minima}} = 5.8, 20.5 \text{ GeV}. \quad (15)$$

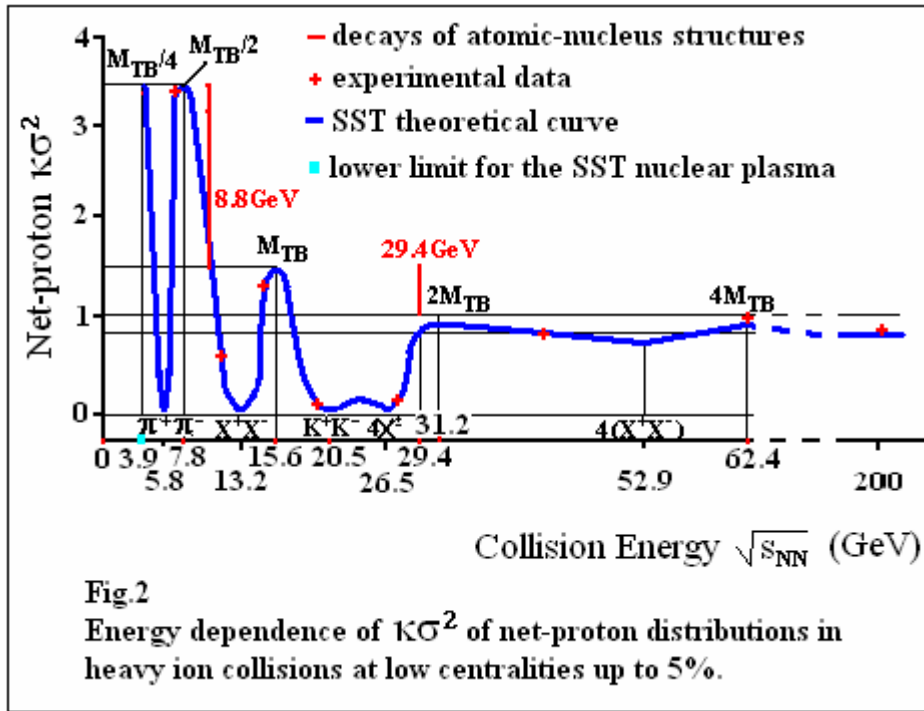
The baryonic chemical potential  $\mu_B = 1417 \text{ MeV}$  relates to the collision energy  $\sqrt{s_{NN}} = 29.4 \text{ GeV}$ , so above such collision energy there is the pure SST nuclear plasma and there dominates the plasma core. It means that the net-proton  $\kappa\sigma^2$  is close to 1 but lower than 1 because of very thin corona composed of the  $X^+X^-$  pairs.

At low energies, thickness of the plasma corona is relatively larger, so the annihilations of the  $X^+X^-$  pairs cause that we observe high particle yields in the low transverse momentum region.

### 2.8. Decays of the atomic-nucleus structures

According to SST, the bases of the cuboids interact due to the nuclear weak interactions, so the characteristic energy is  $Y = 424.12176 \text{ MeV}$  – from formula (11) follows that such energy relates to the collision energy  $\sqrt{s_{NN}} = F Y = 8.8 \text{ GeV}$ . At such collision energy, the cuboids decay to alpha particles but also to the proton-neutron pairs because some of the bases consist of 1 proton and 3 neutrons. It means that the net-proton  $\kappa\sigma^2$  decreases from 3.5 to 1.5.

The upper limit for the nuclear-ionization energy at  $T = 0$  is  $\mu_{B, \text{ch, upper}} = 1417 \text{ MeV}$ , so it relates to  $\sqrt{s_{NN}} = F \mu_{B, \text{ch, upper}} = 29.4 \text{ GeV}$ . At such collision energy, the alpha particles and the  $1p1n$  pairs decay to the single nucleons (more precisely, to the single cores of baryons). It means that the net-proton  $\kappa\sigma^2$  decreases from 1.5 to 1.



### 2.9. Kurtosis

Kurtosis is defined as

$$\kappa = \langle (\delta N)^4 \rangle / \sigma^4, \quad (16)$$

where  $\delta N = N - M$ ,  $M$  is the mean and  $\sigma$  is the standard deviation.

The central values of experimental data in Fig.2 (the red crosses) are from [2] and [3].

### 2.10. The SST chemical freeze-out

In SST, the nuclear strong interactions of nucleons in atomic nuclei at low energy follow from exchanges of the virtual non-relativistic pions  $\pi^0 = 135 \text{ MeV}$  and  $\pi^\pm = 139.6 \text{ MeV}$ .

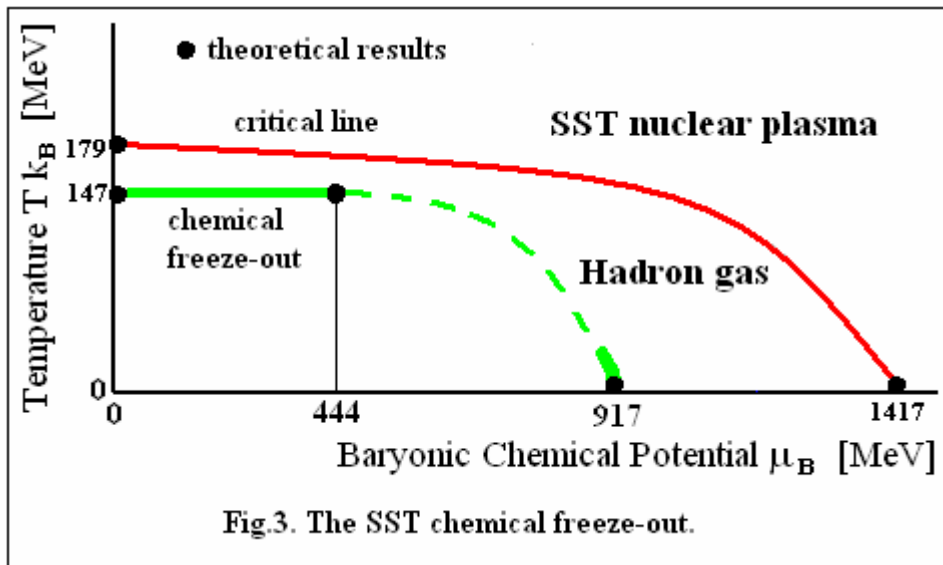
Here we define the SST chemical freeze-out as the baryonic chemical potential,  $\mu_{B,\text{freeze-out}}$ , at which the virtual non-relativistic pions or gluons decouple from nucleons an atomic nucleus consists of.

The virtual  $\pi^+\pi^-$  pairs and gluons appear most frequently at the  $d = 1$  state where is placed the relativistic pion. Radius of it is  $R_{d=1} = 1.199278 \text{ fm}$ . From (1) follows that to emit the virtual non-relativistic charged pion from such a distance (it is the SST nuclear ionization) there is needed the baryonic chemical potential equal to  $\mu_{B,\text{freeze-out,low}} = 917 \text{ MeV}$ . Such freeze-out relates to  $\sqrt{s_{NN}} = 19.1 \text{ GeV}$  (see formula (11)). Such processes dominate at low temperatures.

When the collision energy increases, there appear gluons in the  $d = 1$  state. With increasing collision energy, energy of the gluons increases from zero up to energy of the fundamental gluon loops (FGL,  $m_{FGL} = 67.5444 \text{ MeV}$ ). From (1) follows that to emit the virtual FGLs from such a distance (it is the SST nuclear ionization) there is needed the baryonic chemical potential  $\mu_{B,\text{freeze-out,high}} = 444 \text{ MeV}$ . The baryonic chemical potential increases from zero up to  $444 \text{ MeV}$ . The upper limit of the potential of such freeze-out relates to  $\sqrt{s_{NN}} = 9.2 \text{ GeV}$ . Temperature for such interval of the baryonic chemical potential, i.e.  $0 \text{ MeV} < \mu_{B,\text{freeze-out}} < 444 \text{ MeV}$ , is practically invariant because at high temperatures, the distances between the nucleons are close to  $R_{d=2}$ . We already calculated that the distance  $R_{d=2}$  relates to  $T = 147 \text{ MeV}$ .

For other temperatures, there is a combination of the two described phenomena. At higher temperatures dominate the gluons while at lower temperatures dominate the virtual non-relativistic charged pions (Fig.3).

Our results are consistent with experimental data presented in [2] (see **Figure 2a** in [2]).



### 3. Summary

Orthodox methods used in physics are based on the search for a differential equation, which usually leads to a multi-parameter solution, and then the search for physical phenomena in order to interpret the appearing parameters. In the case of difficult problems such as the one described in this paper, such methods do not work at all because it is like a magician pulling a rabbit out of an empty hat. Often, new experimental data (and even known results) do not match the obtained solution of the equation, which means that practically the entire model is useless.

In the Scale-Symmetric Theory, we use a much more efficient method. Based on known physical phenomena or highly probable phenomena (like, for example, phenomena described by formulae (1) and (11) in this paper), we search for statistical distributions leading to maxima, minima, plateaux, boundaries, resonances, quantized values, and so on, and then combine the obtained points. The advantage of this approach is the fact that the curve obtained in this way has a real physical interpretation right away, and any discrepancies with new experimental data do not make it necessary to reject the entire model, but to search for new phenomena. What's more, unlike looking for solutions to complex differential equations taken out of the empty hat, the SST method applied to extremely non-monotonic changes is practically guaranteed to be 100% effective. It caused that the SST is the superior theory.

Here we showed that the SST core-corona model of nuclear plasma leads to very simple interpretation of the net-proton  $\kappa\sigma^2$  distribution as a function of collision energy. Moreover, obtained theoretical results are consistent with still not numerous experimental data. We calculated the net-proton distribution for collision energies from 3.9 GeV (the SST nuclear plasma starts from such collision energy) up to 200 GeV (see Fig.2), so we can verify our model.

The SST plasma core consists of the cores of baryons packed to maximum, so the net-proton distribution is higher than zero. The SST plasma corona consists of the torus-antitorus pairs ( $X^+X^-$ ), so the net-proton distribution is zero.

In the cores of baryons and the tori/electric-charges are also created, first of all, pions, kaons, gluons and gluon loops, so mass density of the SST nuclear plasma is higher than mean energy density of the cores of baryons ( $> 0.79 \text{ GeV/fm}^3$ ) or mean energy density of the tori ( $> 0.34 \text{ GeV/fm}^3$ ) – it is consistent with experimental data ( $\sim 0.4 - 1 \text{ GeV/fm}^3$ ).

The gluons and exchanged particles quantize the distances between the components of the SST nuclear plasma, so the distances, via the Wien's displacement law, define temperature of the SST plasma.

We showed that at collision energy 29.4 GeV of the plasma core, the nucleons in atomic nuclei decay to the single baryonic cores (they are not entangled but packed to maximum) – it suggests that it is the starting collision energy for the pure SST nuclear plasma.

To explain the discrepancy between QCD and experiment concerning the low transverse momentum region (we observe too much particles), in paper [4] is formulated an alternative core-corona model for nuclear plasma. But we claim that the SST core-corona model much better explains all unsolved problems.

By the way, discovery of very massive early galaxies defies prior understanding of evolution of the Universe [5]. Such discrepancy was predicted within the SST already many years ago. We mentioned it because we cannot separate the evolution of nuclear plasma from evolution of the early Universe. Both descriptions are closely related.

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