Capacitance and Size of the Electron
Based on the CEWL Model

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Abstract

The Charged Electromagnetic Wave Loop (CEWL) model is a novel model of the Electron (developed by this author in 2013), which exactly matches all known values of the electron, including energy, de Broglie frequency, charge, mass, and magnetic moment. The model also later explained the mystery of why the Electron's G factor is 2 rather than one (½ spin) [1] In this paper, further validation of the CEWL Model is explored in two different ways. One method of validation involves using the author's new insight that since the model represents an electromagnet oscillation with zero internal resistance, the capacitive and inductive reactance must match each other, and also match the reactive impedance of free space, leading to a unique value for the Electron's capacitance as well as its' inductance ($3.41912126348 \times 10^{-24}$ Farads and 4.85262×10^{-19} Henries). The Capacitance value can then be used to estimate the probable maximum Width of the charged area relative to Diameter of the electron (~0.53%), which validates that the model produces reasonable values that don't conflict with the known anomalous magnetic moment.

A second validation may stem from the author's new insight that the loop characteristics of the CEWL Model, in which the loop circumference exactly matches the wavelength of a (virtual) photon equal to the Electron's energy, is analogous to the characteristics of a high Q (resonant) loop antennae in which the circumference must also exactly match the wavelength in order to achieve high Q resonance, which leads to a new prediction that the virtual photons of leptons will be generated in the same directions as high Q antennas i.e. in the North and South magnetic directions generated by the CEWL loop (loop antennas without these characteristics have different radiation/absorption patterns). This new insight about probable directionality might guide future research into how and where Neutrinos and virtual photons form near the Electron, Muon, and Tau Leptons.

Keywords

CEWL Electron Model, Charged Electromagnetic Wave Loop, Electron Capacitance, Inductance, Reactance, Impedance of Free Space, General Relativity Mass, Pair Production, g-Factor, Magnetic Moment, Spin, Lepton, Muon, Tau, Neutrino, Loop Antennae, Resonance.

Introduction:

This paper is intended to show the nature of the CEWL Electron Model, show how it has been validated previously, show how the new insights further validate the CEWL Model, and finally to give suggestions about how the model might help research the nature of Leptons in the future.

The paper is composed of 5 sub-sections:

Sections 1 and 2 give a brief description of how the CEWL Model explains the formation of an Electron and Positron pair from a high energy photon, and why the model predicts that the Electron (and Positron) is a charged electromagnetic loop (in order to give context for the remaining sections of the paper).

Section 3 shows the new insight that a unique value for the Capacitance as well as for the Inductance of the Electron can be calculated **($3.41912126348 \times 10^{-24}$ Farads and 4.85262×10^{-19} Henries).**

Section 4 shows how the Capacitance value above can be used to calculate an approximate maximum width "W" of the charged loop relative to the CEWL diameter **(~0.53%),** which further validates the model by producing a believable value that doesn't conflict with the Electron's anomalous magnetic moment.

Section 5 contains discussions about different aspects of the CEWL Model including the width "W" calculation of Section 4, as well some new insights about how the loop in the CEWL Model is similar in nature to a high "Q" resonant loop antenna.

Section 6 contains some conclusions about the new insights as well as suggestions for possible future research.

Note: Reliable references are included for all subject "matter", (pun) but many of the more common concepts can be quickly googled if those expensive references are not available. The "Waveplate" Wiki article for example has excellent diagrams showing how a photon's charge separation rotates as it travels through space, and also shows how a photon's cross-section can be transformed from any form of an elliptical cross-section to any other form of an elliptical cross-section. Likewise, the "Stress-energy tensor" wiki article is a quick way to find Einstein's General Relativity Tensor equation which equates the space time distortions of electromagnetic energies to the space time distortions of mass.

1. The CEWL Electron Model:

The CEWL model [1][2], starts with the premise that since electron-positron pairs form from purely electromagnetic photons (of energy >1.022 Mev [3]), and since the resulting pairs of an electron and a positron (of energy 0.511 Mev each) have the same electromagnetic nature (as witnessed by their de Broglie wavelengths/frequencies), then they must have the same electromagnetic wave nature as the photons from which they originated, except for one detail; The magnetic field lines of electrons and positrons can close back on themselves allowing matter to exist at rest, whereas the magnetic field lines of photons do not (and hence the magnetic and electric fields of photons chase each other forward at the speed of light).

Note: Maxwell was the first to be able to calculate the speed of light "c" with his equation $c^2 = \frac{1}{\epsilon_0 \mu_0}$, where ϵ_0 and μ_0 are the electric and magnetic permittivity constants of free space. Where does the mass come from? One can combine Maxwell's equation above with Einstein's $E = mc^2$ to get $m = E\epsilon_0\mu_0$ where mass can be equated to the purely electromagnetic terms on the right. The electromagnetic energy tensor equations of general relativity theory are also shown to contribute to space time distortion exactly the same as mass does (see below).

2. From Photon to Fermion

2.1. From Photon:

Modern modelling of photons generally focuses on the "potential" E and B fields (Electric and Magnetic fields), but as Maxwell first envisioned a photon, it is composed of a charge separation spiralling through space at the speed of light [4] (the electric permittivity constant of free space ϵ_0 describes the capacitance like ability to induce a charge separation in free space). Fig 1 "before" shows how Maxwell envisioned the charge separation of a photon spiralling through space (the spiral can be either right hand or left polarity). Note: The cross section perpendicular to the direction of travel is of the general form of an ellipse [5], with "circularly" polarized light having a circular elliptical cross section, and regular "polarized" photons having a more elongated elliptical cross section. Circularly polarized photons can be changed into "regular" elliptically polarized photons and, vice versa, "regular" photons can be changed to circularly polarized by sending the photon through non-linear optics such as "quarter wave plates" [6]. The right-hand or left-hand spiral "spin" rotation direction however stays constant unless the photon is reflected by a mirror etc.

2.2. To Fermion

The CEWL model for electron positron pair production is shown below by the transition from a high energy photon in Fig 1 "before" to two charged loops in Fig 2 "after". The positively charged loop is a Positron and the negatively charged loop is an Electron. Due to the original spin rotation of each at formation, the magnetic fields are opposed at the moment of formation, allowing the electron and positron to separate despite their enormous electrostatic attraction at that scale. The original paper [2] contains the math to show that the opposing magnetic field at initial formation of an electron-positron pair would exceed the electrostatic attraction between them.

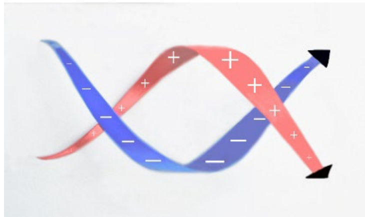

Fig 1.“Before” Photon λ=12.13 x 10^{-13}m

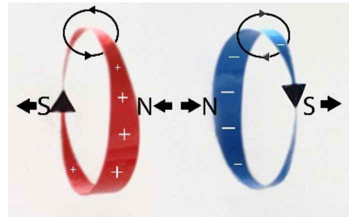

Fig 2. “After” Loop Diam = 7.723 x 10^{-13}m

Fig 1 “Before” and Fig 2 “After” show the transition from a 1.022 Mev photon to positive and negative loops, of 0.511Mev each, that are now closed loops and repelling away from each other magnetically. The positive loop is a positron, and the negative loop is an electron.
Note: The width “W” of the loops in Fig. 2 and 3 are exaggerated to show how the charge is most likely sinusoidally distributed around the loop, but the capacitance calculations (below) indicate a narrower width that is approximately 0.53% of the loop diameter.

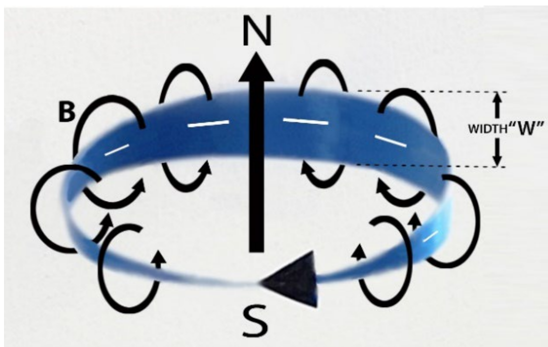

Fig. 3. The Magnetic “B” field lines which are due to the rotation of the charge inside an electron/positron have no component in the direction of the charge rotation and hence can add no rotational energy / mass. An electron or positron has zero internal resistance (or it would lose energy and decay) therefore ½ the mass is “electric” and ½ is “magnetic” [1]. Since only half the mass rotates, it is a ½ spin particle with a gyromagnetic G-factor of 2 rather than 1.

2.3. Electromagnetic Energy and Mass

MIT Physics professors’ emeriti Slater & Frank have solved Maxwell’s Electromagnetic equations for the general case of plane wave propagation of photons to show that the total Electro-Magnetic Energy Density in free space, i.e. with no resistive component is:

Total Electro-Magnetic Energy Density = $U = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$[7] (1)

As further explained by Slater & Frank, the average magnetic component (the B half of this equation) is only greater than the average Electric component when a resistive component is present [7]. In any given rest frame, photons and electrons do not lose energy, i.e., internally they have no resistive component, so therefore if the electrons and positrons maintain the same electromagnetic wave nature as the photons from which they originated, then the average electric energy must exactly equal the average magnetic energy in both cases.

Using $\mathrm{E} = mc^2$, one can divide the above Energy density equation (1) by c^2 to get Mass density:

Mass density = $\frac{U}{c^2} == \frac{1}{2c^2}(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2)$ (2)

Note: Using Maxwell's $c^2 = \frac{1}{\epsilon_0 \mu_0}$ to get rid of μ_0, it is easy to show that this is exactly the same equation as the Electromagnetic Stress-Energy tensor form for mass used in Einstein's General Relativity [8]

Electromagnetic Tensor equation for Mass: $T = \frac{\epsilon_0}{2}\left(\frac{E^2}{c^2} + B^2\right)$ [8] (3)

Since only half the electromagnetic mass contributes to L angular momentum, then we can simply substitute L/2 for L into Feynman's electron gyromagnetic equation:

Feynman: Gyromagnetic ratio= $\frac{\mu}{L} = g_e * (\frac{q}{2m_e})$ [9], Where $g_e = 1$ (4)

With L/2 instead of L:

Gyromagnetic ratio= $\frac{\mu}{L/2} = 2 * (\frac{q}{2m_e})$ (5)

Therefore, with only half the internal mass rotating, the electron's g-factor g_e is 2 instead of one (½ spin).

2.4. No other Diameter model can match reality without violating either Einstein's General relativity or Maxwell's equations.

The various reasons why the CEWL model leads to a unique solution for the Electron diameter can be found in a previous CEWL paper [1], but in general the main reason the CEWL Diameter is unique is as follows:

Due to the energy constraints of Einstein's Relativity, all velocities must be at or under the speed of light, therefore:

A) The charge must rotate at or **inside** the CEWL diameter in order to produce the correct de Broglie frequency/energy.
B) In order to generate the correct magnetic moment, the charge must rotate at or **outside** the CEWL diameter
The only simultaneous solution to both conditions is that the charge rotates at the speed of light at the CEWL diameter.

3. Calculating the Capacitance and Inductance of the Electron

When designing efficient power supplies or antennas, capacitors and/or inductors are generally added to the circuits to increase power factor and to match impedances for maximum energy transfer. The graph below shows a typical reactance graph showing the "real" resistance on the X axis and the positive and negative "impedances" due to the net capacitance and inductance of the circuit on the Y axis. Man-made Inductors have real internal resistance which needs to be allowed for when calculating the best capacitor / inductor to add to the circuit.

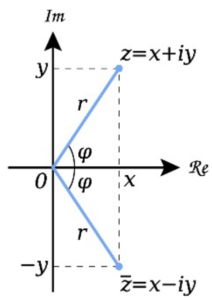

Fig. 4

Electrons and positrons however have zero "real" internal resistance (or they would decay) which simplifies the analysis to the case of simply matching the positive and negative impedances on the Y axis, i.e. the capacitance reactance must match the inductance reactance, i.e. both will match the reactive impedance of free space $\mathbf{X}_0$.

The reactive Impedance of Free Space is:
$\mathbf{X}_0$ = 376.730313668 (57) Ω [10]

The Reactive Impedance X_L of an inductor L, and impedance X_C of a capacitor C depends on Frequency in the following way:

$X_L = \omega L$, and $X_C = 1/(\omega C)$ [11] (6)

Where $\omega = 2\pi F_{Hz}$

The CEWL rotational frequency (which also matches the de Broglie frequency of an Electron) is:
$F_{CEWL} = 1.235590085 \times 10^{20}$ Hz, Therefore:

ω_{CEWL} in this case is $= 2\pi\, F_{CEWL} = 7.76344147 \times 10^{20}$

Solving for L and C of the Electron:

L_{CEWL} (Inductance of the Electron) $= \mathbf{X}_0/\omega = 4.85262 \times 10^{-19}$ Henries (7)

C_{CEWL} (Capacitance of Electron) $= 1/(\omega\, \mathbf{X}_0) = 3.41912126348 \times 10^{-24}$ Farads (8)

We can double check these values by calculating the resonant frequency of a capacitor-inductor (CL) loop:

$F_{Hz} = \mathbf{1} \,/\, \left(\boldsymbol{2\pi\sqrt{CL}}\right) = 1.235590085 \times 10^{20}$ Hz (9)
which exactly matches the Electron's de Broglie frequency (and the CEWL rotational frequency).

4. CEWL Electron Charge Width Calculations:

The Width of the circulating charge in the CEWL Model (see fig. 3) can be estimated by calculating the area of the charged loop required to match the above Electron capacitance.

The capacitance of an isolated object is defined as Q/V, where Q is the total charge on the object when it is at V volt potential relative to infinity. In the case of a conductive sphere, only the exterior surface area affects capacitance due to the Faraday Cage effect. In the case of a sheet of thin conductive material however, the area of both sides of the sheet would be used to calculate Capacitance. The capacitance of shapes other than spheres is generally approximated by applying a "shape factor" to the capacitance of a sphere with equivalent area [12].
Chow and Yovanovich have shown that an elongated needle-like spheroid or a long thin strip of conducting material will both have slightly more capacitance than a sphere of equivalent area and hence require an additional "Shape Factor" correction of approximately 1.2 [12]

Capacitance of a conductive Sphere in free space is well known to be $4\pi\epsilon_0 r$

Area of Sphere (with the same Capacitance as C_{CEWL}) = $1.186647377 \times 10^{-26}$ m^2

The area of both sides of the rotating charge in the CELW model is
= $2\pi D_{CEWL} \times (W_{max}/2)$ (see fig. 3)

Solving for W_{max} = (Area Sphere/1.2) / (πD_{CEWL})

= 4.0757×10^{-15} m (or Approx **0.53% of the Diameter D_{CEWL}**) (10)

5. Discussions:

5.1. Width

The width "W" calculation above is an estimate assuming a capacitive charge distribution similar to a conducting object, which should be a valid assumption since an electron has no internal resistance (or it would decay), but the actual dynamic charge distribution is unknown so a refinement may be necessary in the future. Note: in conductive materials, the charge will be most concentrated near the edges, which suggests that the highest concentration of charge in the CEWL model would be in the upper and lower edges of the band in Fig. 3. This would add a small amount to the magnetic moment due to the fact that the magnetic moment is the product of amperage times area and the area encompassed by the edges is slightly higher. Interestingly, the extra magnetic moment due to the concentration of charge near the edges in the CEWL model is not far from the actual anomalous magnetic moment correction of the electron [9] suggesting that this might be a physical representation of that phenomena.

5.2. No Contradiction with Stern Gerlach Experiment

The Stern Gerlach experiment [13] does not contradict the CEWL model, i.e. it is solely an atomic ecosystem phenomenon. The outermost, lone, electron of the Silver atoms used in the experiment is usually in the lowest energy state, whereby that electron's internal magnetic moment aligns with the magnetic moment created by the same electron as it orbits the nuclei (the Down state), but after being heated in the oven (that sends a stream of silver atoms through a magnetic separator), many of the atoms now have their outermost electron in the next higher energy state i.e. the "Up" state (the only other stable state), which means it's internal magnetic alignment is opposite the direction of the orbital magnetic field. The Stern Gerlach separator then deflects the atoms which have the two magnetic fields aligned more than the atoms in which the alignments are in contradiction. Note: in a previous paper I suggested that the up or down alignment of the outmost electron will either add or subtracted to the nuclei magnetic moment (which is a phenomenon which does happen), but as Quantum Physics Professor J. Shertzer PHD pointed out to me, the nuclei magnetic moment is not strong enough to be detected by the Stern Gerlach experiment. (my own Muon loop model [2] for neutrons and protons predicts an insufficient magnetic moment as well, so that should also have alerted me that I was using the wrong ecosystem, so I apologize for the misdirection). The conclusion is the same however, i.e. the Stern Gerlach experimental results are an atomic ecosystem phenomenon, not a phenomenon of an isolated electron itself, so there is no contradiction with the CEWL model.

5.3. Magnetic moments:

An interesting side note about the internal magnetic moment of an electron vs the (separate) magnetic moment due to its atomic orbit, is that the first orbit of an electron as it orbits around a hydrogen nucleus will generate a magnetic moment that is exactly the same as the magnetic moment of the Electron itself. Then in the second allowable hydrogen orbit (see radius in allowed Rydberg orbits [14]), the magnetic moment of the orbit around the nucleus is now exactly twice the magnetic moment of the electron itself. This suggests that one interpretation of stable orbits is that only the atomic orbits with magnetic moments that are exact multiples of the electron itself will be stable, possibly due to fact that the Larmor frequencies of the atomic orbits [15] and the Larmor frequency of the electron itself [15] need to resonate in order to be stable.

5.4. Resonance:

The circular loop of the CEWL electron model is similar to the nature of an inductive loop antenna [11] in that the circumference around a loop antenna must match the wavelength of the received/transmitted electromagnetic radio wave photons in order to prevent energy loss. Likewise, the circumference of the CEWL model exactly matches the wavelength of a photon of energy equal to that of an electron. Antenna theory subdivides electromagnetic interactions into 3 regions [11]: The “Near Field” where electromagnetic oscillations are induced near the antenna, but the electromagnetic interaction is strictly “reactive” i.e. energy in this near field leads to no net loss of energy from the antenna because all “virtual photon” energy is reabsorbed by the antenna, 2) The “Fresnel” intermediate region and 3) The “Far Field” where photons fully form and propagate away (leading to net energy loss from the antenna). The “virtual photons” (and virtual particles) that start to form near the electron but then get immediately reabsorbed is the basis for how QED originators Feynman, Schwinger, Dyson and Tomonaga (all antennae/resonance experts) were able to calculate the anomalous magnetic moment correction of 1.00116 for the electron’s g-factor [9]. The ability of electrons, positrons and other forms of matter to induce oscillations in the vacuum of free space is what distinguishes modern quantum physics from the previous more “classical” interpretations of physics, i.e. interactions of particles with a pure vacuum cannot be calculated as simply “one-way” interactions, but rather the energy fluctuations of all the virtual photons (and virtual particles) induced near matter must also be calculated, both for their effect back on the original particle as well as for their effect on nearby photons and matter [9][16].

5.5. Q factor of Resonance:

Antenna Theory uses a "Q" resonance factor [11] to characterize the efficiency of an antenna. Since an Electron does not lose energy (or it would decay), therefore the "Q" resonance of an Electron is effectively near infinite. High "Q" loop antennas are ones where the circumference of the loop exactly matches the wavelength of the transmitted or absorbed photon [11]. Loop antennas in which the circumference does Not match the absorbed/transmitted wavelength generally transmit/absorb best in the same plane as the loop (90 degrees away from the rotational axis of the loop) [11]. At perfect "Q" resonance however, when the loop circumference exactly matches the wavelength, the photons are best transmitted and absorbed when their travel direction lines up with the rotational axis of the loop i.e. in the North and South magnetic directions generated by the loop. [11]. To visualize this, refer to Fig. 1 and Fig 2, where either the electron or the positron in Fig. 2 will induce circular "virtual" proto photons like what is shown in Fig. 1, (propagating away from both sides of the electron or positron loop), except due to the extremely high Q of the electron or positron loop, the virtual photons are immediately reabsorbed.

5.6. CEWL Model compatible with Other Leptons:

Other Leptons match the CEWL model. The Muon and Tau forms of the electron have the same CEWL Model characteristics as the electron, except that the charge rotates at a smaller diameter (the Muon rotates at a diameter that is about 207 times smaller than the electron, and the Tau rotates at a diameter about 3,477 times smaller than the Electron). The magnetic moment of a rotating charge is proportional to amperage multiplied by the area enclosed. For a constant speed of light velocity, as the diameter decreases, the amperage increases in inverse proportion to the diameter, but the area enclosed falls off faster due to being proportional to the diameter squared, hence the CEWL model predicts that the magnetic moment for each will be proportional to the diameter of each (which matches reality).

5.7. Neutrinos:

Neutrinos come in 3 flavours, i.e. the Electron, Muon, and Tau, so they most likely have the same diameter as the Electron, Muon, and Tau respectively (but with a much smaller charge). If the rotating charge in Fig. 3 above induces a charge separation loop (offset above and below the Fig. 3 loop), that follows the CEWL charge around the loop, at the same diameter as the CEWL model, but with a much smaller Net charge (that is opposite the CEWL charge), then the resulting oppositely aligned magnetic field of the lesser-charged loop might add a stabilizing effect on the CEWL loop by "guiding" the CEWL magnetic fields back towards the purely reactive "Near Field" thereby preventing any possibility of photon or energy escape. This is just a preliminary guess about the nature of Neutrinos, but whatever the actual form of Neutrinos, it is highly likely to be a necessary part of achieving the near perfect "Q" required for stability. Anybody who can figure out why the Electron, Muon, and Tau (as well as the positively charged antimatter forms of these Leptons) only have the necessary high "Q" resonance precisely at the CEWL diameter and again at ~(1/207) of that diameter (the Muon diameter) and again at ~(1/3,477) of the CEWL diameter (the Tau diameter), will probably receive a Nobel prize 😉. It will probably take an expert in both physics and antenna/resonance theory like Feynman, Schwinger, Dyson and Tomonaga were.

5.8. CEWL Computer Modelling at a Smaller Scale:

The CEWL model doesn't give better energy calculations at the atomic scale than Quantum physics equations do, so one could ask why bother? The main difference is that while Quantum mechanical equations can give very exact energy "quantum jump" solutions etc. for many types of atoms, the solutions can only be localized down to the scale of the entire atom. If one wants to predict behaviour at a smaller scale, closer to the size of the Electron, then an accurate model for the Electron itself will ultimately be necessary. An analogy would be to compare trajectory calculations in the early days of sending rockets into space compared to modern computer modelling. Early trajectory solutions involved using equations that fit while leaving earth but later required switching to using different equations once further away from Earth or in Moon gravity etc. The splicing together of the different equations was problematic and inexact. When computers became more powerful, techniques like Euler's integration made computer modelling of trajectories quicker and more precise. Weather calculations would be another example. A precise model for the Electron and other Leptons opens up the possibility of computer modelling that gives more precise predictions at a smaller scale.

6. Conclusions:

The Charged Electromagnetic Wave Loop (CEWL) model of the Electron has been validated by the fact that it exactly matches all known values of the electron, including energy, de Broglie frequency, charge, mass, and magnetic moment and the model was then further validated when it explained the previous mystery of why the Electron's G factor is 2 rather than one (leading to ½ spin). The new insight that the capacitance and inductance can be uniquely calculated and used to estimate the width "W" of the CEWL loop relative to the diameter (0.53%) is another validation of the model since it produces a realistic width that only produces a small net increase in the magnetic moment, and that net increase is not far from the known anomalous magnetic moment of the Electron.
Other recent insights about A) How the CEWL Model's loop circumference exactly matches the wavelength of a photon of energy and wavelength equal to that of an Electron, B) How high Q loop antennas also share the same characteristic, i.e. the circumference must exactly match the wavelength in order to achieve high Q resonance, and C) How high Q loop antennae theory shows that virtual photons will be generated in the North and South magnetic directions generated by the CEWL loop (loop antennas without these characteristics have different radiation/absorption patterns), all suggest that these new insights may, in the future, provide new research "direction" into how Neutrinos and virtual photons form near the Electron, Muon and Tau Leptons.

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References

[1] Bowen, D., The Real Reason The Electron's Bare g-Factor Is 2 Times Classical, Scientific Research Publishing, scirp.org (Online) June 24, 2016
https://www.scirp.org/journal/PaperInformation.aspx?PaperID=67682

[2] Bowen, D. and Mulkern R.V., An Electron Model Consistent with Electron-Positron Pair Production from High Energy Photons, Scientific Research Publishing, scirp.org (Online) Aug. 27, 2015
http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=59188

[3] J. Gregory Stacy, W. Thomas Vestrand (2003). Pair Production, Encyclopedia of Physical Science 3rd Edition, Academic Press. III.A.3

[4] Maxwell, James Clerk. A Treatise On Electricity & Magnetism Vol. II. London : Clarendon Press, 1873. p. 403.

[5] Feynman, Leighton, Sands. (2011) The Feynman Lectures on Physics, Vol.1, ch.33-1. New York, NY: Basic Books

[6] Hecht, E. (2001). Optics (4th ed.). pp. 352–5.

[7] Slater, John C. and Frank. (2015) Nathaniel H., Electromagnetism, Dover Publications, New York. pg 94-103

[8] Misner, Charles W.; Thorne, Kip S.; Wheeler, John A. (1973). Gravitation. San Francisco, CA: W.H. Freeman and Company. Section 5.7 pp. 141-142

[9] Feynman, Leighton, Sands, (2011) The Feynman Lectures on Physics, Vol. III. New York, NY: Basic Books

[10] Clemmow, P. C. (1973). An Introduction to electromagnetic theory. University Press. p. 183.

[11] Balanis, Constantine A. (2016) Antenna Theory Analysis And Design, 4th Edition. John Wiley & Sons Inc., Hoboken, NJ, Chapters 2,5.

[12] Chow and Yovanovich, The Shape Factor of the Capacitance of a Conductor. J. Appl. Physics. 53(12), December 1982

[13] Eisberg, Robert (1961). Fundamentals of Modern Physics. New York: John Wiley & Sons. pp. 334-338

[14] Gallagher, Thomas F. (1994). Rydberg Atoms. Cambridge University Press

[15] Jackson, J. D. (1999) Classical Electrodynamics, 3rd edition. Wiley. p. 563

[16] Feynman, R. P. (1985) QED, The Strange Theory of Light and Matter. Princeton, NJ : Princeton University Press. pp. 115 – 118 & 152