Quantum X-entropy in Generalized Quantum Evidence Theory

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Abstract

In this paper, a new quantum model of generalized quantum evidence theory is proposed. Besides, a new quantum X-entropy is proposed to measure the uncertainty in generalized quantum evidence theory.

Keywords: Generalized quantum evidence theory, Quantum X-entropy

1. A new quantum model of GQET

Definition 1.1 *Let* $|\Phi\rangle = \{|\phi_1\rangle, \ldots, |\phi_j\rangle, \ldots, |\phi_m\rangle\}$ *be a QFOD. A set of basis events is defined:*

$$BE = \left\{|\emptyset\rangle, |\phi_1\rangle, \ldots, |\phi_j\rangle, \ldots, |\phi_m\rangle\right\}, \tag{1}$$

where $|\emptyset\rangle$ *is an unknown event.*

Definition 1.2 *A vector representation of a basis event is defined:*

$$|e_z\rangle = [\eta_0, \eta_1, \ldots, \eta_g, \ldots, \eta_m]^{\mathbf{T}}, \quad \eta_g = \begin{cases} 1, & g = z, \\ 0, & g \neq z. \end{cases} \tag{2}$$

Definition 1.3 *A pure quantum state of proposition* $|\psi_i\rangle$ *is defined:*

$$|\psi_i\rangle = \sum_z \lambda_z^i |e_z^i\rangle, \tag{3}$$

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where λ_z^i is a complex number with $\sum_z |\lambda_z^i|^2 = 1$.

Definition 1.4 *A density operator of $|\psi_i\rangle$ is defined as:*

$$\rho_i = |\psi_i\rangle\langle\psi_i|. \tag{4}$$

Definition 1.5 *The density operator of a GQBBA is defined as:*

$$\rho_{\mathbb{Q}_{\mathbb{M}}} = \sum_i \mathbb{Q}_{\mathbb{M}}(|\psi_i\rangle)\rho_i. \tag{5}$$

GQET is a quantum express of [1].

2. The proposed quantum X-entropy

Definition 2.1 *The quantum X-entropy is defined as:*

$$X(\mathbb{Q}_{\mathbb{M}}) = -\mathrm{tr}\left(\rho_{\mathbb{Q}_{\mathbb{M}}} \log \frac{\rho_{\mathbb{Q}_{\mathbb{M}}}}{2^d - 1}\right), \tag{6}$$

where d denotes eigenvectors of $\rho_{\mathbb{Q}_{\mathbb{M}}}$.

Let $\mathbb{E}_w$ and d_w be eigenvalues and eigenvectors of $\rho_{\mathbb{Q}_{\mathbb{M}}}$, respectively. The quantum X-entropy is also defined as:

$$X(\mathbb{Q}_{\mathbb{M}}) = -\sum_w \mathbb{E}_w \log \frac{\mathbb{E}_w}{2^{d_w} - 1}. \tag{7}$$

References

[1] F. Xiao, Generalized quantum evidence theory, Applied Intelligence (2022) DOI: 10.1007/s10489–022–04181–0.