Properties of elementary particles, dark matter, and dark energy

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Abstract

This paper discusses physics patterns that associate with properties of objects, elementary particle data, dark matter data, and dark energy phenomena. The patterns also associate with solutions to Diophantine equations that involve s, an integer. Electromagnetic properties (such as charge and magnetic moment) associate with solutions for which |s|=1. Gravitational properties (such as gravitational mass) and inertial properties (such as inertial mass) associate with solutions for which |s|=2. All known elementary particles associate with solutions for which s=0. This paper suggests a new notion – instance – that might pertain regarding the property of charge and a new property – isomer – that might pertain regarding most elementary particles. This paper suggests that nature includes six isomers of most known elementary particles. Five isomers associate with most dark matter. This paper suggests that the notions of instances and isomers help explain data. Possibly explained data include ratios of dark matter effects to ordinary matter effects for some galaxies and other objects. Suggestions regarding dark energy might help resolve tensions regarding the rate of expansion of the universe and regarding large-scale clumpiness. Suggestions regarding dark matter and dark energy might provide insight regarding galaxy formation.

Keywords: elementary particles, dark matter, dark energy, galaxy formation, neutrino masses

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1. Introduction

An objective of this paper is to propose patterns regarding data about elementary particles, dark matter, and dark energy. The patterns have dual associations – known data and solutions to Diophantine equations. Hopes include that the patterns will help physics find new elementary particles, gain insight about cosmology, and increase understanding about the formation of galaxies.

This paper pursues the following goals.

1. Catalog properties of objects.
2. Catalog elementary particles.
3. Use a set of properties and a set of elementary particles to explain data that associate with the two-word term dark matter and to explain data that associate with the two-word term dark energy.

This paper suggests catalogs of the following items.

1. Some properties of objects. Examples of properties include charge, magnetic moment, mass, and angular momentum. (For example, table 4 shows or alludes to properties that associate with electromagnetic interactions and properties that associate with gravitational interactions.)
2. Elementary particles. (For example, so-called intrinsic solution-pairs that table 3 shows associate with all known elementary particles and with some possible elementary particles.)
3. Types of events in which individual elementary particles transform (or decay) into sets of elementary particles. (For example, so-called one-step cascades that table 3 shows associate with types of decays that elementary particles undergo.)
4. Angular momentum aspects of two-component systems. (Discussion related to equation (22) suggests possible bases for cataloging some spin-related aspects of two-component systems.)
5. Some types of events in which multi-component systems participate. (For example, table 5 suggests a basis for cataloging some events in which atomic electron clouds interact with their surroundings.

This paper leaves opportunities to suggest principles that underlie patterns that associate with the catalogs.

This paper suggests a meta-catalog that approximately catalogs the above-mentioned five catalogs. This paper leaves opportunities to suggest principles that underlie the meta-catalog.

The following references provide information about topics that this paper discusses.

- Electromagnetism, gravity, physics constants, and physics properties.

Reference [1] explores notions of a coupling between electromagnetism and gravity. Reference [2] and reference [3] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which have origins in modeling regarding gravitation. References [4], [5], and [6] discuss gravitoelectromagnetism, which suggests similarities between gravity and electromagnetism.

Reference [7] and articles to which reference [7] alludes discuss, at least in the context of general relativity, possible relationships between mass and angular momentum.

Reference [8] discusses notions of repulsive components of gravity.

References [9], [10], [11], and [12] discuss experimental tests of theories of gravity.

- Elementary particles.

Reference [13] provides an overview of elementary particles and the elementary particle Standard Model.

Reference [14] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Reference [15] provides information about some of these types of modeling. References [16], [17], and [18] provide information about modeling and about experimental results. Reference [19] (including reviews numbered 86, 87, 88, 89, 90, and 94) provides other information about modeling and about experimental results.

Reference [20] suggests the notion of an inflaton field.

Reference [21] discusses the notion of a graviton.

Reference [22] discusses the notion of neutrino mass mixing.

Reference [22] discusses notions of sterile neutrinos and heavy neutrinos.

Reference [23] notes that quantum field theory suggests that massless elementary particles cannot have spins that exceed two.

A symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. Reference [24] discusses theory. Reference [18] reviews modeling and experiments regarding magnetic monopoles. Reference [25] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Reference [16] reviews modeling and experiments regarding axions. Reference [16] also notes modeling that suggests that nature might include axions.

Reference [17] reviews modeling and experiments regarding leptoquarks.

Reference [22] discusses modeling and data about neutrino masses and neutrino oscillations.

- Cosmology and astrophysics.

Reference [26] provides an overview of concordance cosmology and related topics regarding general physics, dark matter, and elementary particles. Reference [27] provides an overview of cosmology. References [28], [29], [30], and [31] review aspects of concordance cosmology. Reference [32] discusses observational tests for cosmological models.

Reference [33] discusses possibilities leading up to a Big Bang.

References [34] and [29] discuss inflation.

Reference [35] discusses attempts to explain the rate of expansion of the universe.

References [36] and [37] discuss so-called tensions between cosmology models and cosmology data.

References [30], [38], [39], [40], and [41] discuss the notion that concordance cosmology underestimates recent increases in the rate of expansion of the universe. Reference [30] suggests that possible resolutions regarding such an underestimate might focus on phenomena early in the history of the universe.

Reference [42] and reference [43] discuss possible types of dark matter.

Reference [43] notes that physics has yet to determine directly whether nature includes cold dark matter.

Reference [44] suggests that notions of warm dark matter might reduce discrepancies between data regarding clustering within galaxies and modeling that associates with cold dark matter.

Reference [45] suggests the following notions regarding dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. Some modeling suggests limits on the masses of basic dark matter objects. Observations suggest small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter. (Reference [46] discusses astrophysical and cosmological techniques.)

Reference [47] suggests notions of dark matter charges and dark matter photons. Reference [48] discusses possible effects of dark matter photons.

References [49], [50], and [51] discuss the notion that dark matter might include atom-like objects.

Reference [52] suggests that dark matter might include hadron-like particles.

Reference [53] suggests evidence of non-gravitational interactions - in galaxies and in galactic clusters - between dark matter and ordinary matter.

Reference [54] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [54] discusses parameters for classifying and describing galaxies. Reference [54] seems not to preclude galaxies that have few ordinary matter stars. Reference [54] seems not to preclude galaxies that have little ordinary matter.

Reference [55] suggests that concordance cosmology might not adequately explain gravitational interactions between neighboring galaxies.

- Multipole expansions.

Reference [56] discusses multipole expansions regarding electrostatics and the property of charge. Reference [57] discusses a multipole expansion regarding gravitation and the property of mass. Reference [58] discusses multipole expansions regarding acoustics.

2. Methods

2.1. Motivation for and evolution of the methods

Work leading to the methods that this paper discusses started from two notions.

One notion has roots in inferences that suggest sequential eras - possible inflation, the known slowing down in the rate of expansion, and the recent speeding up in the rate of expansion - in the history of the universe. Multipole expansions of gravitational potentials might explain the eras. (When two objects continue to move away from each other, dominance by quadrupole interactions - between the objects - can give way to dominance by dipole interactions and then dominance by dipole interactions can give way to dominance by monopole interactions. Regarding today's observable universe, pairs of adequately massive objects might exhibit dipole gravitational repelling while pairs of less massive objects exhibit monopole gravitational attracting. Earlier, for pairs of adequately massive objects, quadrupole attracting might have dominated dipole repelling. Yet earlier, octupole repelling might have produced inflation.)

One notion has roots in inferences of a ratio, 5^+ to 1, of dark matter density of the universe to ordinary matter density of the universe. The universe might include six approximately equal copies of a set that consists of most elementary particles. One copy might associate with ordinary matter and possibly some dark matter. The five other copies might associate with (the remaining) dark matter.

For some time, the work exhibited three aspects.

One aspect was that there did not seem to be enough data to adequately confirm or plausibly refute the work.

One aspect was that part of the work seemed to match all known elementary particles and to suggest new elementary particles.

One aspect was that harmonic oscillator mathematics seemed to be useful.

Then, three sets of new developments occurred.

One set of new developments associated with announcements of measured ratios - for other than densities of the universe - of dark matter effects to ordinary matter effects. The work seemed to explain quantitatively a variety of such ratios.

One set of new developments associated with announcements of tensions - between data and modeling - regarding the recent rate of expansion of the universe. The work seemed to point to a basis for the tensions.

One set of new developments associated with increasing emphasis - within the work - on notions based on Diophantine equations and on notions that associate with multipole expansions. The work seemed to gain abilities to point to and catalog electromagnetic properties (such as charge and magnetic moment) of objects, inertial properties (such as angular momentum) of objects, gravitational properties of objects, and so forth.

The technique for cataloging properties became a basis for cataloging elementary particles, for firming up aspects regarding dark matter, and for firming up aspects regarding dark energy (including aspects pertaining to the rate of expansion of the universe).

2.2. Context for the methods

This paper intertwines the following modeling.

1. POST (or, popular space-time modeling). POST has bases in mathematics related to space-time coordinates. POST includes bases for modeling, serves physics branches, and includes hypothesized attributes.
   (a) POST includes the following pair of bases for modeling. CM (or, classical mechanics) includes ND (or, Newtonian dynamics), SR (or, special relativity), and GR (or, general relativity). QM (or, quantum mechanics) includes QFT (or, quantum field theory).
   (b) Within the physics branch of elementary particles, POST includes the SM (or, the elementary particle Standard Model). The SM has bases in QFT.
   (c) Within the physics branch of cosmology and astrophysics, POST includes CC (or, concordance cosmology). CC includes notions about stars, solar systems, black holes, galaxies, galaxy clusters, and so forth. CC has bases in ND, SR, and GR.
   (d) POST includes the following trio of hypothesized attributes. OM (or, ordinary matter) associates (approximately) with stuff that associates directly with observations of light. DM (or, dark matter) associates with notions that suggest more gravitational attracting between objects than the gravitational attracting that POST associates with OM. DE (or, dark energy) associates with notions that suggest gravitational repelling between large objects that POST associates with OM plus DM.
2. SOMA (or, single-object, multiple-attributes modeling). SOMA has bases in integer mathematics. Mathematics that underlies SOMA associates with the two-word term Diophantine equations. (SOMA does not have direct bases in space-time coordinates, POST notions of tangent spaces to space-time spaces, or POST notions of phase spaces. SOMA points to properties - such as velocity - that associate with POST tangent spaces and with POST phase spaces.)
   (a) SOMA outputs a catalog of properties (including charge and mass) of objects and a catalog of elementary particles (including the electron, the Z boson, and all other known elementary particles).
   (b) SOMA suggests properties that POST does not include. One property that SOMA suggests and POST does not include is isomer. SOMA suggests that nature includes six isomers (or, near copies) of all elementary particles except LRI (or, long-range interaction) elementary bosons. LRI elementary bosons include the (known) photon and the (might-be) graviton.

POST and SOMA have a direct link to each other based on notions that associate with POST notions of multipole expansions. POST and SOMA have a direct link to each other based on notions that associate with POST notions of degrees of freedom.

The SM evolved based on proposals for new properties (such as color charge) and proposals for new elementary particles.

SOMA proposes yet other new properties (such as isomer) and yet other new elementary particles. SOMA proposes specifications for DM and DE.

This paper suggests that the combination of the blending of SOMA-suggested properties with POST properties and the blending of SOMA-suggested elementary particles with POST elementary particles might provide insight about elementary particles and might explain (otherwise seemingly unexplained) cosmology-and-astrophysics data.

2.3. A direct link between SOMA and POST

2.3.1. Some multipole aspects of POST

ND associates with three spatial dimensions and includes notions of potential energies V for which equation (1) pertains. Here, r denotes a radial coordinate. n_V is an integer.

$$V(r) \propto r^{n_V} \tag{1}$$

$n_V = -1$ associates with ND notions of monopole potentials and with ND notions of monopole forces. The notion of monopole force pertains - regarding ND - for the property of charge and the electric field component of the electromagnetic field. The notion of monopole force pertains - regarding ND - for the property of mass and the gravitational field.

$n_V = -2$ associates with ND notions of dipole forces. The notion of dipole force pertains - regarding ND - for the property of charge-current and the magnetic field component of the electromagnetic field.

2.3.2. A preview of some aspects of SOMA

SOMA suggests a mathematics-based language for expressing notions that associate with properties and with catalogs of properties. The language has bases in integers, in solutions to Diophantine equations, and in associations with multipole expansions. Notions of space-time coordinates associate with POST and do not directly pertain within the SOMA language.

SOMA mathematics has bases in expressions that involve integers k. Equation 2 specifies the relevant values of k.

$$-2 \leq k \leq 8, \text{ or } \log_2(k) - 3 \text{ is a positive integer} \tag{2}$$

2.3.3. A multipole expansion direct link between SOMA and POST

For an aspect that associates with ND and with $n_V \leq -1$, each relevant value of k is positive and the number, n_Γ, of values of k that pertain is $-n_V$.

Mathematically (but not necessarily physically), for an aspect that associates with ND, $n_\Gamma \leq 6$ pertains and SOMA associates positive values of the integer k with the POST notion of $k\hbar$ units of angular momentum.

Notions of integers k and of positive values of n_Γ underlie SOMA. Notions of ND, n_V, and $V(r) \propto r^{n_V}$ do not underlie SOMA.

SOMA includes a non-POST notion of multipole. Notions of SOMA monopole, dipole, and so forth associate respectively - via $n_\Gamma = -n_V$ - with ND notions of monopole, dipole, and so forth.

2.4. Bases for SOMA catalogs

2.4.1. Mathematical bases for SOMA multipole sums

The symbol Z denotes a set of positive integers.

Equation (3) shows a term in which k is an integer and s_k can be one of minus one, zero, or plus one.

$$ks_k \tag{3}$$

SOMA multipole mathematics has bases in sums of the form that equation (4) shows. An integer k appears no more than once in such a sum.

$$s = \sum_{k \in Z} ks_k \tag{4}$$

Regarding sums of the form that equation (4) shows, the symbol k_{max} denotes the largest value of k for which $|s_k| = 1$.

Equation (5) defines Σ.

$$\Sigma \equiv |s| \tag{5}$$

For each solution that associates with equation (4), there is exactly one different solution for which, for each $k \in Z$, the negative of the value s_k replaces s_k. For the second solution, $-s$ replaces s. SOMA uses the one-element term solution-pair to denote such a pair of solutions.

Equation (6) shows notation that SOMA associates with solution-pairs. The letter g is a convenience regarding notation. (Some applications of SOMA associate $\Sigma = 1$ with electromagnetic properties of objects and $\Sigma = 2$ with gravitational properties of objects. Regarding $\Sigma = 1$ and the letter g, one might think of the two-word term gamma rays. Regarding $\Sigma = 2$ and the letter g, one might think of the word gravity.) The symbol Γ denotes a list - in ascending order - of the positive integers k for which $k \in Z$ and $|s_k| = 1$.

$$\Sigma \mathrm{g} \Gamma \tag{6}$$

Regarding equation (6), SOMA uses the symbol Z_Γ to denote the set of positive integers k for which $k \in Z$ and $|s_k| = 1$. The symbol n_Γ denotes the number of positive integers k for which $k \in Z$ and $|s_k| = 1$.

Table 1 alludes to all $s = \sum_{k \in Z}(ks_k)$ expressions for which $1 \leq k \leq k_{max} \leq 4$.

SOMA includes solution-pairs for which integers k for which $k \geq 5$ pertain. For each of those solution-pairs, $k_{max} \geq 5$ pertains. In general, the following notions pertain.

SOMA suggests that each relevant solution-pair comports with equation (7).

Table 1: $\Sigma = |s| = |\sum_{k\in Z}(ks_k)|$ solution-pairs for which $1 \leq k \leq k_{max} \leq 4$. The columns labeled $1\cdot s_1$ through $4\cdot s_4$ show contributions that associate with terms of the form ks_k. Each entry in the column with the label Σ alludes to a unique solution-pair. Regarding table 1, the integer n_0 equals the number of k for which $1 \leq k \leq k_{max} \leq 4$ and $s_k = 0$. The integer n_Γ equals the number of k for which k appears in the list Γ. The number n_{sp} equals $2^{n_\Gamma - 1}$ and states the number of solution-pairs. The column for which the one-element label is SOMA-pole associates mathematically with the number of solution-pairs. For a row for which exactly one solution-pair pertains, the column shows the word monopole. For a row for which exactly two solution-pairs pertain, the column shows the word dipole. For a row for which exactly four solution-pairs pertain, the column shows the word quadrupole. For a row for which exactly eight solution-pairs pertain, the column shows the word octupole. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solution-pairs. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

k_{max}	Γ	$1\cdot s_1$	$2\cdot s_2$	$3\cdot s_3$	$4\cdot s_4$	Σ	n_0	n_Γ	n_{sp}	SOMA-pole
1	1	±1	-	-	-	1	0	1	1	Monopole
2	2	0	±2	-	-	2	1	1	1	Monopole
2	1‘2	±1	±2	-	-	1,3	0	2	2	Dipole
3	3	0	0	±3	-	3	2	1	1	Monopole
3	1‘3	±1	0	±3	-	2,4	1	2	2	Dipole
3	2‘3	0	±2	±3	-	1,5	1	2	2	Dipole
3	1‘2‘3	±1	±2	±3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	±4	4	3	1	1	Monopole
4	1‘4	±1	0	0	±4	3,5	2	2	2	Dipole
4	2‘4	0	±2	0	±4	2,6	2	2	2	Dipole
4	3‘4	0	0	±3	±4	1,7	2	2	2	Dipole
4	1‘2‘4	±1	±2	0	±4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	±1	0	±3	±4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	±2	±3	±4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	±1	±2	±3	±4	0,2,2,4,4,6,8,10	0	4	8	Octupole

$$1 \in Z_\Gamma \text{ or } 2 \in Z_\Gamma \text{ or } 3 \in Z_\Gamma \text{ or } 4 \in Z_\Gamma \tag{7}$$

For each solution-pair ΣgΓ, equation (8) defines k_{n_0}.

$$k_{n_0} \equiv \max\{k|1 \leq k \leq 4 \text{ and } k \in Z_\Gamma\} \tag{8}$$

For each solution-pair ΣgΓ, equation (9) computes n_0.

$$n_0 = \text{ the number of } k \text{ for which } 1 \leq k \leq k_{n_0} \text{ and } k \notin Z_\Gamma \tag{9}$$

Equation (7) and equation (9) imply that the range $0 \leq n_0 \leq 3$ pertains regarding n_0.

For $n_\Gamma \geq 4$, each one of some combinations of Γ and Σ associates with more than one solution-pair. For a combination of Γ and Σ that associates with more than one solution-pair, equation (10) shows a symbol that SOMA uses.

$$\Sigma\text{g}\Gamma\text{x} \tag{10}$$

2.4.2. Some sets of SOMA multipole solution-pairs

SOMA uses symbols of the form Σg' to denote solution-pairs for which the integer Σ is positive and $\Sigma \in Z_\Gamma$. For example, 1g1‘2 associates with 1g'.

SOMA uses symbols of the form Σg" to denote solution-pairs for which the integer Σ is positive and $\Sigma \notin Z_\Gamma$. For example, 3g1‘2 associates with 3g". (Here, one solution associates with $s = +1 + 2 = +3$.)

SOMA uses the symbol 0g to denote solution-pairs for which the integer Σ is zero. The solution-pair 0g1‘2‘3 provides an example. (Here, one solution associates with $s = -1 - 2 + 3 = 0$.) Arithmetically, 0g associates with $n_\Gamma \geq 3$. SOMA associates the symbol n_{Γ^0} with n_Γ that associate with 0g solution-pairs.

2.4.3. Cascades that interrelate SOMA multipole solution-pairs

SOMA includes the notion of adding - to one Γ - one new positive integer and thereby producing a new Γ. For an original solution-pair Σ_1gΓ_1, SOMA requires that a resulting solution-pair Σ_2gΓ_2 satisfies $\Sigma_2 = \Sigma_1$.

SOMA associates the word cascade with that notion.

For one original solution-pair, more than one cascade solution-pair might pertain.

SOMA also associates the word cascade with a network of solution-pairs that cascade (from each other) based on multiple cascade steps that ensue from one solution-pair. The solution-pair 1g1'2 associates with a first step in a cascade that starts with the solution-pair 1g1. A next cascade step provides the 1g1'2'4 solution-pair. A next cascade step produces two 1g1'2'4'6 solution-pairs and the 1g1'2'4'8 solution-pair.

2.4.4. Some direct links between SOMA and POST

Equation (4) and the equation $\Sigma = 2 \in Z_\Gamma$ provide direct links between SOMA and POST. SOMA suggests that $s = +2$ associates with the POST notion of the left-circular polarization mode of the gravitational field. SOMA suggests that $s = -2$ associates with the POST notion of the right-circular polarization mode of the gravitational field. SOMA suggests that solution-pairs that associate with $\Sigma = 2 \in Z_\Gamma$ associate with gravitational properties of objects. SOMA suggests that solution-pairs that associate with $\Sigma = 2 \in Z_\Gamma$ associate with inertial properties of objects.

2.4.5. Gravitational properties and inertial properties

ND includes three notions of mass - (gravitationally) active mass, (gravitationally) passive mass, and inertial mass.

ND tends to assume that active mass, passive mass, and inertial mass equal each other.

SOMA interprets GR as replacing the notions of active mass, passive mass, and inertial mass with notions of, respectively, (gravitationally) active energy, (gravitationally) passive energy, and inertial energy.

SOMA assumes that (gravitationally) active energy equals (gravitationally) passive energy. Based on the notion of more than one isomer, SOMA suggests that gravitational energy does not necessarily equal inertial energy.

2.4.6. Notions that associate with objects and with systems of objects

POST includes various modeling techniques and a notion of physical entities.

Regarding POST modeling that treats an entity as not having components, SOMA associates the word object with the entity.

Regarding POST modeling that treats an entity as having components, SOMA associates the word system (or, the three-word phrase system of objects) with the entity and associates the word object with a specific component of the entity.

SOMA suggests that objects and systems associate with observations and with $\Sigma = 0$ (and, thus, with 0g). Arithmetically, $\Sigma = 0$ associates with $n_{\Gamma^0} \geq 3$. (Table 3 pertains regarding elementary particles. Discussion related to equation (22) pertains regarding systems that model as being comprised of two objects.)

2.4.7. Notions that associate with events

SOMA associates the word event with (at least) interactions between the two objects in a two-object system.

QM associates interactions with intermediation by bosons. SOMA suggests that interactions mediated by LRI bosons can associate with Σg" solution-pairs for which $\Sigma \geq 1$ and $n_\Gamma \geq 3$. Here, $\Sigma = 1$ associates with electromagnetically mediated interactions. (Table 5 pertains.) Here, $\Sigma = 2$ associates with gravitationally mediated interactions.

2.4.8. An approximate catalog of SOMA catalogs

Table 2 previews SOMA suggestions regarding associations between solution-pairs and physics.

Table 2 approximately catalogs most of the catalogs that SOMA suggests.

2.5. Aspects that associate with degrees of freedom

2.5.1. Physical notions

POST does not necessarily include notions that parallel SOMA uses of integers k, s_k, and ks_k.

The following notions pertain for each k that is a member of Z_Γ.

- Mathematically, $|s_k| = 1$ pertains.
- Physically, SOMA associates three similar aspects with $|s_k| = 1$. For example, in some cases, the three aspects associate with POST notions of three DOF (or, degrees of freedom). In some cases, the three aspects associate with POST notions of three related states.

Table 2: SOMA suggestions regarding associations between solution-pairs and physics. The leftmost two columns specify a set of solution-pairs. The third column shows the associated Σg$\cdots$ notation. The rightmost column describes physics notions (usually POST-related notions) that SOMA suggests associate with some of the solution-pairs. Notions of inertia-related associate with the first $\cdots$g' row. Notions of LRI-related associate with the remaining $\cdots$g' rows. Notions of inertia-related also associate with the last $\cdots$g' row. The notions of $\Sigma = 2$ and $\Sigma \in Z_\Gamma$ pertain for each one of the first row and the third row. For the first case of $\Sigma = 2$ and $\Sigma \in Z_\Gamma$, other properties include momentum and angular momentum. For the case of $\Sigma = 1$ and $\Sigma \in Z_\Gamma$, another property is magnetic moment. For the cases of $3 \leq \Sigma \leq 4$ and $\Sigma \in Z_\Gamma$, properties associate with POST notions of elementary-particle handedness and with SOMA notions of isomer. For the case of $\Sigma = 3$ and $\Sigma \notin Z_\Gamma$ and $n_\Gamma = 2$, an anomalous property is anomalous magnetic moment.

Σ	Other constraints	$\cdots$g$\cdots$	Associations
$= 2$	$\Sigma \in Z_\Gamma$	2g'	Inertial properties (such as energy) of objects
$= 1$	$\Sigma \in Z_\Gamma$	1g'	Electromagnetic properties (such as charge) of objects
$= 2$	$\Sigma \in Z_\Gamma$	2g'	Gravitational properties (such as energy) of objects
$= 3$	$\Sigma \in Z_\Gamma$	3g'	SOMA-suggested (isomer-related) properties of objects
$= 4$	$\Sigma \in Z_\Gamma$	4g'	SOMA-suggested (isomer-related) properties of objects
$1, .., 4$	$\Sigma \in Z_\Gamma$	$(1, .., 4)$g'	LRI-related conservation laws (such as conservation of energy)
$= 0$	$5 \notin Z_\Gamma$, $7 \notin Z_\Gamma$	0g	Elementary particles (such as the electron and the Z boson)
$= 0$	$5 \in Z_\Gamma$ or $7 \in Z_\Gamma$	0g	Spin-centric states of multi-object systems
≥ 1	$\Sigma \notin Z_\Gamma$, $n_\Gamma = 2$	(≥ 1)g"	QFT anomalous properties of objects
≥ 1	$\Sigma \notin Z_\Gamma$, $n_\Gamma \geq 3$	(≥ 1)g"	Events via which multi-object systems transit between states

- This paper associates the two-element term DOF-like triad with any such set of three similar aspects.

The following notions pertain for each k for which k comports with equation (2), $1 \leq k \leq k_{max}$, and k is not a member of Z_Γ.

- Mathematically, $s_k = 0$ pertains.
- Physically, SOMA associates one aspect with $s_k = 0$. For example, in some cases, the aspect associates with POST notions of one DOF. In some cases, the aspect associates with a lack of applicability of any of the three aspects that SOMA associates with $|s_k| = 1$.

For a DOF-like monad, DOF-like dyad, or DOF-like triad, this paper associates the two-element term DOF-like aspects with the elements of the associated set.

For integers k for which $-2 \leq k \leq 0$, the following notions pertain. SOMA associates $|s_k| = 1$ with three DOF-like aspects. SOMA associates $s_k = 0$ with one DOF-like aspect. SOMA does not associate k with $|k|\hbar$ units of angular momentum.

2.5.2. Harmonic oscillator mathematics that might associate with the physical notions

Per reference [59], for integers n for which $n \geq 2$, mathematics associates $SU(n)$ symmetry with the ground state of an n-dimensional isotropic harmonic oscillator. SOMA uses the expression gen(*group*) to denote the number of generators of the group *group*. Mathematics provides, for integers $n \geq 2$, the result $\text{gen}(SU(n)) = n^2 - 1$.

Mathematics associates a $U(1)$ symmetry with a one-dimensional harmonic oscillator. Mathematics provides that $\text{gen}(U(1)) = 1$.

SOMA suggests the following possible associations regarding $|s_k| = 1$-related occurrences of three DOF-like aspects.

- For a positive value of k, mathematically (but not necessarily physically), SOMA associates $s_k = +1$ with a notion of a left-circular polarization contribution toward a net polarization that associates with s.
- For a positive value of k, mathematically (but not necessarily physically), SOMA associates $s_k = -1$ with a notion of a right-circular polarization contribution toward a net polarization that associates with s.
- For any SOMA-relevant value of k, SOMA suggests that $s_k = +1$ associates with mathematics that associates with a one-dimensional harmonic oscillator.
- For any SOMA-relevant value of k, SOMA suggests that $s_k = -1$ associates with mathematics that associates with a one-dimensional harmonic oscillator.

- SOMA associates a two-dimensional isotropic harmonic oscillator with the two one-dimensional harmonic oscillators that associate, respectively, with $s_k = +1$ and $s_k = -1$.
- Mathematics associates $SU(2)$ symmetry with the ground state of the two-dimensional isotropic harmonic oscillator.
- SOMA associates the three generators of the $SU(2)$ group with the three DOF-like aspects.

SOMA suggests the following possible associations regarding $s_k = 0$-related occurrences of one DOF-like aspect.

- SOMA suggests that $s_k = 0$ associates with mathematics that associates with a one-dimensional harmonic oscillator.
- Mathematics associates $U(1)$ symmetry with the one-dimensional isotropic harmonic oscillator.
- SOMA associates the one generator of the $U(1)$ group with the one DOF-like aspect.

2.6. Intrinsic properties of objects and extrinsic properties of objects

SOMA suggests using the word intrinsic and the word extrinsic to label properties of objects.

Regarding CM, the SOMA notion of intrinsic property can associate with an object. Examples of intrinsic properties include charge and mass. Regarding CM, the SOMA notion of extrinsic property can associate with motion sensed by observers of the object. Examples of extrinsic properties include charge-current and momentum.

For a SOMA-labeled extrinsic property, there is (at least) one associated intrinsic property. For example, for charge-current, the associated intrinsic property is charge. SOMA notions of such an extrinsic property associate with a one-step cascade from the intrinsic property. An extrinsic property can associate with three additional (compared to a related intrinsic property) DOF-like aspects.

2.7. Inertial properties of objects

This unit associates with the row - in table 2 - that associates with 2g' and mentions the two-word term inertial properties.

2.7.1. Inertial properties

Regarding CM notions of (spatial) position, two notions pertain. For one notion, the extrinsic property of position (of the object) associates with three DOF-like aspects. The three DOF-like aspects associate with the possible relative positions of an observer and the object. For the other notion, the object senses itself as being where the object is. The intrinsic property of position (of the object) associates with zero DOF-like aspects.

Regarding velocity, two notions pertain. For one notion, the extrinsic property associates with the (instantaneously linear) velocity (of the object) with respect to an observer. Extrinsic velocity associates with three additional (compared to extrinsic position) DOF-like aspects. For one notion, the intrinsic property associates with the angular velocity (of the object) and associates with three additional (compared to intrinsic position) DOF-like aspects.

Regarding acceleration, the following notions pertain.

- An extrinsic property of acceleration (of the object) would associate with a change in the velocity (of the object) relative to an observer. POST associates such an acceleration with notions of nonzero forces that act on at least one of the object and the observer and on the notion that the effect of such forces on the object differs from the effect of such forces on the observer. In effect, at least one of the object and the observer models as being part of a multi-object system and, thereby, models as (at least partly) losing its identity.
- An intrinsic property of acceleration (of the object) would associate with a change in the angular velocity of the object. POST associates such an acceleration with notions of nonzero forces that act on the object. In effect, the object models as being part of a multi-object system and, thereby, models as (at least partly) losing its identity.
- More generally, any attempt to add (with respect to velocity) three new DOF-like aspects associates with a loss (or, at least partial loss) of identity.

Via the property of inertial mass, ND associates intrinsic position with mass, extrinsic velocity with momentum, and intrinsic angular velocity with angular momentum.

SOMA associates intrinsic use of the solution-pair 2g2 with the property of energy.

SOMA associates extrinsic use of the solution-pair 2g2‘4 with the property of momentum.

Regarding SR, the combination of intrinsic use of 2g2 and extrinsic use of 2g2‘4 associates with a 4-vector.

SOMA associates intrinsic use of the solution-pair 2g2‘4 with the property of angular momentum. For the object, SOMA suggests that extrinsic use of the solution-pair 2g2‘4‘8 associates with (linear) motion of the angular momentum. To not lose the identity of the object, the intrinsic property of angular momentum needs to move along with the intrinsic property of energy. SOMA associates extrinsic use of the solution-pair 2g2‘4‘8 with the (linear) velocity of the intrinsic property of angular momentum and with zero new (compared to extrinsic use of 2g2‘4) DOF-like aspects.

SOMA associates intrinsic use of 2g2‘4‘8 with changes in intrinsic angular momentum. For such a change to be nonzero, POST suggests that the object interacts (perhaps via fields) with other objects. The object would (at least partly) lose its identity. Within the SOMA notion of object, SOMA suggests that intrinsic 2g2‘4‘8 associates with no effect and with zero new DOF-like aspects.

2.7.2. A SOMA limit that associates with inertial properties

Equation (11) reflects a limit that SOMA suggests regarding inertial properties. (Discussion above regarding inertial properties and the 2g2‘4‘8 solution-pair motivate equation (11).)

$$\text{For each } \Sigma \in Z_\Gamma \text{ solution-pair, } k_{max} \leq 8 \tag{11}$$

2.8. Electromagnetic properties of objects

This unit associates with the rows - in table 2 - that associate with 1g’.

2.8.1. An electromagnetism direct link between POST and SOMA

For electromagnetism, equation (4) and the equation $\Sigma = 1 \in Z_\Gamma$ provide a direct link between POST and SOMA. SOMA suggests that $s = +1$ associates with the POST notion of the left-circular polarization mode of the electromagnetic field. SOMA suggests that $s = -1$ associates with the POST notion of the right-circular polarization mode of the electromagnetic field. SOMA suggests that solution-pairs that associate with $\Sigma = 1 \in Z_\Gamma$ associate with electromagnetic properties of objects. ($\Sigma = 1 \in Z_\Gamma$ associates with 1g’.)

2.8.2. Electromagnetic properties

SOMA associates intrinsic use of the solution-pair 1g1 with the property of charge.

SOMA associates extrinsic use of the solution-pair 1g1‘2 with the property of charge-current.

Regarding SR, intrinsic 1g1 and extrinsic 1g1‘2 associate with a 4-vector.

SOMA associates intrinsic use of the solution-pair 1g1‘2 with the property of magnetic moment.

SOMA associates extrinsic use of the solution-pair 1g1‘2‘4 with the velocity that associates with the object. SOMA associates extrinsic use of the solution-pair 1g1‘2‘4 with zero (new, compared to extrinsic 1g1‘2) DOF-like aspects.

SOMA associates intrinsic use of the solution-pair 1g1‘2‘4 with three new DOF-like aspects and with a property of rotation of the axis that associates with the magnetic moment. (The related three new DOF-like aspects associate with a lack of equality between the axis of the magnetic moment and the axis of rotation of the object.) For example, for the Earth, that rotation has a period of one day. (The POST term Larmor precession does not pertain. Larmor precession associates with effects of an external magnetic field.)

SOMA associates extrinsic use of the solution-pair 1g1‘2‘4‘8 with motion of the property that associates with 1g1‘2‘4. SOMA associates extrinsic use of the solution-pair 1g1‘2‘4‘8 with zero (new, compared to extrinsic 1g1‘2) DOF-like aspects.

2.8.3. An association between two types of contributions to the electromagnetic field

An observer that senses an object as non-moving and non-rotating observes - regarding contributions by the object to the electromagnetic field - that $E \propto |q|$ and $B = 0$. $|q|$ denotes the magnitude of the charge of the object. E denotes the magnitude of the electric field. B denotes the magnitude of the magnetic field.

SOMA associates an observer-perceived contribution - by an object - to the electric field with the observer's notion of intrinsic 1g1 for the object. SOMA associates observer-perceived contributions - by an object - to the magnetic field with the observer's notions of extrinsic 1g1'2 for the object and intrinsic 1g1'2 for the object.

For an observer that senses an object for which $|q| > 0$ as moving and not rotating, $B > 0$.

SR associates equation (12) with Lorentz invariance. c denotes the speed of light.

$$E^2 - c^2B^2 = \text{ a constant } \geq 0 \tag{12}$$

Per equation (12), an observer that senses the object as moving senses a larger value of E than does an observer that senses the object as not moving.

SOMA suggests that - in effect - effects of extrinsic 1g1'2 subtract from effects of intrinsic 1g1. Similarly, SOMA suggests that - in effect - effects of intrinsic 1g1'2 subtract from effects of intrinsic 1g1.

2.9. Associations between multipole contributions and monopole contributions

SOMA suggests the following generalization, based on discussion related to equation (12).

For any one value of Σ, equation (13) and equation (14) pertain.

$$\text{dipole, octupole, } \cdots \text{ effects subtract from monopole effects} \tag{13}$$

$$\text{quadrupole, 16-pole, } \cdots \text{ effects add to monopole effects} \tag{14}$$

2.10. Some properties of all known and some SOMA-suggested elementary particles

This unit associates with the row - in table 2 - that associates with 0g, $5 \notin Z_\Gamma$, and $7 \notin Z_\Gamma$.

The notion of 0g pertains. Equation (11) does not pertain.

SOMA uses the following notions to catalog elementary particles. A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the POST expression $S(S+1)\hbar^2$ that associates with angular momentum. Regarding POST, known values of S include 0, 0.5, and 1. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of the charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, SOMA omits - from the symbols for families - the symbol Q.

Table 3 catalogs all known elementary particles and some elementary particles that SOMA suggests nature might include.

SOMA suggests the following notions regarding properties of elementary particles that associate with intrinsic solution-pairs that table 3 shows.

- Boson or fermion? - Equation (15) pertains.

$$\text{The object is a fermion } \Leftrightarrow \; 6 \in (\text{intrinsic})Z_\Gamma \tag{15}$$

- Charge or no charge? - Equation (16) pertains.

$$Q = 1 \Leftrightarrow 4 \notin Z_\Gamma \;\text{ and }\; Q = 0 \Leftrightarrow 4 \in Z_\Gamma \tag{16}$$

- Mass or no mass? - Equation (17) pertains. The symbol m associates with the property of mass.

$$m = 0 \Leftrightarrow (6 \notin Z_\Gamma \text{ and } 8 \in Z_\Gamma) \tag{17}$$

- Magnitude of spin? - SOMA associates the symbol n_{Γ^o} with n_Γ that associate with 0g solution-pairs. For boson elementary particles, intrinsic use of a solution-pair associates with the spin that equation (18) computes. For fermion elementary particles, intrinsic use of a solution-pair associates with the spin that equation (19) computes.

$$\text{If the object is a boson, } S = |n_{\Gamma^o} - 4| \tag{18}$$

$$\text{If the object is a fermion, } S = |n_{\Gamma^o} - 4.5| \tag{19}$$

Table 3: All known elementary particles and some elementary particles that SOMA suggests nature might include. The leftmost three columns provide information about elementary particles. The three charged leptons are the electron, the muon, and the tau. n_{EP} denotes the number of elementary particles. Q is a magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron). m denotes mass. Regarding the $0.5\mathrm{Q}_{2/3}$ family of three quarks, a notion of two-thirds times the $Q = 1$ intrinsic solution-pair plus one-third times the $Q = 0$ intrinsic solution-pair pertains. Regarding the $0.5\mathrm{Q}_{1/3}$ family of three quarks, a notion of one-third times the $Q = 1$ intrinsic solution-pair plus two-thirds times the $Q = 0$ intrinsic solution-pair pertains. The symbol † denotes that the elementary particles are as-yet unfound. The word inflaton associates with POST notions of a possible inflaton elementary particle. 2L cascades from 1L, 3L cascades from 2L, and so forth. The acronym TBD abbreviates the three-word phrase to be determined. The following notation pertains regarding the series 2L, 3L, and so forth. The symbol $+32\hat{}$ denotes the series $+32$, $-32+64$, $-32-64+128$, and so forth. The symbol $+64\hat{}$ denotes the series $+64$, $-64+128$, $-64-128+256$, and so forth.

Intrinsic	Names	Families	n_{EP}	Q	m	One-step cascades
$\lvert -1-3+4 \rvert$	Z	1Z	1	0	>0	$\lvert +1-2-3+4 \rvert$; $\lvert -1-3-4+8 \rvert$; $\lvert +1-3-4+6 \rvert$.
$\lvert -1-2+3 \rvert$	W	$1\mathrm{W}_1$	1	1	>0	$\lvert +1-2-3+4 \rvert$; $\lvert -1-2-3+6 \rvert$.
$\lvert +1-2-3+4 \rvert$	Higgs boson	0H	1	0	>0	$\lvert +1-2-3-4+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$.
$\lvert -1-3-4+8 \rvert$	Inflaton	0I	1 †	0	=0	$\lvert +1-2-3-4+8 \rvert$; $\lvert -1-3-4-8+16 \rvert$; $\lvert -1+3-4-6+8 \rvert$.
$\lvert +1-3-4+6 \rvert$	Neutrinos	0.5N	3	0	>0	$\lvert -1+3-4-6+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$.
$\lvert -1-2-3+6 \rvert$	Charged leptons	$0.5\mathrm{C}_1$	3	1	>0	$\lvert -1+3-4-6+8 \rvert$; $\lvert -1+2-3-4+6 \rvert$.
$\lvert -1+3-4-6+8 \rvert$	Arcs	0.5R	3 †	0	>0	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$.
$\lvert -1+2-3-6+8 \rvert$ and $\lvert -1+2-3-4+6 \rvert$	Quarks	$0.5\mathrm{Q}_{2/3}$; $0.5\mathrm{Q}_{1/3}$	6	1, 0	>0	$\lvert -1-2-3+4-6+8 \rvert$, $\lvert +1-2+3-4-6+8 \rvert$.
$\lvert +1-2-3-4+8 \rvert$	Gluons	1G	8	0	=0	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$.
$\lvert +1-2-3-4+8 \rvert$	Jay	1J	1 †	0	=0	$\lvert +1-2+3-4-6+8 \rvert$, $\lvert -1-2-3+4-6+8 \rvert$.
$\lvert -1-3-4-8+16 \rvert$	Photon	1L	1	0	=0	$\lvert +1-2-3-4-8+16 \rvert$, $\lvert -1-3-4-8-16+32 \rvert$, $\lvert -1+3-4-6-8+16 \rvert$.
$\lvert -2-4+6 \rvert$	TBD	1.5R	3 †	0	>0	$\lvert -2+4-6+8 \rvert$
$\lvert -2-6+8 \rvert$ and $\lvert -2-4+6 \rvert$	TBD	$1.5\mathrm{Q}_{>0}$	≥3 †	1, 0	>0	$\lvert -2+4-6+8 \rvert$
$\lvert -1-3-4-8-16+32\hat{} \rvert$	Graviton, TBD, …	2L, 3L, …	1 †, 1 †, …	0, 0, …	=0, =0, …	$\lvert +1-2-3-4-8-16+32\hat{} \rvert$, $\lvert -1+3-4-6-8-16+32\hat{} \rvert$, $\lvert -1-3-4-8-16-32+64\hat{} \rvert$

- Number of fermion flavours? - For fermion elementary particles, each intrinsic use of a solution-pair associates with $6 \in Z_\Gamma$ and with three flavours.

- LRI elementary particle or not an LRI elementary particle? Equation (20) pertains.

$$\text{An elementary particle is an LRI elementary particle } \Leftrightarrow \ 16 \in (\text{intrinsic}) Z_\Gamma \tag{20}$$

The three might-be arc fermion elementary particles associate with the pair of one-step-cascade solution-pairs with which quarks associate. SOMA suggests that arcs and gluons might form hadron-like particles. Arcs-plus-gluons hadron-like particles would have no charge and might measure as being dark matter.

Each one of the might-be arcs, the quarks, the gluons, and the might-be jay boson associates with the same pair of one-step-cascade solution-pairs and associates with the strong interaction. Regarding the two relevant boson intrinsic solution-pairs, the two one-step-cascade solution-pairs associate with eight gluons and one might-be jay boson. SOMA suggests that the jay boson might associate with repulsive aspects of the residual strong force. SOMA suggests that the jay boson might associate with Pauli repulsion between like fermions, whether the like fermions are elementary fermions or are not elementary fermions.

The notion of 0g and results that equation (19) suggest do not necessarily rule out possibilities for 1.5R and $1.5\text{Q}_{>0}$ elementary particles. (The following notions might suggest that the might-be 1.5R and $1.5\text{Q}_{>0}$ elementary particles do not exist. For all other elementary particles to which table 3 alludes, $1 \in Z_\Gamma$ and $3 \in Z_\Gamma$. For the might-be 1.5R and $1.5\text{Q}_{>0}$ elementary particles, neither $1 \in Z_\Gamma$ nor $3 \in Z_\Gamma$. Notions that associate with equation (29) and notions that associate with equation (35) might extrapolate to suggest that the might-be 1.5R and $1.5\text{Q}_{>0}$ elementary particles would associate with negative masses. Regarding the existence of might-be $1.5\text{Q}_{>0}$ elementary particles and not necessarily regarding the existence of the might-be 1.5R elementary particle, people have not found evidence of the might-be $1.5\text{Q}_{>0}$ elementary particles, which would have charge and might measure as OM.) The relevant one-step cascade shares - with the one-step cascades that associate with the elementary particles that associate with the strong interaction - the notion that each one of two, four, six, and eight is a member of Z_Γ. Each other elementary particle that does not associate with the strong interaction does not associate with the notion that each one of two, four, six, and eight is a member of each one-step-cascade Z_Γ. SOMA does not necessarily yet rule out the notion that some dark matter consists of hadron-like particles that include gluons and 1.5R elementary particles.

SOMA suggests the following statements regarding transformations between elementary particles.

- Any elementary boson that associates with a one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ can transform (or, regarding some POST terminology, decay) into a pair of elementary bosons that are similar to each other and not similar to the original elementary boson. For example, a Z boson can transform into two photons. For the W boson, there is no one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. The W boson cannot transform into two (hypothetical) elementary particles for which each of the two produced elementary particles would associate with $Q = 0.5$.

- Any elementary boson that does not associate with a one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ cannot directly transform into a pair of elementary bosons that are similar to each other and not similar to the original elementary boson. For the gluons, there is no one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. Gluons do not transform directly into, for example, pairs of photons.

- Any elementary boson that associates with a one-step-cascade solution-pair for which $6 \in Z_\Gamma$ can transform into a pair of elementary fermions. For example, a Z boson can transform into two elementary fermions that are antiparticles to each other. The W boson can transform into a pair of fermions (for example, an electron and a neutrino). The W boson is the only elementary boson that cannot transform into two elementary fermions that are antiparticles to each other.

SOMA associates the symbol that equation (21) shows with a possible maximum spin S for LRI elementary particles.

$$S_{max,\text{L}} \tag{21}$$

Each one of the following two notions might suggest that $S_{max,\text{L}}$ is no greater than four. The first notion associates with the following sentence. Discussion related to equation (24) suggests the limit $n_0 \leq 3$ and hence a limit of $\Sigma \leq 4$ regarding the relevance of $\Sigma\text{g}\Gamma$ solution-pairs for which Σ is the only element in the list Γ. The second notion associates with the following sentences. Equation (32) suggests that the solution-pair 1g1 associates with an interaction strength that includes a factor of four and that the solution-pair 2g2 associates with an interaction strength that includes a factor of three. Extrapolation suggests that the solution-pair 3g3 associates with an interaction strength that includes a factor of two, that the solution-pair 4g4 associates with an interaction strength that includes a factor of one, and that the solution-pair 5g5 would associate with an interaction strength that includes a factor of zero.

QFT associates with the SOMA notion of just one isomer. Reference [23] notes that QFT suggests that zero-mass elementary particles do not have spins that exceed two. SOMA does not necessarily address the possible applicability - regarding might-be multi-isomer analogs to QFT - of such a limit.

2.11. Some properties of a two-object system and its two objects

This unit associates with the row - in table 2 - that associates with 0g and with $5 \in Z_\Gamma$ or $7 \in Z_\Gamma$.

2.11.1. Spins S for two-object systems

The notion of 0g pertains. Equation (11) does not pertain. The notion of an upper limit on k_{max} does not pertain. Regarding spin S, the notion that $2S$ is a nonnegative integer pertains. The range $0 \leq S < \infty$ pertains.

Per equation (18) and equation (22), the solution-pair 0g1‘3‘5‘7 associates with $S = 0$.

$$0 = |+1-3-5+7| \tag{22}$$

The following three sets of solution-pairs associate with $\Sigma = 0$ and with $4 \notin Z_\Gamma$. The symbol 8ˆ associates with the series $+8 = +8$, $+8 = -8+16$, $+8 = -8-16+32$, and so forth. The symbol 16ˆ associates with the series $+16 = +16$, $+16 = -16+32$, $+16 = -16-32+64$, and so forth. Regarding the notions of 0g and ΣgΓ, for each solution-pair, the integers shown below (or alluded to by the series just above) appear in Γ and no other integers appear in Γ. For each set, for other than the first solution-pair, each solution-pair cascades from the first solution-pair.

1. $|-1-2-5+8|$, $|-1-2-5-8+16\hat{}|$,$|+1+2-5-6+8|$, and $|+1+2-5-6-8+16\hat{}|$
2. $|+2-3-7+8|$, $|+2-3-7-8+16\hat{}|$, $|+2+3-6-7+8|$, and $|+2+3-6-7-8+16\hat{}|$.
3. $|+1-3-5+7|$, $|-1-3+5-7+8\hat{}|$, $|-1-3+5-6+7|$, and $|-1-3-5+6-7+8\hat{}|$.

For each set, SOMA suggests that equation (18) pertains regarding the first two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an even integer. For each set, SOMA suggests that equation (19) pertains regarding the second two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an odd integer.

SOMA suggests, regarding a system that models as consisting of two objects, that $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with one object, $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the other object, and $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the system.

Removal of (just) the criterion that $4 \notin Z_\Gamma$ results in the following notions. Regarding $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, $n_{\Gamma^0} = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-2-4+5|$, $|-1-2-5+8|$, $|+1-4-5+8|$, and $|+2-3-4+5|$. Regarding $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, $n_{\Gamma^0} = 4$ solution-pairs that might associate with $S = 0$ associate with $|-1-2-4+7|$, $|+2-3-7+8|$, and $|+3-4-7+8|$. Regarding $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$, $n_{\Gamma^0} = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-3-5+7|$ and $|+2-4-5+7|$. The numbers of $S = 0$ solution-pairs are four for the case of $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, three for the case of $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, and two for the case of $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$.

2.11.2. Spins S regarding atoms

For an atom, each one of the nucleus and the electron cloud has nonzero charge. Based on the notion that $4 \in Z_\Gamma$ associates with zero charge, SOMA suggests that the following notions might pertain. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|-1-2-5+8|$ associate with spins S of the electron cloud. The solution-pair that associates with $|+1-2-4+5|$ associates with the spin $S = 0$ state of the electron cloud if the atom is an ion that has no electrons. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-3-7+8|$ associate with spins S of the nucleus. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-4-5+7|$ associate with spins S of the atom, if the atom (is not an ion and thus) has a charge of zero. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+1-3-5+7|$ associate with spins S of the atom, if the atom (is an ion and thus) has a nonzero charge.

2.12. *Dark matter and the notion of six isomers of most elementary particles*

POST does not yet provide an established description of dark matter.

POST associates with exactly one use of the set of all known elementary particles.

SOMA uses the two-word term isomeric set to denote the set of all (known and yet-to-be-found) elementary particles except the elementary particles that do (if known) intermediate LRI or might (if found) intermediate LRI. (Table 3 points to all known elementary particles and to all yet-to-be-found elementary particles that SOMA suggests. 1L is known; associates with LRI; and is, thus, not a member of the isomeric set. Each one of 2L, 3L, and 4L is not known, would associate with LRI, and would not be a member of the isomeric set.)

SOMA suggests that most dark matter associates with five new (compared to POST) approximate copies of the isomeric set (of elementary particles).

SOMA associates the word isomer with each of the six copies of the isomeric set. SOMA uses the one-element term isomer-zero to denote the isomer that mostly associates with OM. SOMA suggests some yet-to-be-found elementary particles that would - for each of the six copies of the isomeric set - associate with objects that would measure as DM. Thus, SOMA suggests that isomer-zero might associate with some DM.

SOMA suggests that - generally (but not necessarily always) - effects that associate with the five copies of the isomeric set other than the isomer-zero copy measure - from the perspective of POST - as effects of POST notions of DM.

Table 10, table 11, and table 12 associate with the SOMA suggestion that SOMA notions regarding dark matter might help explain data that POST otherwise does not necessarily explain.

2.13. *Isomer-related properties of elementary particles*

Regarding each LRI field, for each Σg' solution-pair, one solution associates with $s > 0$ and with the POST notion of a left-circular polarization mode of the field and one solution associates with $s < 0$ and with the POST notion of a right-circular polarization mode of the field.

POST notions of left-handedness pertain for (at least) all known elementary particles - possibly except for neutrinos - that have nonzero mass and nonzero spin. (LRI elementary particles associate with zero mass. POST associates the notion of polarization - and not necessarily the notion of handedness - with all known elementary particles that have zero mass.)

For each 0g solution-pair that associates with a non-LRI elementary particle, SOMA suggests that the notion of two solutions associates with a notion of a pair of isomers. SOMA associates the one-element term isomer-pair with such a pair of isomers. SOMA suggests that one isomer associates with POST notions of left-handedness and that one isomer associates with POST notions of right-handedness.

SOMA suggests the notion of three-isomer pairs.

SOMA names the isomers with one-element terms - isomer-zero, isomer-one, ..., and isomer-five. The three isomer-pairs associate, respectively, with isomer-zero and isomer-three, isomer-one and isomer-four, and isomer-two and isomer-five. The notion of left-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-zero, isomer-two, and isomer-four. The notion of right-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-one, isomer-three, and isomer-five.

SOMA suggests that POST associates with isomer-zero. For example, POST non-LRI elementary particles that have nonzero mass and nonzero spin associate with left-handedness. POST does not associate with the other five isomers that SOMA suggests.

SOMA suggests that the masses of counterpart non-LRI elementary particles do not vary between isomers. Per discussion related to table 7, SOMA suggests the following notions about flavours regarding isomers of known isomer-zero nonzero-charge fermion elementary particles.

- For each one of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton equals the remaining quark flavour.

- For each one of isomer-one and isomer-four, the flavour of the lowest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton equals the remaining quark flavour.

- For each one of isomer-two and isomer-five, the flavour of the intermediate-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the highest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the lowest-mass charged lepton equals the remaining quark flavour.

2.14. Reaches that associate with SOMA multipole solution-pairs

2.14.1. Physical notions

SOMA uses the word instance and the word reach to describe aspects of the extent to which $\Sigma \geq 1$ LRI solution-pairs associate with interactions within and between isomers.

SOMA suggests that equation (23) pertains for each $\Sigma \geq 1$ LRI solution-pair. The positive integer n_I denotes a number of instances of a solution-pair. The positive integer R_I denotes the reach - in number of isomers - that associates with one instance of the solution-pair.

$$n_I R_I = 6 \tag{23}$$

POST suggests that, to a first approximation, DM appears - to OM - to be electromagnetically dark. SOMA suggests that, to a first approximation, each isomer appears - to each other isomer - to be electromagnetically dark. For the solution-pair 1g1, SOMA suggests that $n_I = 6$ and $R_I = 1$. SOMA points to six instances of the property of charge.

POST suggests that gravity associates with interactions between OM and OM, interactions between OM and DM, and interactions between DM and DM. For the solution-pair 2g2, SOMA suggests that $n_I = 1$ and $R_I = 6$. SOMA points to one instance of the property of energy.

SOMA suggests the following reaches for intrinsic uses of LRI solution-pairs - $R_I = 1$ for $n_0 = 0$, $R_I = 6$ for $n_0 = 1$, $R_I = 2$ for $n_0 = 2$, and $R_I = 1$ for $n_0 = 3$. (Equation (9) defines n_0.)

2.14.2. Harmonic oscillator mathematics that might associate with reaches

SOMA suggests - based in part on information that table 1 provides - the following extrapolations. (Discussion related to equation (9) pertains.) For $n_0 = 0$, $R_I = 1$ pertains. For $1 \leq n_0 \leq 3$, equation (24) pertains.

$$R_I = \mathrm{gen}(SU(7))/\mathrm{gen}(SU(2n_0 + 1)) \tag{24}$$

2.14.3. Applications of the physical notions

SOMA suggests that the R_I for an extrinsic use of an LRI solution-pair equals the R_I for the intrinsic use of the solution-pair from which the extrinsic solution-pair cascades in one step. (Otherwise, the notion of object would not pertain.)

Equation (25) shows SOMA notation for the notion that a reach of R_I associates with each instance of a $\Sigma \geq 1$ solution-pair ΣgΓ.

$$\Sigma(R_I)\mathrm{g}\Gamma \tag{25}$$

SOMA extends the use of the notation (that equation (25) shows) to include $\Sigma(R_I)\Phi\Gamma$, in which Φ associates with the notion of a family of elementary particles. (Table 3 and discussion related to table 3 provide details regarding families of elementary particles.) Φ =L associates with LRI fields. Other than for Φ =L, SOMA suggests that $R_I = 1$ for $0(R_I)\Phi\Gamma$.

2.15. Object-properties and conservation laws that associate with long-range interactions

This unit associates with the rows - in table 2 - that associate with $\cdots$g'.

POST includes the notion of conservation of charge. For SOMA intrinsic use of 1g1, there are six instances of 1g1. SOMA suggests that - in effect - each isomer associates with its own instance of the property of charge. Conservation of charge pertains for each isomer.

POST includes the notion of conservation of energy. For SOMA intrinsic use of 2g2, there is one instance of 2g2. SOMA suggests that conservation of energy pertains for the combination of the six isomers and the set of LRI elementary particles.

Equation (26) shows notation that SOMA uses to describe a reach that includes all six isomers and all LRI phenomena.

$$R_I = 6\uplus \tag{26}$$

POST includes the notion of conservation of momentum. For SOMA extrinsic use of 2g2‘4, there is one instance of 2g2‘4. (Extrinsic use of 2g2‘4 inherits its reach from intrinsic use of 2g2.) SOMA suggests that conservation of momentum associates with extrinsic use of 2g2‘4 and with $R_I = 6⊎$.

POST includes the notion of conservation of angular momentum. Based on its relationship to extrinsic use of 2g2‘4, SOMA suggests that intrinsic use of 2g2‘4 associates with angular momentum. SOMA suggests that - for intrinsic use of 2g2‘4 - $n_0 = 2$ associates with three instances of 2g2‘4 and with three isomer-pairs. Conservation of angular momentum would need to embrace angular momentum that reflects phenomena that associate with LRI elementary particles as well as phenomena that associate with stuff based on isomers of non-LRI elementary particles. Regarding inertial properties and conservation of angular momentum (but not regarding LRI reaches), SOMA suggests that - in effect - the DOF-like aspects that associate with the notion of three isomer-pairs combine with the DOF-like aspects that associate with $n_0 = 2$ to produce a result that associates with $n_0 = 1$. (Discussion related to equation (48) suggests that $k = 0$ might associate with the notion of three isomer-pairs.) SOMA suggests that conservation of angular momentum associates with intrinsic use of 2g2‘4 and with $R_I = 6⊎$.

Table 4 shows cascades that associate with Σg’ solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, $5 \notin Z_\Gamma$, and $7 \notin Z_\Gamma$. Discussion immediately above associates with properties and conservation laws that associate with $1 \leq \Sigma \leq 2$. Discussion immediately below associates with properties and conservation laws that associate with $3 \leq \Sigma \leq 4$.

SOMA suggests relevance for intrinsic use of the 3g3 solution-pair and related extrinsic use of the 3g3‘6 solution-pair. There are three instances of 3g3. Each instance associates with its own isomer-pair. Per discussion regarding table 3, SOMA suggests that extrinsic use of the 3g3‘6 solution-pair associates with interactions in which a 3L boson transforms into two somewhat similar elementary fermions. One of the fermions associates with left-handedness. The other fermion associates with right-handedness. SOMA suggests that conservation of net-left-minus-right (as in the number of left-handed fermion elementary particles minus the number of right-handed fermion elementary particles) pertains for each one of the three isomer-pairs. SOMA suggests that - in association with the notion that $6 \in Z_\Gamma$ for 3g3‘6 - an approximate conservation of flavour symmetry pertains. Conservation of flavour pertains for interactions mediated by single elementary bosons. QM suggests that conservation of flavour does not necessarily pertain for interactions mediated by pairs of W bosons. (QFT associates interactions mediated by pairs of W bosons with weak-interaction CP symmetry violation. Here, C symmetry associates with notions of charge conjugation - or charge reversal. P symmetry associates with notions of parity reversal. CP symmetry associates with notions of combined charge conjugation and parity reversal.)

SOMA suggests relevance for intrinsic use of the 4g4 solution-pair and related extrinsic use of the 4g4‘8 solution-pair. There are six instances of 4g4. Each instance associates with its own isomer. Per discussion regarding table 3, SOMA suggests that extrinsic use of the 4g4‘8 solution-pair associates with interactions in which a 4L boson transforms into two somewhat similar elementary bosons. SOMA suggests that conservation of isomer pertains in the following sense regarding interactions between boson elementary particles. If all the incoming boson elementary particles identify with just one isomer, each of the outgoing boson elementary particles that identifies with just one isomer identifies with the same isomer as did the incoming boson elementary particles.

2.16. Gravitational properties of objects

This unit associates with the rows - in table 2 - that associate with 2g’. This unit also associates with the row - in table 2 - that associates with 1g’.

Discussion above (regarding inertial properties) suggests a lack of a quadrupole intrinsic inertial property. Discussion above (regarding electromagnetic properties) points to a quadrupole intrinsic electromagnetic property. Table 4 suggests that objects can exhibit intrinsic properties that associate with the notions of quadrupole intrinsic gravitational property, octupole intrinsic gravitational property, and so forth.

GR associates a notion of repulsion between objects with a notion of pressure. POST associates a notion of DE with such a pressure.

Per table 4, equation (13), and equation (14), SOMA suggests that an excitation of a field (such as the 2L field - or, the gravitational field) encodes knowledge of the (isomeric-set related) instances of the properties that associate with the excitation. In effect, the gravitational field carries knowledge of the isomers that associate with the excitation.

Regarding the active gravitational properties of an object A, the following notions can pertain. The number of instances, n_I, of gravitationally attractive intrinsic monopole properties is one. The number of instances of gravitationally diluting intrinsic dipole properties can be as many as three. The number of

Table 4: Cascades that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, $5 \notin Z_\Gamma$, and $7 \notin Z_\Gamma$. The column with the one-word label extrinsic shows solution-pairs that cascade in one step from the intrinsic solution-pairs. The symbol † alludes to the notion that the intrinsic solution-pairs do not cascade from other intrinsic solution-pairs that the table shows. The symbol ‡ alludes to the notion that the solution-pairs appear more than once in the column that lists extrinsic solution-pairs. The next-to-rightmost column designates rows that show SOMA-relevant pairs of one intrinsic solution-pair and one extrinsic solution-pair. (An intrinsic solution-pair can associate with more than one extrinsic solution-pair.) Here, the symbol SL associates with a known LRI elementary particle or a might-be LRI elementary particle and with the value of S equals Σ. The symbol $\oslash$ associates with the notion that - for the intrinsic solution-pairs - no extrinsic solution-pairs pertain. SOMA assumes that each one of extrinsic $\oslash$ and intrinsic $6 \in Z_\Gamma$ associates with the notion that intrinsic uses of the solution-pairs that the column labeled intrinsic lists are not relevant regarding SOMA. (Per equation (15), SOMA suggests that - regarding elementary particles - intrinsic use of solution-pairs for which $6 \in Z_\Gamma$ associates with fermion elementary particles and does not associate with boson elementary particles. The notion of SL associates with boson elementary particles and does not associate with fermion elementary particles.) The rightmost column shows the reach for each SOMA-relevant pair of one intrinsic solution-pair and one extrinsic solution-pair. The two columns with labels that include the word property show properties that associate with conservation laws. (The symbol * alludes to the notion that the table does not name some properties, including magnetic moment.) The symbol $\uplus$ abbreviates the notation $R_I = 6\uplus$. The one-element symbol F: associates with the word fermion. The one-element symbol B: associates with the word boson.

†	Intrinsic	Property *	Extrinsic	Property *	SL	R_I
†	1g1	Charge	1g1‘2	-	1L	1
-	1g1‘2	-	1g1‘2‘4	-	1L	1
-	1g1‘2‘4	-	1g1‘2‘4‘8, 1g1‘2‘4‘6x	-	1L	6
-	1g1‘2‘4‘8	-	1g1‘2‘4‘6‘8x ‡	-	1L	6
-	1g1‘2‘4‘6‘8x ‡	-	$\oslash$	-	-	-
-	1g1‘2‘4‘6x	-	1g1‘2‘4‘6‘8x ‡	-	-	-
†	1g1‘4‘6	-	1g1‘4‘6‘8	-	-	-
-	1g1‘4‘6‘8	-	1g1‘2‘4‘6‘8x ‡	-	-	-
†	2g2	Energy $\uplus$	2g2‘4	Momentum $\uplus$	2L	6
-	2g2‘4	Angular momentum $\uplus$	2g2‘4‘8	-	2L	2
-	2g2‘4‘8	-	$\oslash$	-	-	-
†	2g1‘2‘3	-	2g1‘2‘3‘4x, 2g1‘2‘3‘6	-	2L	1
-	2g1‘2‘3‘4x	-	2g1‘2‘3‘4‘8x	-	2L	1
-	2g1‘2‘3‘4‘8x	-	2g1‘2‘3‘4‘6‘8x ‡	-	2L	1
-	2g1‘2‘3‘4‘6‘8x ‡	-	$\oslash$	-	-	-
-	2g1‘2‘3‘6	-	2g1‘2‘3‘6‘8x	-	-	-
-	2g1‘2‘3‘6‘8x	-	2g1‘2‘3‘4‘6‘8x ‡	-	-	-
†	3g3	F: Net-left-minus-right	3g3‘6	F: Flavour	3L	2
-	3g3‘6	-	$\oslash$	-	-	-
†	3g2‘3‘4	-	3g2‘3‘4‘8, 3g2‘3‘4‘6	-	3L	6
-	3g2‘3‘4‘8	-	3g2‘3‘4‘6‘8 ‡	-	3L	6
-	3g2‘3‘4‘6	-	3g2‘3‘4‘6‘8 ‡	-	-	-
-	3g2‘3‘4‘6‘8 ‡	-	$\oslash$	-	-	-
†	4g4	-	4g4‘8	B: Isomer	4L	1
-	4g4‘8	-	$\oslash$	-	-	-
†	4g1‘2‘3‘4x	-	4g1‘2‘3‘4‘6x	-	4L	1
-	4g1‘2‘3‘4‘6x	-	4g1‘2‘3‘4‘6‘8x ‡	-	-	-
-	4g1‘2‘3‘4‘6‘8x ‡	-	$\oslash$	-	-	-
†	4g1‘2‘3‘4‘8x	-	4g1‘2‘3‘4‘6‘8x ‡	-	4L	1

instances of gravitationally additive intrinsic quadrupole properties can be as many as six. The number of instances of gravitationally diluting intrinsic octupole properties can be as many as six. The number of instances of gravitationally additive intrinsic 16-pole properties can be as many as six.

Regarding a one-isomer object, there are six possibilities regarding isomer. The following examples pertain regarding interactions between a sun (as a one-isomer object A) and a planet (as a one-isomer object C).

- To the extent that the sun exhibits only gravitationally active $R_I = 6$ aspects (or, monopole intrinsic gravitational properties), SOMA suggests that POST modeling is not necessarily inadequate.
- To the extent that the sun exhibits gravitationally active $R_I \neq 6$ aspects, SOMA suggests that POST modeling might be inadequate. Nonzero rotation of an isomer-zero sun provides a basis for an example. A one-isomer planet that associates with isomer-zero or with isomer-three senses the rotation of the sun. The rotation affects the trajectory of the planet. A one-isomer planet that associates with isomer-one, isomer-two, isomer-four, or isomer-five does not sense the rotation of the sun. The rotation does not affect the trajectory of the planet.

In POST, the effective active gravitational properties of object A depend only on aspects of object A.

SOMA suggests that the effective active gravitational properties of object A depend on aspects of object A and on aspects of objects C that interact with (or, observe) object A.

For a general case of a point-like (and possibly multi-isomer) object C interacting with the 2L field that associates with an object A, object C senses all (nonzero value of property) 2gΓ solution-pair components that associate with the 2L field that associates with object A. The weighting that associates with any one intrinsic solution-pair associates with the geometric factor of the pole (monopole, dipole, or so forth) that associates with the intrinsic solution-pair and with an orientation factor that associates with a tensor-like notion (scalar for monopole, vector for dipole, and so forth) that associates with the intrinsic solution-pair. (SOMA uses the word weighting to avoid possibly inappropriate conflation with POST notions such as probability and amplitude. This paper does not operationally define the one-word term weighting.) For ND, the geometric factor associates with r^{-n_Γ}. Generally, possibly, effects that associate with one geometric factor or with two geometric factors dominate compared to effects that associate with other geometric factors.

2.17. Electromagnetic events that associate with atoms or stars

This unit associates with the row - in table 2 - that associates with (≥ 1)g" and $n_\Gamma \geq 3$. This unit also associates with the row - in table 2 - that associates with 1g'.

2.17.1. Electromagnetic events that associate with two-object systems such as atoms

SOMA suggests that, for $\Sigma \geq 1$ and $n_\Gamma \geq 3$, Σg" solution-pairs can associate with properties of objects that model as being parts of multi-object systems. Regarding the individual objects, conservation laws that discussion related to table 4 suggests do not necessarily pertain.

Arithmetically, no Σg" solution-pairs associate with $n_\Gamma = 1$. SOMA suggests that Σg" solution-pairs that associate with $n_\Gamma = 2$ associate with anomalous properties.

Table 5 shows some aspects of some cascades that associate with some Σg" solution-pairs for which $n_\Gamma \geq 3$, $\Sigma = 1$, and $1 \leq k_{max} \leq 8$. (Discussion related to equation (22) pertains. The notion that $4 \in Z_\Gamma$ associates with zero charge pertains regarding solution-pairs for which $\Sigma = 0$ and does not necessarily pertain regarding table 5.)

2.17.2. Electromagnetic events that associate with stars

SOMA suggests that stars tend to associate with single isomers. For a one-isomer star, SOMA suggests the following contributions to the electromagnetic field.

SOMA suggests that stellar radiation (such as thermal radiation) can associate with intrinsic 1g3‘5‘7 and extrinsic solution-pairs such as 1g2‘3‘5‘7, 1g3‘4‘5‘7, 1g3‘5‘6‘7, and 1g3‘5‘7‘8. The notion of 1g" pertains. The relevant reach, R_I, for the emitted electromagnetic radiation is one isomer.

If the star has a net nonzero charge, some contributions to the electromagnetic field associate with the star's intrinsic 1g1 and extrinsic 1g1‘2. The notion of 1g' pertains. The relevant reach, R_I, for the emitted electromagnetic radiation is one isomer.

Table 5: Some aspects of some cascades that associate with some Σg" solution-pairs for which $n_\Gamma \geq 3$, $\Sigma = 1$, and $1 \leq k_{max} \leq 8$. The column with the one-word label intrinsic shows intrinsic solution-pairs. The column with the one-word label extrinsic shows extrinsic solution-pairs. The symbol † alludes to the notion that the intrinsic solution-pair does not cascade from other intrinsic solution-pairs that the table shows. The column with the one-element label R_I shows the reach for each intrinsic solution-pair and for each extrinsic solution-pair. SOMA suggests that notions - in this table - of intrinsic and extrinsic associate with properties of objects that model as being parts of multi-object systems. For example, for an atom, $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with internal energy-related and angular-momentum-related properties of the atom's electron cloud. $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with possible external influences on the atom's electron cloud. Intrinsic use of 1g2'4'7 can associate with the atom's net charge. Intrinsic use of 1g2'4'7'8 can associate with the atom's hyperfine state. The column with the two-element label atom-related event suggests phenomena - related to an atom - that can associate with excitations or de-excitations of the 1L (or, electromagnetic) LRI field.

†	Intrinsic	Extrinsic	SL	R_I	Atom-related event
†	1g2'4'5	1g2'4'5'8, 1g2'4'5'6	1L	2	An electron transits to a new principal energy.
-	1g2'4'5'8	1g2'4'5'6'8x	1L	2	An atomic fine-structure change occurs.
†	1g2'4'7	1g2'4'7'8, 1g2'4'6'7	1L	2	The atom adds or subtracts an electron.
-	1g2'4'7'8	1g2'4'6'7'8x	1L	2	An atomic hyperfine transition occurs.

2.17.3. Implications regarding cosmic background radiation and sensing dark matter

Per table 5, components of cosmic (electromagnetic) background radiation that associate with creation (of electromagnetic radiation) via atomic phenomena can associate with a reach R_I of two isomers. Components of cosmic (electromagnetic) background radiation that associate with creation via other phenomena can associate with a reach R_I of one isomer.

SOMA suggests that electromagnetic radiation that associates with creation via single-isomer atomic phenomena can associate with a reach R_I of two. SOMA suggests that OM detectors can detect such radiation that was created by isomer-zero atomic phenomena or by isomer-three atomic phenomena. Discussion related to table 10 suggests that OM detectors may have detected electromagnetic radiation that was created by isomer-three atomic phenomena.

3. Results

3.1. Elementary particles

Table 3 catalogs all known elementary particles and some as-yet-unfound elementary particles that SOMA suggests nature might include.

3.2. Relationships among properties of boson elementary particles

SOMA suggests that equation (27) pertains regarding the masses of the nonzero-mass elementary bosons.

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \tag{27}$$

Equation (27) is not inconsistent with data. Based on information that reference [60] provides, the following notions pertain. The most accurately known of the three masses is m_{Z}. Based on the nominal value of m_{Z}, the nominal value (that equation (27) suggests) for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}. Based on the nominal value of m_{Z}, the nominal value (that equation (27) suggests) for m_{W} is within 3.6 experimental standard deviations of m_{W}. Based on information that reference [61] provides, the following notions pertain. Based on the nominal value of m_{Z}, the nominal value (that equation (27) suggests) for m_{W} is within 1.1 experimental standard deviations of m_{W}. (Reference [61] does not provide new information about m_{Higgs}.) Based on the nominal value of m_{Z} that reference [61] suggests and on information that reference [60] provides about m_{Higgs}, the nominal value that equation (27) suggests for m_{Higgs} is within 0.5 experimental standard deviations of m_{Higgs}.

SOMA suggests that equation (27) might point to possible insight regarding - and a possible extension to - the POST notion of the weak mixing angle.

For each known elementary boson, equation (28) and equation (29) define, respectively, the integer l_{ms} and the number j_m. The symbol Q denotes the magnitude (in units of the magnitude $|q_e|$ of the charge - q_e - of the electron) of the charge of the elementary boson. The symbol m' denotes the mass (in units of $m_{\mathrm{Z}}/3$) of the elementary boson.

$$m' > 0 \Leftrightarrow l_{ms} = -1 \; ; \quad m' = 0 \Leftrightarrow l_{ms} = 0 \tag{28}$$

$$(j_m)^2 \equiv (m')^2 + S^2 + Q(Q+1) + l_{ms} \tag{29}$$

For each elementary boson to which table 3 alludes, SOMA suggests that j_m is an integer. For each known elementary boson, the notion that j_m is an integer is not inconsistent with data. SOMA suggests that equation (30) pertains for boson elementary particles.

$$m' > 0 \;\Leftrightarrow\; j_m = (\text{intrinsic})n_{\Gamma^0} \;; \quad m' = 0 \;\Leftrightarrow\; j_m = (\text{intrinsic})n_{\Gamma^0} - 4 \tag{30}$$

3.3. Relationships among properties of fermion elementary particles

Regarding charged leptons, SOMA suggests a link between the strength of electromagnetism and the strength of gravity.

Equation (31) and equation (32) define, respectively, β' and β. m_τ denotes the mass of the tau particle (which is a charged lepton). m_e denotes the mass of the electron (which is a charged lepton). The right-hand side of equation (32) is the ratio of the electrostatic repelling between two electrons to the gravitational attracting between the two electrons. The ratio does not depend on the distance between the two electrons.

$$\beta' \equiv m_\tau / m_e \tag{31}$$

$$(4/3) \cdot (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{32}$$

Based on data, $\beta \approx 3477.1891 \pm 0.0226$. (Reference [60] provides the relevant underlying data.) The standard deviation associates almost entirely with the standard deviation for G_N, the gravitational constant.

Equation (33) shows an equality that SOMA suggests.

$$\beta' = \beta \tag{33}$$

Equation (34) results from equation (33). The standard deviation associates almost entirely with the standard deviation for G_N.

$$m_{\tau,\text{ calculated}} \approx 1776.8400 \pm 0.0115 \text{ MeV}/c^2 \tag{34}$$

Equation (34) comports with data. More than eight standard deviations fit within one standard deviation from the data-based nominal value for m_τ.

SOMA suggests a formula that might approximately link the masses of all elementary fermions.

Equation (35) defines $m(l_m, l_q)$ and has bases in the equations that immediately follow equation (35). Equation (36) defines the fine-structure constant. Equation (41) has bases in trying to fit data.

$$m(l_m, l_q) \equiv m_e \cdot (\beta^{1/3})^{l_m + (j''_{l_m})d''} \cdot (\alpha^{-1/4})^{g(l_q)\cdot(1+l_m) + j'_{l_q} d'(l_m)} \tag{35}$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \tag{36}$$

$$j''_{l_m} = 0, +1, 0, -1 \text{ for, respectively, } l_m \bmod 3 = 0, 1, 3/2, 2\text{; with } 3/2 \bmod 3 \equiv 3/2 \tag{37}$$

$$d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840613 \times 10^{-2} \tag{38}$$

$$g(l_q) = 0, 3/2, 3/2, 3/2, 3/2, \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{39}$$

$$j'_{l_q} = 0, -1, 0, +1, +3 \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{40}$$

$$d'(0) \sim 0.324,\; d'(1) \sim -1.062,\; d'(2) \sim -1.509 \tag{41}$$

$$d'(l_m) = 0 \text{ for } l_m \leq -1 \text{ and for } l_m \geq 3 \tag{42}$$

Table 6: Approximate values of $\log_{10}(m(l_m, l_q)/m_e)$ for all known charged fermion elementary particles. Regarding flavour, this table generalizes, based on POST terminology that associates with charged leptons and with neutrinos. (For example, POST uses the term electron-neutrino.) In table 6, the symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain for fermions in the 0.5C$_1$ family. The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$. The "l_f (0.5Q$_{>0}$)" terms pertain for quarks (or, elementary particles in the two families 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$). l_m is an integer parameter. The domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q = 1$. The family 0.5Q$_{2/3}$ associates with $Q = 2/3$. The family 0.5Q$_{1/3}$ associates with $Q = 1/3$. Regarding table 6, $l_q = 3Q$ pertains. Regarding the rightmost four columns, items show $\log_{10}(m(l_m, l_q)/m_e)$ and - for particles that nature includes - the name of an elementary fermion. For each † case, no particle pertains. Each number in the column with the label $Q = 1/2$ equals the average of the number in the $Q = 2/3$ column and the number in the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding each † case, equation (35) provides the number $m(l_m, l_q)$.

l_f (0.5C$_1$)	l_f (0.5Q$_{>0}$)	l_m	$Q = 1$	$Q = 2/3$	$Q = 1/2$	$Q = 1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.63 Up	0.80 †	0.97 Down
-	2 (Charm, Strange)	1	1.23 †	3.40 Charm	2.83 †	2.26 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.53 Top	4.72 †	3.91 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

Table 6 shows information about properties of all known charged fermion elementary particles. (Reference [60] provides the data that underlie table 6.) Regarding similar tables for each one of isomer-one, isomer-two, isomer-four, and isomer-five, SOMA suggests (per table 7) that the values of l_f that table 6 shows for the charged leptons are not appropriate. For example, for isomer-two, the l_f values in the leftmost column would be 3 (for the row for which - for quarks - $l_f = 1$), blank (for the row for which - for quarks - $l_f = 2$), 1 (for the row for which - for quarks - $l_f = 3$), and 2 (for the remaining row).

For each charged elementary fermion except the top quark, equation (35) suggests a mass that is within one experimental standard deviation of the nominal mass that reference [60] reports. Reference [60] alludes to three estimates for the mass of the top quark. Equation (35) provides a mass (for the top quark) that is within 4.4 standard deviations below the nominal mass that associates with direct measurements, within 4.3 upward standard deviations above the nominal mass that associates with cross-section measurements, and within 1.6 standard deviations below the nominal mass that associates with the four-element phrase pole from cross-section measurements.

The count of independent irrational numbers input into the above calculations of nine fermion elementary particle masses is seven. For example, the list consisting of m_e, m_μ, β, α, $d'(0)$, $d'(1)$, and $d'(2)$ includes seven irrational numbers.

SOMA suggests neutrino masses.

Reference [19] suggests that data point to the notion that the sum of the three neutrino rest energies is at least approximately 0.06 eV and not more than approximately 0.12 eV. Reference [62] discusses data and modeling regarding upper bounds for the sum of the masses of the three neutrinos. Reference [63] discusses a lower bound of 0.06 eV, an upper bound of 0.15 eV, and a possible upper bound of 0.12 eV. Reference [60] suggests that an upper bound might be approximately 0.10 eV.

Neutrinos associate with $Q = 0$. SOMA suggests that some $m(l_m, 0)$ solutions associate with neutrino masses. For $l_m \leq -1$ and for $l_m \geq 3$, no quarks pertain and SOMA suggests that $d'(l_m) = 0$.

Equation (43) shows a result from equation (35).

$$mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2} \text{ eV} \tag{43}$$

SOMA suggests the following two possibilities.

1. $mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of the three neutrinos.
2. $mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6, 0)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5, 0)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.

SOMA suggests that interactions that associate with 2g' solution-pairs conserve mass but do not necessarily conserve flavour. SOMA suggests that interactions that associate with 3g' solution-pairs conserve flavour but do not necessarily conserve mass. SOMA suggests that these notions regarding conservation of properties might associate with the POST notion that mass eigenstates for neutrinos do not necessarily equal flavor eigenstates for neutrinos.

POST suggests notions that associate with the two-word term neutrino oscillations and with the two-word term neutrino mixing. SOMA suggests interactions - between neutrinos and the environments through which neutrinos pass - that might explain neutrino oscillations and POST notions of mass-mixing. Examples include interactions intermediated by the might-be jay boson and (paralleling notions that table 5 discusses) interactions that associate with events that associate with intrinsic use of the 3g1‘2‘4 solution-pair. Even if all three neutrino flavours associate with the same mass, SOMA might explain data that POST interprets as suggesting differences between neutrino masses.

Per equation (35), SOMA suggests that the three flavors of the might-be arcs might associate respectively with the following rest energies - $m(0,0)c^2 \sim 10.7$ MeV, $m(1,0)c^2 \sim 6.8$ MeV, and $m(2,0)c^2 \sim 102$ MeV. (References [64] and [65] discuss lower limits regarding masses for POST notions of heavy neutrinos. Based in part on such limits, SOMA is reluctant to try to associate the SOMA-suggested arcs with POST notions of heavy neutrinos.)

3.4. Anomalous magnetic moments of charged leptons

This unit associates with the row - in table 2 - that associates with (≥ 1)g" and $n_\Gamma = 2$.

QFT associates with two complementary aspects of magnetic moment - nominal magnetic moment and anomalous magnetic moment. QFT calculates anomalous magnetic moments that match data regarding the electron and the muon. The calculations feature notions of virtual photons.

SOMA associates two solution-pairs - 1g1‘2 and 3g1‘2 - with the Γ that equals 1‘2.

SOMA suggests the possibility that intrinsic use of the solution-pair 1g1‘2 associates with the intrinsic property of nominal magnetic moment and that intrinsic use of the solution-pair 3g1‘2 associates with the intrinsic property of anomalous magnetic moment.

Two extrinsic solution-pairs associate with intrinsic use of the 3g1‘2 solution-pair. The 3g1‘2‘6 extrinsic solution-pair associates with $6 \in Z_\Gamma$. SOMA suggests the possibility that the strength of 3g1‘2‘6 can vary based on elementary fermion flavour. The 3g1‘2‘4 extrinsic solution-pair associates with $6 \notin Z_\Gamma$. SOMA suggests the possibility that the strength of 3g1‘2‘4 does not vary based on elementary fermion flavour.

SOMA suggests the possibility that equation (44) approximates a_{cl}, the anomalous magnetic moment for the cl charged lepton. Here, each one of a_4 and a_6 is a constant with respect to a choice between $cl = e$ (for the electron), $cl = \mu$ (for the muon), and $cl = \tau$ (for the tau). a_4 associates with 3g1‘2‘4. a_6 associates with 3g1‘2‘6.

$$a_{cl} \approx a_4 + a_6 t_{cl} \tag{44}$$

Aspects of equation (32) feature squares of properties. (Regarding equation (31), equation (32), and equation (33), $\beta^2 = (G_N(m_\tau)^2)/(G_N(m_e)^2)$.) Squares of properties might associate with notions of self-interactions.

SOMA suggests the possibility that t_{cl} is $(\log(m_{\mathrm{cl}}/m_e))^2$.

Based on data that reference [60] provides regarding the electron and the muon, SOMA calculates a_4 and a_6. Then, SOMA calculates a value, $a_{\tau,\mathrm{SOMA}}$, for a_τ. Reference [66] provides, based on the SM, a first-order result - which SOMA calls $a_{\tau,\mathrm{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard Model. The value of $a_{\tau,\mathrm{SOMA}}$ results in a value of $(a_{\tau,\mathrm{SOMA}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.00228. Each one of $a_{\tau,\mathrm{SOMA}}$ and $a_{\tau,\mathrm{SM}}$ comports with data that reference [60] provides.

3.5. Differences - between isomers - regarding properties of fermion elementary particles

If the stuff that associates with each of the five all-DM isomers evolved similarly to the stuff that associates with isomer-zero, SOMA suggestions regarding DM might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. (Discussion - below - that cites reference [67] provides more information.)

SOMA uses the symbol l_i to number the isomers. The notion of isomer-l_i pertains.

Per discussion (including discussion regarding table 6) above, regarding each l_i that is at least one, SOMA suggests that the instances (of elementary particles) that associate with isomer-l_i match - with respect to mass - the instances (of the counterpart elementary particles) that associate with isomer-zero.

For modeling regarding flavours (and not - for $0 \leq l_i \leq 5$ - for modeling regarding masses), SOMA associates the quarks in isomer-l_i with three values of l_m. The values are $3l_i + 0$, $3l_i + 1$, and $3l_i + 2$. (Table 6 shows the associations for $l_i = 0$.) Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, SOMA suggests that - within isomer-l_i - flavour 1 associates with $l_m = 3l_i$, flavour 2 associates with $l_m = 3l_i + 1$, and flavour 3 associates with $l_m = 3l_i + 2$.

Aspects of table 6 point to the notion that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer-zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates with $l_m = 3$. SOMA suggests that - for each l_i - a charged lepton associates with each of $l_m = 3l_i + 0$, $l_m = 3l_i + 2$, and $l_m = 3l_i + 3$.

Table 7: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol $0.5Q_{>0}$ denotes the pair $0.5Q_{1/3}$ and $0.5Q_{2/3}$. As in table 6, here the symbol l_f numbers the three flavours.

Isomer	l_m ($0.5Q_{>0}$)	Respective l_f ($0.5Q_{>0}$)	l_m ($0.5C_1$)	Respective l_f ($0.5C_1$)
0	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

SOMA suggests that - for each isomer-l_i such that $1 \leq l_i \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_i) + 0$ equals the flavour that associates with the isomer-$(l_i - 1)$ charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_i - 1) + 3$. SOMA suggests that, across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 7 shows, for isomers of charged elementary fermions, matches between masses and flavours.

3.6. Possibilities for conversions between isomers

SOMA suggests that conversions between isomers might occur based on interactions mediated by LRI fields.

For each one of 1L and 3L, table 4 points to at least one intrinsic solution-pair for which both $R_I \geq 2$ and there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$. Table 4 also suggests the notion - for elementary fermions - of conservation of net-left-minus-right.

An isomer-zero pair of elementary fermions for which one fermion is the antiparticle of the other fermion could annihilate to excite a 1L field or a 3L field. That excitation could de-excite to produce one isomer-zero left-handed fermion elementary particle and one isomer-three right-handed fermion elementary particle. (For elementary fermions, table 4 notes that conservation of net-left-minus-right pertains regarding isomer-pairs and does not necessarily pertain regarding individual isomers.) Equation (45) symbolizes results of such an excitation and de-excitation. The symbol FLH denotes a left-handed fermion elementary particle and the symbol FRH denotes a right-handed fermion elementary particle.

$$FLH_{l_i=0} + FRH_{l_i=0} \to FLH_{l_i=0} + FRH_{l_i=3} \tag{45}$$

For each one of 1L, 2L, and 3L, extending table 5 would result in at least one intrinsic solution-pair for which both $R_I \geq 2$ and there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$. Examples - for which $R_i = 6$ - of such intrinsic solution-pairs include 1g2‘3‘4, 2g1‘3‘4, and 3g1‘2‘4. Excitations that associate with such solution-pairs do not necessarily associate with single isomers. Such an excitation might de-excite to produce a particle-and-antiparticle fermion pair that associates with any one of the six isomers. Equation (46) symbolizes results of one such type of excitation and de-excitation. The symbol X denotes a system-state that de-excites to a system-state that the symbol Y denotes.

$$X \to Y + FLH_{l_i} + FRH_{l_i} \tag{46}$$

3.7. Known and POST-assumed eras in the history of the universe

CC points to three eras in the rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

SOMA suggests using the notion of eras regarding the separating from each other of clumps - that, today, POST would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and even larger clumps. SOMA suggests (per discussion above) that, for a pair of similar objects that always move away from each other, the dominating gravitational effects transit (over time) all or a portion of the following sequence: 16-pole attracting, octupole repelling, quadrupole attracting, dipole repelling, and monopole attracting.

Table 8 discusses three possible eras in the rate of separating of large clumps. (References [68], [20], [34], and [69] discuss the possible inflationary epoch. References [70], [71], [72], and [73] provide data and discussion about the two multi-billion-years eras. Reference [35] discusses attempts to explain the rate of expansion of the universe.)

Table 8: Three possible eras regarding the rate of separating of large clumps. The rightmost two columns suggest eras. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. In table 8, subsequent rows associate with later eras. The word inflation (or, the two-word term inflationary epoch) names the era that associates with the first row in the table. CC suggests inflation and the next two eras. Regarding inflation, CC hypothesizes this era. CC suggests notions of a Big Bang (or - at least - of a time that CC associates with the two-word term Big Bang). CC suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. CC suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for the inflationary epoch. CC interpretations of data support the notions of the two billions-of-years eras. The leftmost four columns describe phenomena that SOMA suggests as noteworthy causes for the eras. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before an era starts.

Force	Intrinsic s-p	SOMA-pole	R_I	$\rightarrow$	Rate of separating	Duration
Repulsive	2g1'2'3'4x	Octupole	1	$\rightarrow$	Increases rapidly	Fraction of a second
Attractive	2g1'2'3	Quadrupole	1	$\rightarrow$	Decreases	Billions of years
Repulsive	2g2'4	Dipole	2	$\rightarrow$	Increases	Billions of years

3.8. Evolution of stuff that associates with dark matter isomers

SOMA uses the two-element term isomer-l_i stuff to denote objects (including hadron-like particles, atom-like objects, and stars) that associate with the isomer-l_i.

3.8.1. Evolution of isomer-1, isomer-2, isomer-4, and isomer-5 stuff

Here, SOMA uses the one-element term alt-isomer to designate an isomer other than isomer-zero and isomer-three.

For each one of the six isomers, a charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, the hadron that includes just two tops and one bottom has a larger total mass than does the hadron that includes just one top and two bottoms.)

Per table 6 and table 7, alt-isomer flavour 3 charged leptons are less massive than isomer-zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the stuff that associates with an alt-isomer converts more charged baryons to zero-charge baryons than does the stuff that associates with isomer-zero. Eventually, regarding the stuff that associates with the alt-isomer, interactions that entangle multiple W bosons result in the stuff that associates with the alt-isomer having more neutrons and fewer protons than does the stuff that associates with isomer-zero. The sum of the mass of a proton and the mass of an alt-isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer-zero neutrons, alt-isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt-isomer scarcely interacts with itself via electromagnetism.

3.8.2. Evolution of isomer-3 stuff

The following two possibilities pertain. In one possibility, the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff. In the second possibility, the evolution of isomer-three stuff does not parallel the evolution of isomer-zero stuff. The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer-three and isomer-zero.

This paper nominally assumes that the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff.

3.9. Tensions - among data and models - regarding large-scale phenomena

SOMA suggests means to resolve tensions - between data and CC - regarding the rate of expansion of the universe, regarding large-scale clumping of matter, and regarding gravitational interactions between neighboring galaxies.

3.9.1. The rate of expansion of the universe

Table 8 lists two known eras in the history of the universe.

CC underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (References [38], [39], [40], [41], [74], [75], [76], and [77] provide further information. Reference [78] suggests that the notion that DM is similar to OM might help resolve the relevant tension. Reference [79] discusses various possible resolutions.)

SOMA suggests the following explanation for such underestimates.

When using modeling based on GR, CC might try to extend the use of an equation of state (or the use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding the first multi-billion-years era, SOMA suggests dominance by attractive effects that associate with intrinsic use of the 2g1‘2‘3 component of gravity. The notion of a reach of one pertains. The symbol 2(1)g1‘2‘3 pertains. SOMA suggests that - later in the first multi-billion-years era - repulsive effects that associate with intrinsic use of 2(2)g2‘4 become significant. Dominance by 2(2)g2‘4 pertains by the time the second multi-billion-years era starts. However, use of an equation of state that has roots in the time period in which 2(1)g1‘2‘3 dominates might - at best - extrapolate based on a notion of 2(1)g2‘4 (and not based on a notion of 2(2)g2‘4) and would underestimate the strength of the key gravitational driver - of expansion - by a factor of two.

SOMA points - conceptually - to the following possible remedy.

CC might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repelling and the 2g2‘4 component of gravity. The contribution - to the pressure - that associates with intrinsic use of 2g2‘4 might need to double (compared to the contribution that would associate with intrinsic use of 2(1)g2‘4).

3.9.2. Large-scale clumping of matter

CC overestimates large-scale clumping of matter - OM and DM. (References [80], [81], [82], and [41] provide data and discussion.)

SOMA suggests that CC modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. SOMA suggests that 2(2)g2‘4 pertains. (That is, for each instance of 2g2‘4, a reach of two isomers pertains.) The additional (compared to CC modeling) repelling might explain the overestimating that CC suggests.

3.9.3. Effects - within galaxies - of the gravity associated with nearby galaxies

CC might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (Reference [55] provides further information.)

SOMA suggests that CC modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. SOMA suggests that 2(2)g2‘4 pertains. The additional (compared to CC modeling) repelling might explain at least some aspects of the data that reference [55] discusses.

3.10. Formation and evolution of galaxies

3.10.1. Mechanisms regarding the formation and evolution of galaxies

Reference [43] suggests that galaxies form around early clumps of stuff. Reference [43] associates the word halo with such clumps.

SOMA suggests that each one of many early halos associates with one isomer. SOMA associates with such early halos the three-element term one-isomer original clump. Clumping occurs based on gravitational effects. Differences - between the evolution of stuff associating with any one of isomer-zero and isomer-three and the evolution of stuff associating with any one of isomer-one, isomer-two, isomer-four, and isomer-five are not necessarily significant regarding this gravitationally based clumping. The six isomers might form such clumps approximately equally.

Table 9 discusses SOMA suggestions regarding the formation and evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all the clumps associate with just one isomer. Possibly, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

3.10.2. Aspects regarding the evolution of galaxies

Table 9 suggests three eras regarding the evolution of galaxies. The first era associates with the first two rows in table 9. The second era associates with the 2g2 attractive force that associates with the third row in table 9. The third era associates with collisions between and mergers of galaxies.

SOMA suggests the possibility that some galaxies do not exit the first era and do not significantly collide with other galaxies.

SOMA suggests that some galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in table 9. Here, this paper discusses three cases. (Mixed cases and other cases might pertain.)

Table 9: Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The next-to-rightmost column describes aspects of the stage. The leftmost three columns in the table describe a component of 2L that is a noteworthy cause for the stage. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. The symbol $\rightarrow$ associates with the notion that a noteworthy cause may gain prominence before a stage starts. Table 9 associates with a scenario in which a galaxy forms based on one original one-isomer clump and initially does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer. The rightmost column in table 9 suggests terminology regarding the evolution of galaxies. (A galaxy can include stuff from more than one earlier galaxy.)

Force	Intrinsic s-p	R_I	$\rightarrow$	Stage	Aspects of the stage	Era
Attractive	2g1'2'3	1	$\rightarrow$	1	A one-isomer original clump forms.	First
Repulsive	2g2'4	2	$\rightarrow$	2	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.	First
Attractive	2g2	6	$\rightarrow$	3	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.	Second
Attractive	2g2	6	$\rightarrow$	4	Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.	Third

Table 10: Ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. The three-word phrase cosmic optical background associates with radiation that - recently - measures as optical radiation or measures as close (with respect to wavelengths) to optical radiation. The acronym CMB associates with radiation that - recently - measures as cosmic microwave background radiation. DM:OM denotes a ratio of DM effects to OM effects that this paper suggests.

Aspect	Observed : POST-CC-expected	SOMA-suggested DM:OM
Amount of cosmic optical background	2 : 1	1 : 1
Some absorption of CMB	2 : 1	1 : 1

- Each one of some era-one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attracting) stuff associating with various isomers that have representation in nearby IGM. The galaxy becomes an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.
- Each one of some era-two galaxies merges (via 2g2 attracting) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses it status of being a galaxy. The resulting larger object is an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.
- Each one of some era-one or era-two galaxies merges (via 2g2 attracting) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era-three galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

3.11. Ratios of dark matter effects to ordinary matter effects

3.11.1. Ratios that might pertain regarding the cosmic electromagnetic background

Table 10 lists ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. The acronym CMB abbreviates the three-word term cosmic microwave background (or, the four-word term cosmic microwave background radiation). (References [83], [84] and [85] provide data and discussion regarding the amount of cosmic optical background. References [86], [87], and [88] provide data and discussion regarding absorption of CMB.)

The following two paragraphs provide SOMA-suggested explanations for the observations to which table 10 alludes.

The three-word phrase cosmic optical background associates with now nearly-optical light remaining from early in the universe. CC suggests that atomic transitions produced radiation that today measures as cosmic (optical and microwave) background radiation. SOMA associates POST atomic transitions with

Table 11: Suggested explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that associate with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The two-element term DM galaxy denotes a galaxy that contains much less OM than DM. Possibly, people have yet to directly detect early DM galaxies. Table 9 provides information about the explanations.

Objects	DM:OM	Examples	Explanation
Some early galaxies	$0^+ : 1$	Reported	OM original clump. Stage 1 or 2.
Some later galaxies	$0^+ : 1$	Reported	OM original clump. Stage 1 or 2.
Some early galaxies	$1 : 0^+$	No known reports	DM-isomer(s) original clump. Stage 1 or 2.
Some later galaxies	$1 : 0^+$	Reported	DM-isomer(s) original clump. Stage 1 or 2.
Some later galaxies	$\sim 4 : 1$	Reported	Non-isomer-three original clump. Stage 3.
Many later galaxies	$5^+ : 1$	Reported	Any-isomer(s) original clump(s). Stage 4.

isomer-zero. Observations found twice as much light as CC expected. SOMA suggests that isomer-one, isomer-two, isomer-four, and isomer-five stuff did not result in much stuff that is similar to isomer-zero atoms. SOMA suggests that isomer-three stuff evolved similarly to isomer-zero stuff. For four types of changes in atomic energy levels, table 5 alludes to 1L-producing events that associate with $R_I = 2$. SOMA suggests that such events explain the two-to-one observed-to-expected ratios regarding the cosmic optical background. Isomer-zero (or, OM) stuff produced half of the observed light. Isomer-three (or, DM) stuff produced half of the observed light.

The four-element phrase some absorption of CMB associates with the notion that measurements of some specific depletion of CMB indicate twice as much depletion as CC expected based solely on hyperfine interactions with (isomer-zero) hydrogen atoms. SOMA suggests (per table 5) that isomer-three (or, DM) hydrogen-like atoms account for the half of the absorption for which isomer-zero (or, OM) hydrogen atoms do not account.

3.11.2. Ratios that pertain for some galaxies

Table 11 suggests explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. (References [89] and [90] provide data and discussion. Reference [89] influenced the choice - that this paper reflects - of a time range to associate with the word early. Regarding the combination of 0^+:1 and later, references [91], [92], [93], [94], [95], [96], and [97] provide data and discussion. Reference [98] discusses a galaxy that might have started as containing mostly OM. Regarding observed DM galaxies, references [43], [99], [100], and [101] provide data and discussion. Current techniques might not be capable of observing early DM galaxies. References [102] and [103] suggest, regarding galaxy clusters, the existence of clumps of DM that might be individual galaxies. Extrapolating from results that references [43] and [104] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM $1 : 0^+$ later galaxies. Reference [105] discusses a trail of galaxies for which at least two galaxies have little DM. Reference [105] suggests that the little-dark-matter galaxies result from a collision that would have some similarities to the Bullet Cluster collision. Regarding galaxies for which DM:OM ratios of ~4:1 pertain, references [106] and [107] provide data and discussion. Regarding later galaxies for which DM:OM ratios of 5^+:1 pertain, reference [43] provides data and discussion. References [108] and [109] provide data about collisions of galaxies.)

Table 11 does not rule out the notion that galaxies somewhat fully populate DM:OM ranges within the interval of $0 : 1$ to, say, $6 : 1$. For DM:OM ratios of less than (say) ten, table 9 suggests that each range of DM:OM ratios to which table 11 alludes might stand out statistically (in terms of numbers of galaxies) from ranges (of positive-number ratios) near to the range to which table 11 alludes.

Table 11 does not rule out the notion that galaxies somewhat fully populate a DM:OM range of $10^{p_1} : 1$ to $10^{p_2} : 1$ for which SOMA does not suggest a value of p_1; p_1 exceeds, say, three; p_2 exceeds p_1; and SOMA does not suggest a value of p_2.

3.11.3. Ratios that pertain regarding phenomena that are bigger than galaxies

SOMA suggests two possible contributions toward the notion that measurements of large-scale presences of DM might exceed five times measurements of large-scale presences of OM.

- Asymmetric evolution. Here, the term alt-isomer refers to isomer-one, isomer-two, isomer-four, or isomer-five. The evolution of alt-isomer stuff might deviate - compared to the evolution of isomer-zero stuff - early enough that (nominally) isomer-zero high-energy excitations of the electromagnetic

Table 12: Suggested explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	Each one of asymmetric evolution and asymmetric measurement might pertain. To the extent that only asymmetric measurement pertains, counterpart (to the isomer-zero stuff that measures as DM) stuff across all six isomers associates with the plus in DM:OM $5^+ : 1$.
Some galaxy clusters	$5^+ : 1$	SOMA suggests that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe. (Similarity to DM:OM ratios for many stage 4 galaxies also pertains.)

field produce alt-isomer stuff significantly more copiously than (nominally) alt-isomer excitations of the electromagnetic field produce isomer-zero stuff. (Discussion related to equation (46) pertains.)

- Asymmetric measurement. Isomer-zero stuff might include hadron-like particles that include just zero-charge elementary fermions and gluons. (Table 3 points to - for example - might-be arc fermion elementary particles.) These isomer-zero hadron-like particles might measure as DM. The stuff associated with isomers other than isomer-zero might also include hadron-like particles that include just zero-charge elementary fermions and gluons. Those non-isomer-zero hadron-like particles would measure as DM.

Table 12 suggests explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. (Reference [60] provides data and discussion regarding densities of the universe. References [110], [111], [112], and [113] provide data and discussion regarding galaxy clusters.)

3.11.4. Aspects related to collisions of pairs of galaxy clusters

Reference [67] discusses the Bullet Cluster collision of two galaxy clusters.

CC suggests two general types of trajectories for stuff. Most DM - from either one of the clusters - exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Also, OM stars - from either cluster - exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, OM IGM - from either cluster - lags the cluster's OM stars and DM. CC suggests that the OM IGM interacted electromagnetically with the other cluster's OM IGM, as well as gravitationally with the other cluster.

SOMA suggests that SOMA might comport (regarding each cluster) with the interpretations of data, with one possible exception. The possible exception associates with the notion that SOMA suggests that isomer-three IGM interacts electromagnetically and follows trajectories that are consistent with OM IGM trajectories.

Regarding the possible exception, at least three possibilities arise.

For one possibility, per table 5, the light that CC associates with OM IGM might include light that SOMA associates with OM IGM and light that SOMA associates with isomer-three IGM.

For one possibility, isomer-three IGM measures as DM and CC does not adequately report (or otherwise account for) lagging isomer-three IGM.

For one possibility, isomer-three IGM follows trajectories that are consistent with other DM trajectories.

SOMA suggests that interpretations of data may not be sufficient to rule out each one of the first two possibilities or to rule out a combination of the first two possibilities.

SOMA notions of DM are not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - regarding DM.

4. Discussion

4.1. Possible elementary particles that POST has yet to include

SOMA might provide insight regarding the existence and properties of some as-yet-unfound elementary particles that POST hypothesizes or that SOMA might suggest.

4.1.1. Elementary particles that SOMA suggests

Table 3 catalogs as-yet-unfound elementary particles that SOMA suggests.

4.1.2. Right-handed W boson

Reference [114] discusses a fraction of decays - of OM top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Reference [19] provides a confidence level of 90 percent that the rest energy of a might-be W_R (or, right-handed W boson) exceeds 715 GeV. Reference [115] provides other information.

SOMA suggests that W_R bosons associate only with isomers one, three, and five. SOMA suggests possibilities for inter-isomer interactions and conversions.

Aspects of SOMA might approximately reproduce the above result that SM modeling suggests.

Aspects related to equation (35) suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, SOMA does not suggest that $m(5,3)$ associates with the inertial mass of an isomer-one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_i \leq 4$ and $0 \leq l'_f \leq 3$, $m(3(l_i+1)+l'_f,3) = \beta m(3(l_i+0)+l'_f,3)$. One might conjecture that isomer-zero observations of some aspects of isomer-one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer-zero elementary particles (and that are β times inertial masses for the counterpart isomer-one elementary particles).

Based on notions of scaling that might calculate non-inertial mass-like quantities, SOMA might suggest that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [114] discusses. A notion of $m_{\text{non-inertial,W}_R\text{isomer one}}c^2 = \beta m_{\text{W}} c^2 \approx 2.8 \times 10^5$ GeV might pertain. Here, the notion of a non-inertial mass-like quantity might associate with data that associate with interactions that associate with 1L or 1W_1. The interactions do not necessarily associate directly with 2L.

4.1.3. Axion

POST suggests the possibility for an axion - a zero-spin, nonzero-mass elementary particle. POST suggests that axions might decay into photons or interact with magnetic fields. POST suggests that axions might measure as being dark matter. POST generally suggests that an axion would have a mass that would be much less than the mass of an electron. Data have yet to point to evidence for axions. POST suggests that axions might associate with strong-interaction violations of CP-symmetry.

SOMA suggests that a non-isolated decay of an alt-isomer nonzero-mass boson elementary particle might - as measured by isomer-zero equipment - associate with notions that POST might associate with axions. Per discussion related to table 4, an isolated decay could produce a pair of photons for which the reach for each photon is just the alt-isomer. Per an extension of discussion related to table 5, for a non-isolated decay, the reach of each photon in an outgoing pair of photons could be more than one isomer. Regarding the mass of the alt-isomer nonzero-mass boson elementary particle, SOMA might suggest a non-inertial mass-like quantity that is - for some integer l_{ai} - $\beta^{-l_{ai}}$ times the mass of the alt-isomer nonzero-mass boson elementary particle. For example, if the alt-isomer nonzero-mass boson elementary particle is an isomer-three Higgs boson, l_{ai} might be three and the corresponding non-inertial rest-energy-like quantity might be approximately 3×10^{-3} eV. For this example, larger values of l_{ai} - and, hence, smaller non-inertial rest-energy-like quantities - might pertain.

4.1.4. Magnetic monopole

Table 1 seems not to suggest a 1L interaction with a monopole other than an electric monopole. SOMA does not suggest a property that would associate with a magnetic monopole.

4.2. Phenomena that might involve the SOMA-suggested jay boson elementary particle

SOMA points to interactions that might involve the might-be jay boson.

4.2.1. Pauli repulsion

POST includes the notion that two identical fermions cannot occupy the same state. Regarding QM, one notion is that repelling between identical fermions associates with overlaps of wave functions. Another QM notion features wave functions that are anti-symmetric with respect to the exchange of two identical fermions.

Table 13: Six possible eras regarding the rate of separating of large clumps. The rightmost two columns suggest eras. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. In table 13, subsequent rows associate with later eras. The word inflation (or, the two-word term inflationary epoch) names the era that associates with the third row in the table. Regarding eras that would precede inflation, SOMA points to the possibility for the two eras that the table discusses. Intrinsic use of the solution-pair 0g1‘2‘3‘4‘8 associates with the Pauli exclusion principle (and with the might-be jay boson). The other intrinsic solution-pairs to which table 13 alludes associate with gravitation. TBD denotes to be determined. The symbol † denotes a possible association between the relevant era and some CC notions of a Big Bang. The leftmost four columns describe phenomena that SOMA suggests as noteworthy causes for the eras. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol → associates with the notion that a noteworthy cause may gain prominence before an era starts.

Force	Intrinsic s-p	SOMA-pole	R_I	→	Rate of separating	Duration
Attractive	2g1‘2‘3‘4‘8x	16-pole	6	→	Is negative	TBD
Repulsive	0g1‘2‘3‘4‘8	16-pole	1	→	Turns positive †	TBD
Repulsive	2g1‘2‘3‘4x	Octupole	1	→	Increases rapidly	Fraction of a second
Attractive	2g1‘2‘3	Quadrupole	1	→	Decreases	Billions of years
Repulsive	2g2‘4	Dipole	2	→	Increases	Billions of years
Attractive	2g2	Monopole	6	→	Would decrease	-

SOMA might be compatible with such aspects of POST and, yet, not necessitate - regarding POST dynamics modeling - the use of wave functions. QM based on jay bosons might suffice. CM based on potentials that would associate with effects of jay bosons might suffice.

QM or CM based on jay bosons might suggest that the prevention of two identical fermions from occupying the same state might associate with, in effect, interactions - mediated by jay bosons - that try to change aspects related to the fermions. Notions of changing a spin orientation might pertain. For elementary fermions, notions of changing a flavour might pertain.

4.2.2. Energy levels in positronium

Reference [116] discusses the transition - between two states of positronium - characterized by the expression that equation (47) shows.

$$2^3S_1 \rightarrow 2^3P_0 \tag{47}$$

Four standard deviations below the nominal observed value of the energy that associates with the transition approximately equals four standard deviations above the nominal value of the energy that POST suggests.

SOMA notions regarding jay bosons might explain the might-be discrepancy regarding positronium. Compared to QFT, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that QFT does not suffice to explain positronium energy levels, SOMA notions related to the jay boson might help to close the gap between observations and modeling.

4.2.3. Pauli crystals

Reference [117] reports detection of Pauli crystals. SOMA suggests that modeling based on the notion of jay bosons might help explain relevant phenomena.

4.3. Possible pre-inflation eras in the evolution of the universe

Reference [118] discusses CC notions regarding cyclic cosmology. SOMA includes the possibility that the present universe arose from an implosion of energy. SOMA does not yet consider either aspects that may have created the energy that would have imploded or whether the present universe might eventually implode.

Reference [33] discusses possibilities that might lead to a Big Bang.

Table 13 discusses six possible eras regarding the rate of separating of large clumps. Table 13 includes information that table 8 includes.

SOMA suggests that some SOMA notions regarding eras that start with and follow the inflationary epoch might not necessarily depend significantly on SOMA notions regarding eras that might precede the inflationary epoch.

This paper does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repelling based on 2g2‘4 to attracting based on 2g2 might occur.

4.4. Baryon asymmetry

The two-word term baryon asymmetry associates with the POST notion that - regarding known stuff - there are many more left-handed (or matter) fermion elementary particles than right-handed (or antimatter) fermion elementary particles. CC suggests that baryon asymmetry arose early in the history of the universe. From the perspective of SOMA, such known stuff associates with isomer-zero.

Discussion related to equation (45) points to a means that may have produced baryon asymmetry. Possibly, POST notions of lasing pertained regarding relevant excitations of LRI fields. SOMA suggests that processes leading to baryon asymmetry led to isomer-three stuff having fewer left-handed (or anti-matter, from the perspective of isomer-three) fermion elementary particles than right-handed (or matter, from the perspective of isomer-three) fermion elementary particles.

This paper does not address the topic of the extent to which steps leading to a predominance in isomer-zero stuff of left-handed elementary particles (and not to a predominance of right-handed elementary particles) have a basis other than random chance.

4.5. Some phenomena that associate with galaxies

4.5.1. Some quenching of star formation

Some galaxies seem to stop forming stars. (Reference [119] and reference [120] discuss examples.) Such quenching might take place within three billion years after the Big Bang, might associate with a lack of hydrogen atoms, and might (per reference [120]) pertain to half of the galaxies that associate with the notion of a certain type of galaxy.

SOMA suggests that some such quenching might associate with repelling that associates with 2(2)g2‘4. Some quenching might associate with galaxies for which original clumps featured isomer-zero stuff or isomer-three stuff.

4.5.2. Some stopping of the accrual of matter

Reference [121] discusses a galaxy that seems to have stopped accruing both OM and DM about four billion years after the Big Bang.

The galaxy that reference [121] discusses might (or might not) associate with the notion of significant presence early on of one of isomer-zero and isomer-three, one of isomer-one and isomer-four, and one of isomer-two and isomer-five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

4.5.3. Aspects regarding stellar stream GD-1 in the Milky Way galaxy

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-OM clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (References [122] and [123] provide data and discussion regarding the undetected object. Reference [123] cites reference [124] and reference [125].) SOMA suggests that the undetected object might be a clump of DM.

4.6. Some possibilities for detecting non-isomer-zero dark matter

Table 5 points to electromagnetic phenomena that associate with reaches of two and, thereby, suggests that OM equipment might be able to catalyze or detect transitions within isomer-three atoms. Discussion related to table 10 suggests that data point to detection, by OM equipment, of light emitted by transition events that associate with isomer-three atoms. Presumably, some isomer-three atoms pass (essentially unimpeded by isomer-zero stuff) through isomer-zero stuff that is near to and includes the Earth. SOMA suggests that experiments - based on OM-produced light and OM-detected light - might be able to detect (via transition events that associate with isomer-three atoms) isomer-three atomic stuff. This paper does not discuss notions regarding whether techniques are now or when techniques might become sufficiently sensitive that such experiments would be feasible.

4.7. Some information that gravitational waves might convey

Reference [126] discusses opportunities for research regarding gravitational waves.

Extending notions that associate with table 5 suggests intrinsic uses of 2g” solution-pairs that might associate with the producing of gravitational waves.

SOMA suggests that intrinsic use of the 2g1‘3‘4‘5‘7 solution-pair might be relevant. Intrinsic use of the 2g1‘3‘4‘5‘7 solution-pair associates with a reach of six.

Intrinsic use of 2g4‘5‘7 associates with a reach of one. Intrinsic use of any one of 2g1‘4‘5, 2g3‘4‘5, and 2g1‘4‘7 associates with a reach of two. To the extent that intrinsic use of at least one such 2g” solution-pair is relevant regarding the producing of gravitational waves, SOMA suggests that adequately detailed analyses of the gravitational signatures that associate with collisions of objects - such as black holes - might enable the development of data that associate with the extents to which the colliding objects include stuff that associates with more than one isomer or more than one isomer-pair.

4.8. Modeling regarding gravity

Present GR is not necessarily adequately compatible with SOMA notions of DM. SOMA suggests that POST modeling based on GR can be less than adequately accurate.

Tests of GR have featured phenomena that associate with the isomer-pair that includes isomer-zero and isomer-three. Each one of the Sun, the planet Mercury, and the Earth associates with isomer-zero. Relevant radiation from distant stars and galaxies associates essentially just with isomer-zero stuff and isomer-three stuff.

For cases in which POST suggests that uses of general relativity adequately (or nearly adequately) comport with data, SOMA suggests that the following notions - about uses of SOMA solution-pairs and about the GR stress-energy tensor - might help bridge from SOMA to GR or from GR to SOMA. (The GR stress-energy tensor is symmetric.) Intrinsic 2g2 associates with the one stress-energy-tensor component (T^{00}) that associates with energy density. Extrinsic 2g2'4 associates with the three components (T^{01}, T^{02}, and T^{03}) that associate with momentum density and also associates with the three components (T^{10}, T^{20}, and T^{30}) that associate with energy flux. Intrinsic 2g2'4 associates with the three components (T^{11}, T^{22}, and T^{33}) that associate with pressure. Extrinsic 2g2'4'8 associates with the three components (T^{21}, T^{31}, and T^{32}) that associate with momentum flux and also associates with the three components (T^{12}, T^{13}, and T^{23}) that associate with shear stress. (This paper does not further explore the usefulness of such notions.)

SOMA does not (yet) suggest associations between 2L solution-pairs that associate with $R_I = 1$ and the stress-energy tensor.

4.9. Classical mechanics, quantum mechanics, and SOMA solution-pairs

This unit associates with table 2.

SOMA suggests - at least approximately - the following notions. SOMA Σg' aspects directly link with CM. QM evolved based, in part, on CM. SOMA 0g aspects associate with notions of discrete (in the sense of not continuous). SOMA 0g aspects directly link with QM. SOMA Σg" aspects associate with notions of discrete. SOMA Σg" aspects directly link with QM.

4.10. Aspects that might associate with SOMA uses of nonpositive integers k

The notion of n_Γ does not necessarily associate with nonpositive values k.

For $k \leq 0$, SOMA suggests that equation (48) might associate k with some aspects of POST.

$$k = -n_V \tag{48}$$

$n_V = 2$ can associate with harmonic oscillator mathematics. Per equation (48), $k = -2$ pertains.

$n_V = 1$ might associate with the POST notion - regarding the strong interaction - of asymptotic freedom. Per equation (48), $k = -1$ pertains.

$n_V = 0$ associates with a POST notion of zero force. Per equation (48), $k = 0$ pertains.

SOMA suggests that SOMA notions - that, for $k > 0$, associate with s_k and with DOF-like aspects - can also pertain regarding $k \leq 0$.

4.10.1. Three spatial dimensions and one temporal dimension

SOMA suggests that the three DOF-like aspects that associate with $|s_{-2}| = -1$ might associate with POST notions of three spatial dimensions.

SOMA suggests that the one DOF-like aspect that associates with $s_{-2} = 0$ might associate with POST notions of one temporal dimension.

(This paper does not explore the notion that modeling might associate the properties of extrinsic position, extrinsic velocity, and so forth with $k = -2$. Such modeling might, for example, associate with terms of the form $|k|^n s_{k_n}$. This paper does not explore the notion that modeling might associate CM inertial properties with $k = -2$.)

4.10.2. Strong interaction properties of objects

SOMA suggests that the three DOF-like aspects that associate with $|s_{-1}| = -1$ might associate with the three POST color charges - red, blue, and green.

SOMA suggests that the one DOF-like aspect that associates with $s_{-1} = 0$ might associate with the POST notions of no color charge and clear (or white) color charge.

Paralleling notions that pertain to electromagnetism and gravity, strong-interaction fields associate with two circular polarization modes - left-circular polarization and right-circular polarization.

4.10.3. Three isomer-pairs

SOMA suggests that the three DOF-like aspects that associate with $|s_0| = 1$ might associate with the SOMA notion of three isomer-pairs (of non-LRI elementary particles).

SOMA suggests that the one DOF-like aspect that associates with $s_0 = 0$ might associate with the SOMA notion of (an LRI elementary particle's) not being associated with just one specific isomer.

4.11. Patterns regarding elementary particles

Table 3 and discussion related to table 3 underlie this unit.

For this discussion, Z_Γ associates only with 0g intrinsic solution-pairs that associate with known elementary particles or with SOMA-suggested elementary particles. For this discussion, m_b denotes a notion of boson mass and m_f denotes a notion of fermion mass. For this discussion, S_b denotes a notion of boson spin and S_f denotes a notion of fermion spin.

For SOMA modeling regarding the W boson, the following aspects pertain for the one relevant intrinsic solution-pair.

- The notion of $4 \notin Z_\Gamma$ pertains. The notion of $Q = 1$ pertains.
- The notion of $6 \notin Z_\Gamma$ pertains. The notion of $m_b > 0$ pertains. The notion ofm_f does not pertain.
- The notions of $6 \notin Z_\Gamma$ and $n_{\Gamma^0} \neq 4$ pertain. The notion of $S_b > 0$ pertains. The notion of S_f does not pertain.
- The notion of $8 \notin Z_\Gamma$ pertains. The notion of $m_b = 0$ does not pertain.
- The notion of $16 \notin Z_\Gamma$ pertains. The notion of $n_I = 6$ pertains. The notion of $R_I = 1$ pertains.
- The notions of $6 \notin Z_\Gamma$ and $8 \notin Z_\Gamma$ pertain. POST models can treat the elementary particle as modeling as an object that is not part of a system of objects.

Compared to aspects that pertain regarding the W boson, the following aspects pertain for intrinsic solution-pairs that associate with known elementary particles or with SOMA-suggested elementary particles.

- A change to $4 \in Z_\Gamma$ removes the notion of $Q = 1$ and installs the notion of $Q = 0$.
- A change to $6 \in Z_\Gamma$ removes the notion of m_b and installs the notion of $m_f > 0$.
- A change to $6 \notin Z_\Gamma$ and $n_{\Gamma^0} = 4$ removes the notion of $S_b > 0$ and installs the notion of $S_b = 0$.
- A change to $6 \in Z_\Gamma$ removes the notion of S_b and installs the notion of $S_f > 0$.
- A change to $8 \in Z_\Gamma$ removes the notion of $m_b > 0$.
- A change to $16 \in Z_\Gamma$ and $128 \notin Z_\Gamma$ removes the notion of n_I and installs the notion of $R_I > 1$
- A change to $6 \in Z_\Gamma$ and $8 \in Z_\Gamma$ removes the notion that POST models treat the elementary particle as modeling as an object that is not part of a system of objects.

4.12. Some possible applications of harmonic oscillator mathematics

4.12.1. Some harmonic oscillator mathematics

Modeling for a j-dimensional isotropic harmonic oscillator can feature j linear coordinates $x_{k'}$ - each with a domain $-\infty < x_{k'} < \infty$ - and an operator that is the sum - over k' - of j operators of the form that equation (49) shows. The number C is positive and is common to all j uses of equation (49). The word isotropic associates with the commonality - across all j uses of equation (49) - of the number C.

$$-\frac{\partial^2}{\partial(x_{k'})^2} + C \cdot (x_{k'})^2 \tag{49}$$

For $j \geq 2$, one can split the overall operator into pieces. Equation (50) associates with a split into two pieces. Here, each of j_1 and j_2 is a positive integer.

$$j = j_1 + j_2 \tag{50}$$

In discussion below, the symbol D might be any one of j, j_1, and j_2.

For $D \geq 2$, mathematics related to isotropic harmonic oscillators can feature partial differential equations, a radial coordinate, and $D-1$ angular coordinates. Equation (51) defines a radial coordinate.

$$x = (\sum_{k'} (x_{k'})^2)^{1/2} \tag{51}$$

SOMA suggests replacing x via the expression that equation (52) shows. Here, r_{HO} denotes the radial coordinate and has dimensions of length. The parameter η has dimensions of length. The parameter η is a nonzero real number. The magnitude $|\eta|$ associates with a scale length. (Here, r_{HO} associates with mathematics for HO - or, harmonic oscillators - and does not necessarily associate with uses of r elsewhere - for example, in equation (1) - in this paper.)

$$x = r_{HO}/\eta \tag{52}$$

Applications of equations (53) and (54) can associate with POST. Each of ξ and ξ' is an as-yet unspecified constant. The symbol $\phi_R(r_{HO})$ denotes a function of r_{HO}. The symbol ${\nabla_{r_{HO}}}^2$ denotes a Laplacian operator. Ω associates with aspects that associate with angular coordinates. (For $D = 3$, reference [127] shows a representation for Ω in terms of an operator that is a function of spherical coordinates.)

$$\xi \phi_R(r_{HO}) = (\xi'/2)(-\eta^2 {\nabla_{r_{HO}}}^2 + (\eta)^{-2} r_{HO}^2) \phi_R(r_{HO}) \tag{53}$$

$${\nabla_{r_{HO}}}^2 = r_{HO}^{-(D-1)} (\partial/\partial r_{HO})(r_{HO}^{D-1})(\partial/\partial r_{HO}) - \Omega r_{HO}^{-2} \tag{54}$$

SOMA applications assume that the symbol Ω is a constant. SOMA applications do not necessarily require that D is a positive integer for which $D \geq 2$. SOMA applications include solutions that pertain for the domain that equation (55) shows. With respect to the domain $0 \leq r_{HO} < \infty$, ϕ_R associates with the mathematics notion of having a definition almost everywhere. (Some aspects of POST applications associate with the following notions. D is a nonnegative integer. ϕ_R associates with a radial factor that is part of a representation of a wave function. For $D = 1$, equation (54) might not be appropriate. For $D > 1$, a representation of a wave function may need to include a factor for which angular coordinates play roles. The domain for a representation of such a wave function needs to include $r_{HO} = 0$. For SOMA applications, ϕ_R does not necessarily associate with the notion of a factor in a representation for a wave function and does not necessarily need to have a definition that associates with $r_{HO} = 0$.)

$$0 < r_{HO} < \infty \tag{55}$$

In discussion below, the symbol D might be any real number.

SOMA considers solutions of the form that equation (56) shows. (In POST, solutions that associate with equation (49) and with $D = 1$ have the form $H(x)\exp(-x^2)$, in which $H(x)$ is a Hermite polynomial. Mathematics that SOMA suggests can allow for a SOMA-adequately-useful set of solutions for which each solution associates with - in effect - a one-term polynomial.)

$$\phi_R(r_{HO}) \propto (r_{HO}/\eta)^{\nu} \exp(-r_{HO}^2/(2\eta^2)), \text{ with } \eta^2 > 0 \tag{56}$$

Equations (57) and (58) characterize solutions. The parameter η does not appear in these equations.

$$\xi = (D + 2\nu)(\xi'/2) \tag{57}$$

$$\Omega = \nu(\nu + D - 2) \tag{58}$$

$\phi_R(r_{HO})$ normalizes if and only if equation (59) pertains. The symbol $(\phi_R(r_{HO}))^*$ denotes the complex conjugate of $\phi_R(r_{HO})$.

$$\int_0^\infty (\phi_R(r_{HO}))^* \phi_R(r_{HO}) r_{HO}^{D-1} dr_{HO} < \infty \tag{59}$$

Equation (60) associates with the domains of D and ν for which normalization pertains for $\phi_R(r_{HO})$. For $D + 2\nu = 0$, normalization pertains in the limit $\eta^2 \to 0^+$. Regarding mathematics relevant to normalization for $D+2\nu = 0$, the delta function that equation (61) shows pertains. Here, $(x')^2$ associates with r_{HO}^2 and 4ϵ associates with η^2. (Reference [128] provides equation (61).) The difference in domains, between $-\infty < x' < \infty$ and equation (55), is not material here.

$$D + 2\nu \geq 0 \tag{60}$$

$$\delta(x') = \lim_{\epsilon \to 0^+} (1/(2\sqrt{\pi\epsilon}))e^{-(x')^2/(4\epsilon)} \tag{61}$$

4.12.2. Possible associations with DOF-like aspects

In equation (54), Ω associates with the radial aspects of the Laplacian operator that associates with D dimensions. For $2P$ being a nonnegative integer, the notions of equation (58) and $P = \nu$ combine to produce $\Omega = P(P + D - 2)$. (For $D = 3$ and $P = S$ and $2S$ being a nonnegative integer, Ω can associate with the POST notion $S(S+1)\hbar^2$ that POST associates with angular momentum.) SOMA suggests that D might associate with a number of DOF-like aspects.

4.12.3. Possible associations with relationships among properties of elementary particles

For boson elementary particles, equation (29) suggests links between mass, spin, and charge. The following notions might provide useful insight.

Each one of m' and S is always non-negative. Perhaps, regarding each one of m' and S, some modeling associates with two DOF-like aspects - positive quantity and zero quantity. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for each of $P = m'$ and $P = S$, the factor $P^2 = P(P + D - 2)$ pertains regarding aspects of the mathematics. Equation (29) includes a term $(m')^2$. Equation (29) includes a term S^2.

Charge - which is a basis for Q - can be positive, zero, or negative. Perhaps some modeling associates with three DOF-like aspects - positive quantity, zero quantity, and negative quantity. Regarding mathematics associated with Laplacian operators, for $D = 3$ and for $P = Q$, the factor $P(P+1) = P(P+D-2)$ pertains regarding aspects of the mathematics. Equation (29) includes a term $Q(Q + 1)$.

Whether or not l_{ms} is nonzero associates with the POST notion of whether longitudinal polarization can pertain. Regarding mathematics associated with Laplacian operators, for $D = 2$ and for $P = (l_{ms})^{1/2}$, the factor $P^2 = P(P + D - 2)$ pertains regarding aspects of the mathematics. Equation (29) includes a term $l_{ms} = ((l_{ms})^{1/2})^2$.

Regarding the masses of boson elementary particles, $D = 2$ associates with two DOF-like aspects of which one aspect is $m_b > 0$ and the other aspect is $m_b = 0$. Regarding the masses of fermion elementary particles, possibly, $D = 1$ associates with one DOF-like aspect of $m_f > 0$. Here, the factor $P(P + D - 2) = P(P - 1)$ might pertain. Possibly, modeling can consider that a total of three (as in two plus one) DOF-like aspects associate with the notion of elementary particle mass.

Regarding the spins of boson elementary particles, $D = 2$ associates with two DOF-like aspects of which one aspect is $S_b > 0$ and the other aspect is $S_b = 0$. Regarding the spins of fermion elementary particles, possibly, $D = 1$ associates with one DOF-like aspect of $S_f > 0$. Here, the factor $P(P+D-2) = P(P-1)$ might pertain. Possibly, modeling can consider that a total of three (as in two plus one) DOF-like aspects associate with the notion of elementary particle spin.

This insight might provide a basis for modeling that would underlie SOMA. This paper does not further discuss uses for the insight.

4.12.4. Possible associations with the Higgs mechanism

The SM includes the notion of a Higgs mechanism.

Regarding positive integers D and negative integers 2ν, equation (60) points to solutions that POST might consider to associate with harmonic-oscillator-state energies that are less than ground-state energies. For example, $D = 3$, $\nu = -1$, $\phi_R(r_{HO}) \propto (r_{HO}/\eta)^{-1} \exp(-r_{HO}^2/(2\eta^2))$, and a lack of angular dependence might associate with $\Omega = 0$ and with a wave function that associates with an energy for which $(D + 2\nu)/2 = 0.5$ pertains, whereas a POST ground state might associate with $D = 3$, $\nu = 0$, $\Omega = 0$, a lack of angular dependence, and a wave function that associates with an energy for which $(D + 2\nu)/2 = 1.5$ pertains.

This paper does not further discuss the notion that this insight - about two solutions that associate with $\Omega = 0$, no dependence on angular coordinates, and $S = 0$ - might associate with the SM notion of the Higgs mechanism.

4.12.5. Some other possible applications of harmonic oscillator mathematics

POST includes notions in which non-integer numbers of dimensions D associate with the expression $\lim_{D \to j}(\cdots)$, in which j is a nonnegative integer such as a number of spatial dimensions. (This paper does not further discuss the notion that aspects such as equation (58) might prove useful regarding such POST notions.)

POST includes notions that associate with the two-element phrase point-like particle. Discussion related to equation (60) and equation (61) points to the possibility that notions that associate with $\lim_{(D+2\nu) \to 0^+}$ might associate with notions of point-like. (This paper does not further discuss the notion that aspects that associate with $\lim_{(D+2\nu) \to 0^+}$ might prove useful regarding POST notions of point-like.)

5. Concluding remarks

Each of the following sentences describes a physics challenge that has persisted for the most recent eighty or more years. Interrelate physics models. Interrelate physics properties, properties of objects, and physics constants. Provide, for elementary particles, an analog to the periodic table for chemical elements. Describe bases for phenomena that POST (or, modeling that has bases in popular modeling notions of space-time coordinates) associates with the two-word term dark matter. Describe bases for phenomena that POST associates with the two-word term dark energy. Explain the overall evolution of the universe.

Physics amasses data that people can use as bases for developing and evaluating modeling aimed at addressing the challenges.

SOMA (or, modeling that - for a single object - points to multiple attributes) addresses those physics challenges and has bases in the following mathematics - integer arithmetic, multipole expansions, Diophantine equations, and multidimensional harmonic oscillators.

SOMA unites and decomposes aspects of electromagnetism and gravity. For each of those two long-range interactions, the decomposition associates with properties - of objects - that people can measure and that POST features. For electromagnetism, the properties include charge and magnetic moment. For gravity, the properties include energy.

SOMA points to all known elementary particles and to some might-be elementary particles. SOMA includes a notion of isomers of elementary particles that do not mediate long-range interactions. SOMA features a notion of instances of components of long-range interactions.

SOMA suggests explanations for data regarding dark matter. SOMA points to possible resolutions for tensions - between data and POST - regarding effects of dark energy. SOMA suggests insight regarding galaxy formation and evolution.

SOMA matches data that POST matches, suggests explanations for data that POST seems not to explain, suggests results regarding data that people have yet to gather, and points to possible opportunities to develop models that unite aspects of physics and physics modeling.

In summary, SOMA suggests augmentations - to POST - that might achieve the following results. Extend the list of elementary particles. Predict masses for at least two neutrinos. Predict masses - that would be more accurate than known masses - for some other elementary particles. Describe dark matter. Explain ratios of dark matter effects to ordinary matter effects. Provide insight regarding galaxy formation. Describe bases for phenomena that associate with the two-word term dark energy. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models.

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