Octonion Dynamics

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Abstract

The relativistic 4D space-time cover of Electrodynamics gives a roadmap for the creation of a potential function-based field theory within the algebraic structure of Octonion Algebra. Understanding the true algebraic nature of axial and polar vector types tells us which six Octonion basis components cover the electric and magnetic field types. Increasing the total dimension count from space-time 4 to Octonion 8 provides additional degrees of freedom required to accommodate Gravitation within the same structure. The Octonion framework for mathematical physics is presented in an algebraic covariant fashion. The Octonion Algebra forms for the analogous Maxwell's Equations, physical fields, forces, work, energy and momentum conservation are presented. The importance of structuring the Octonion forms in an algebraic orientation covariant fashion is abundantly clear in this presentation.

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The proper covariant derivative definition for Octonion Algebra is the Ensemble Derivative **E** defined in references [1], [6] as:

$\mathbf{E}(\mathbf{A}(\mathbf{v})) = 1/J\ \partial/\partial v_i\ [\ C_{ij}\ \ T_{kl}\ \ A_k\ ]\ \ e_j * e_l$

J is the Jacobian of general basis transformation T from the Octonion intrinsic basis e to a new basis $g_k = T_{kl}\ e_l$ with position **v** specified as $v_r\ g_r$. C_{ij} holds the cofactor of T_{ij} and $\mathbf{A} = A_s\ g_s$ is the functional algebraic element operated on. With nothing special about T other than its Jacobian is non-zero, the functional is best carried as an e basis algebraic element $A`_l\ e_l$, where $A`_l = T_{kl}\ A_k$. Then all results are e basis algebraic elements with clearly defined algebraic manipulation. Do understand the partial differentiation is on the complete […] and it is *this* which leads to a covariant definition. Result covariance requires whole applications of this form; the *ensemble* of vector analysis notions like gradient, divergence and curl, which must no longer be taken as individually fundamental. This is not to say they have no separate meaning; they do, but this understanding must come *from*, and not lead *to* the greater fundamental ensemble structure. They must not be separately manipulated then indiscriminately combined if covariance is required, which it essentially always is.

The covariant Ensemble Derivative in Octonion Algebra's intrinsic e basis with position algebraic element $u_i\ e_i$ defines $\mathbf{u} = \mathbf{v}$, $T_{kl} = \delta_{kl}$, $C_{ij} = \delta_{ij}$, $J = +1$ so we can write the intrinsic basis covariant derivative as

$\mathbf{E}(\mathbf{A}(\mathbf{u})) = \partial/\partial u_i\ [\ A_k\ ]\ e_i * e_k = \nabla_{(\mathbf{u})} * \mathbf{A}(\mathbf{u})$.

Highlighting what is covered in detail within reference [11], when T is an algebraic basis gauge transformation to gauge basis g defined to be an algebraic isomorphism with the Octonion intrinsic e basis, if $e_a * e_b = s_{abc}\ e_c$ where s_{abc} are the structure constants defining the particular Octonion Algebra orientation, then g defined to be $g_k = T_{kl}\ e_l$ with basis products also requiring $g_a * g_b = s_{abc}\ g_c$. If g is restricted to an algebraic isomorphism, T is required to be a lower block diagonal limited member of SO(7).

Limiting T to J = +1 orthonormal, matrix C will equal matrix T. The covariant derivative form may then be written as

$\mathbf{E}(\mathbf{A}(\mathbf{v})) = \partial/\partial v_i\ [\ A_k\ g_i * g_k\ ]$

If the g basis description is independent of the gauge transformation position algebraic element **v**, that is the gauge transformation is a global gauge, we can take g_i and g_k outside the differentiation. In this case, the differentiation over **v** can be written as a simple g system * product of the algebraic element del operator given by $\nabla_{(\mathbf{v})} = g_i\ \partial/\partial v_i$ acting on the g basis functional algebraic element $A_k\ g_k$, and this may be written as $\nabla_{(\mathbf{v})} * \mathbf{A}(\mathbf{v})$. This is seen to be form invariant with the identity transformation intrinsic basis covariant derivative.

If we allow the parametrization of the SO(7) portion of T to vary with **v** position, we now have a *local algebraic basis gauge transformation*. The * isomorphism is still required to hold at each **v** position, but for the covariant derivative, we can no longer take g_i and g_k out of the differentiation, adding terms to the form invariant portions.

Whether we use the intrinsic e basis, or a local or global algebraic basis gauge transformation, we can define left and right application partial covariant derivatives in like fashion:

$E_i(\mathbf{A}(\mathbf{u})) = \partial/\partial u_i\ [\ A_k\ e_i * e_k\ ]$ $\qquad (\mathbf{A}(\mathbf{u}))E_i = \partial/\partial u_i\ [\ A_k\ e_k * e_i\ ]$

$E_i(\mathbf{A}(\mathbf{v})) = \partial/\partial v_i\ [\ A_k\ g_i * g_k\ ]$ $\qquad (\mathbf{A}(\mathbf{v}))E_i = \partial/\partial v_i\ [\ A_k\ g_k * g_i\ ]$

We can assume by context which algebraic basis is in play, and simply write $E_i(\mathbf{A})$ or $(\mathbf{A})E_i$. Expressing mathematical physics expressions in terms of the Ensemble Derivative singularly provides the form for Octonion Algebra dynamics independent of basis choice.

To cover natural dynamics with Octonion Algebra I chose a path patterning after Electrodynamics. This requires us to formulate Octonion Algebra representations for expected physical fields built from first order differentiations on potential functions. These set the foundation for expressing the notion of observable flow, an Octonion 8-current formed from another differentiation on the physical fields. A further scalar result differentiation of this 8-current must provide a continuity equation indicating the divergence of the flow in/out of any prescribed volume is balanced by the time derivative of the total amount of flow material within the volume. Total flow material is conserved. Notions of force and work must be formed by Octonion products of physical fields and the 8-current. Restructuring the 8–work–force equation with an outside differentiation on all terms, forming an equation equating the two equivalent representations, then integrating over any prescribed volume gives equations for the conservation of energy and momentum. All will be provided below.

In reference [11] it was shown necessary to include structure constants if we hoped to represent equations with what I call *algebra orientation covariance*. This requires attaching two characteristics to each product term, odd/even parity of total variant product count throughout any product term's full product history, and a final disposition of either no Quaternion subalgebra product rule in play or defined by one of seven specified in its +1 structure constant orientation s_{abc} for some chosen proper Octonion orientation which then may be singularly used. With the mechanics of this representation understood, it might be nice to consolidate the parity with the triplet designation. Repeating the optimal Quaternion subalgebra partitioning in reference [11] we have Q_n defined as

$Q_1 = \{e_2\ e_4\ e_6\}$ $\quad$ $Q_2 = \{e_1\ e_4\ e_5\}$ $\quad$ $Q_3 = \{e_3\ e_4\ e_7\}$
$Q_4 = \{e_1\ e_2\ e_3\}$ $\quad$ $Q_5 = \{e_2\ e_5\ e_7\}$ $\quad$ $Q_6 = \{e_1\ e_6\ e_7\}$ $\quad$ $Q_7 = \{e_3\ e_5\ e_6\}$

I will use the following composite structure constant s_n understandably and contextually separable from s_{abc} where n is a single integer in the range 0 to 15 rather than a triplet of single digit indexes. Index n will be an odd integer if the product history count of variant products (parity) is odd, and an even integer if the parity is even. Dividing n by two with truncation gives either 0 representing no triplet orientation rule in play, or the triplet index for Q as above. This succinctly gives us the Octonion Algebra orientation variation sieve results. The result s_0 implies an algebraic invariant, and need not be shown. The other 15 will optimally index the algebraic variant partitions.

Take n for s_n to be a 4-bit binary number. We can use the common computer language Boolean logic operator ^ for exclusive-or to determine the product $s_a\ s_b = s_c$. *Only* because our Quaternion triplets were partitioned with the index exclusive-or to zero scheme, we enumerated them as Q_i above, and agreed to reduce the +1 orientation of the Quaternion subalgebra structure constants s_{abc} to s_n, we have a very simple computer language friendly representation for the product of two composite structure constants: $s_a\ s_b = s_{a\wedge b}$.

Define potential functions in our bases of interest here as $\mathbf{A}(\mathbf{u}) = A_i\ e_i$ or $\mathbf{A}(\mathbf{v}) = A_i\ g_i$. Since Octonion Algebra is not generally commutative, we need to define left and right physics type fields in any algebraic basis as

$\mathbf{F}_L = \mathbf{E}(\mathbf{A})$
$\mathbf{F}_R = (\mathbf{A})\mathbf{E}$

Just as with Electrodynamics, we neither need nor want any scalar content within these, so we must remove the scalar content produced, but without implying it sums to zero since this would unfavorably collide with the analogous Lorentz Condition discussed below.

It will be more illuminating if we use the intrinsic basis del operator algebraic element here simply as $e_n\ \nabla_n$ understanding it will be form invariant with any global algebraic basis gauge transformation defining the del operator partial derivatives appropriate for the position algebraic element in the g basis.

$\mathbf{F}_L =$
$\{+\nabla_0(A_1) + \nabla_1(A_0) + (\nabla_5(A_4) - \nabla_4(A_5))\, s_5 + (\nabla_2(A_3) - \nabla_3(A_2))\, s_9 + (\nabla_7(A_6) - \nabla_6(A_7))\, s_{13}\}\, e_1$
$\{+\nabla_0(A_2) + \nabla_2(A_0) + (\nabla_6(A_4) - \nabla_4(A_6))\, s_3 + (\nabla_3(A_1) - \nabla_1(A_3))\, s_9 + (\nabla_5(A_7) - \nabla_7(A_5))\, s_{11}\}\, e_2$
$\{+\nabla_0(A_3) + \nabla_3(A_0) + (\nabla_7(A_4) - \nabla_4(A_7))\, s_7 + (\nabla_1(A_2) - \nabla_2(A_1))\, s_9 + (\nabla_6(A_5) - \nabla_5(A_6))\, s_{15}\}\, e_3$
$\{+\nabla_0(A_4) + \nabla_4(A_0) + (\nabla_2(A_6) - \nabla_6(A_2))\, s_3 + (\nabla_1(A_5) - \nabla_5(A_1))\, s_5 + (\nabla_3(A_7) - \nabla_7(A_3))\, s_7\}\, e_4$
$\{+\nabla_0(A_5) + \nabla_5(A_0) + (\nabla_4(A_1) - \nabla_1(A_4))\, s_5 + (\nabla_7(A_2) - \nabla_2(A_7))\, s_{11} + (\nabla_3(A_6) - \nabla_6(A_3))\, s_{15}\}\, e_5$
$\{+\nabla_0(A_6) + \nabla_6(A_0) + (\nabla_4(A_2) - \nabla_2(A_4))\, s_3 + (\nabla_1(A_7) - \nabla_7(A_1))\, s_{13} + (\nabla_5(A_3) - \nabla_3(A_5))\, s_{15}\}\, e_6$
$\{+\nabla_0(A_7) + \nabla_7(A_0) + (\nabla_4(A_3) - \nabla_3(A_4))\, s_7 + (\nabla_2(A_5) - \nabla_5(A_2))\, s_{11} + (\nabla_6(A_1) - \nabla_1(A_6))\, s_{13}\}\, e_7$

$\mathbf{F}_R$ is simply $\mathbf{F}_L$ with all algebra structure constants s_j replaced with $-s_j$, essentially negating all rotational physics fields. In terms of our Octonion Algebra presentation, irrotational forms are not subjected to ordered permutation triplet multiplication rules, and thus are algebraic invariants. Each of the rotational forms within parentheses are defined using the rules of the same ordered permutation triplet, so are algebraic variants.

The proper form for 8-current in any algebraic basis is (ref [1] et.al.)

$\mathbf{j} = ½ \{ \mathbf{E}(\mathbf{F}_L) + (\mathbf{F}_R)\mathbf{E} \}$

Within the intrinsic e basis, the result is an algebraic invariant as expected, since it is an observable.

$\mathbf{j} =$

$- \{- \nabla^2_0(A_0) + \nabla^2_1(A_0) + \nabla^2_2(A_0) + \nabla^2_3(A_0) + \nabla^2_4(A_0) + \nabla^2_5(A_0) + \nabla^2_6(A_0) + \nabla^2_7(A_0)\}\, e_0$
$- \nabla_0\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_0$

$- \{- \nabla^2_0(A_1) + \nabla^2_1(A_1) + \nabla^2_2(A_1) + \nabla^2_3(A_1) + \nabla^2_4(A_1) + \nabla^2_5(A_1) + \nabla^2_6(A_1) + \nabla^2_7(A_1)\}\, e_1$
$+ \nabla_1\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_1$

$- \{- \nabla^2_0(A_2) + \nabla^2_1(A_2) + \nabla^2_2(A_2) + \nabla^2_3(A_2) + \nabla^2_4(A_2) + \nabla^2_5(A_2) + \nabla^2_6(A_2) + \nabla^2_7(A_2)\}\, e_2$
$+ \nabla_2\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_2$

$- \{- \nabla^2_0(A_3) + \nabla^2_1(A_3) + \nabla^2_2(A_3) + \nabla^2_3(A_3) + \nabla^2_4(A_3) + \nabla^2_5(A_3) + \nabla^2_6(A_3) + \nabla^2_7(A_3)\}\, e_3$
$+ \nabla_3\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_3$

$- \{- \nabla^2_0(A_4) + \nabla^2_1(A_4) + \nabla^2_2(A_4) + \nabla^2_3(A_4) + \nabla^2_4(A_4) + \nabla^2_5(A_4) + \nabla^2_6(A_4) + \nabla^2_7(A_4)\}\, e_4$
$+ \nabla_4\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_4$

$- \{- \nabla^2_0(A_5) + \nabla^2_1(A_5) + \nabla^2_2(A_5) + \nabla^2_3(A_5) + \nabla^2_4(A_5) + \nabla^2_5(A_5) + \nabla^2_6(A_5) + \nabla^2_7(A_5)\}\, e_5$
$+ \nabla_5\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_5$

$- \{- \nabla^2_0(A_6) + \nabla^2_1(A_6) + \nabla^2_2(A_6) + \nabla^2_3(A_6) + \nabla^2_4(A_6) + \nabla^2_5(A_6) + \nabla^2_6(A_6) + \nabla^2_7(A_6)\}\, e_6$
$+ \nabla_6\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_6$

$- \{- \nabla^2_0(A_7) + \nabla^2_1(A_7) + \nabla^2_2(A_7) + \nabla^2_3(A_7) + \nabla^2_4(A_7) + \nabla^2_5(A_7) + \nabla^2_6(A_7) + \nabla^2_7(A_7)\}\, e_7$
$+ \nabla_7\{\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7)\}\, e_7$

The analogous Lorentz Condition is

$\nabla_0(A_0) + \nabla_1(A_1) + \nabla_1(A_1) + \nabla_2(A_2) + \nabla_3(A_3) + \nabla_4(A_4) + \nabla_5(A_5) + \nabla_6(A_6) + \nabla_7(A_7) = 0$

Just as it does for 4D classical Electrodynamics, assigning a value of zero to the analogous Lorentz Condition uncouples the **j** component dependence on multiple components of the 8-potential, allowing each 8-current dimension to be produced solely from the Octonion D'Alembertian operating on only the same dimension's coefficient of the 8-potential. Later, when we derive the conservation equations, we will keep the analogous Lorentz condition non-zero and in place, since these equations do not require it to be 0. Moreover, making it zero will just complicate things.

Now is the perfect time to dive into the Octonion form for Maxwell's Equations. We can simplify the presentation by mapping our differentiated potential function field forms to singular field types representing the irrotational and rotational fields. Define the full complement of each as

$\mathbf{I} = \sum_{k=1 \text{ to } 7} (-\nabla_k A_0 - \nabla_0 A_k)\, e_k$ 7 irrotational field components
$\mathbf{R} = \sum_{r=1 \text{ to } 6,\ s=r+1 \text{ to } 7} s_{rs(r\hat{}s)} (\nabla_r A_s - \nabla_s A_r)\, e_{r\hat{}s}$ 21 rotational field component indexes oriented $+R_{uv}\, e_{u\hat{}v}$

We can then write the left physics field as

$\mathbf{F}_L =$
$\{-I_1 + R_{54}\, s_5 + R_{23}\, s_9 + R_{76}\, s_{13}\}\, e_1$
$\{-I_2 + R_{64}\, s_3 + R_{31}\, s_9 + R_{57}\, s_{11}\}\, e_2$
$\{-I_3 + R_{74}\, s_7 + R_{12}\, s_9 + R_{65}\, s_{15}\}\, e_3$
$\{-I_4 + R_{26}\, s_3 + R_{15}\, s_5 + R_{37}\, s_7\}\, e_4$
$\{-I_5 + R_{41}\, s_5 + R_{72}\, s_{11} + R_{36}\, s_{15}\}\, e_5$
$\{-I_6 + R_{42}\, s_3 + R_{17}\, s_{13} + R_{53}\, s_{15}\}\, e_6$
$\{-I_7 + R_{43}\, s_7 + R_{25}\, s_{11} + R_{61}\, s_{13}\}\, e_7$

$\mathbf{F}_R$ again is $\mathbf{F}_L$ with all variant terms negated. From these form $\mathbf{j} = \frac{1}{2}\{\, \mathbf{E}(\mathbf{F}_L) + (\mathbf{F}_R)\mathbf{E}\,\}$.

$\mathbf{j} =$

$\{+\nabla_1(I_1) + \nabla_2(I_2) + \nabla_3(I_3) + \nabla_4(I_4) + \nabla_5(I_5) + \nabla_6(I_6) + \nabla_7(I_7)\}\, e_0$

$\{-\nabla_0(I_1) + \nabla_2(R_{12}) - \nabla_3(R_{31}) - \nabla_4(R_{41}) + \nabla_5(R_{15}) - \nabla_6(R_{61}) + \nabla_7(R_{17})\}\, e_1$
$\{-\nabla_3(R_{57}) + \nabla_5(R_{37}) + \nabla_7(R_{53})\}\, s_2\, e_1$
$\{+\nabla_2(R_{65}) + \nabla_5(R_{26}) - \nabla_6(R_{25})\}\, s_6\, e_1$
$\{-\nabla_3(R_{64}) - \nabla_4(R_{36}) - \nabla_6(R_{43})\}\, s_{10}\, e_1$
$\{+\nabla_2(R_{74}) - \nabla_4(R_{72}) + \nabla_7(R_{42})\}\, s_{14}\, e_1$

$\{-\nabla_0(I_2) - \nabla_1(R_{12}) + \nabla_3(R_{23}) - \nabla_4(R_{42}) + \nabla_5(R_{25}) + \nabla_6(R_{26}) - \nabla_7(R_{72})\}\, e_2$
$\{+\nabla_3(R_{76}) + \nabla_6(R_{37}) - \nabla_7(R_{36})\}\, s_4\, e_2$
$\{-\nabla_1(R_{65}) + \nabla_5(R_{61}) + \nabla_6(R_{15})\}\, s_6\, e_2$
$\{+\nabla_3(R_{54}) - \nabla_4(R_{53}) + \nabla_5(R_{43})\}\, s_{12}\, e_2$
$\{-\nabla_1(R_{74}) - \nabla_4(R_{17}) - \nabla_7(R_{41})\}\, s_{14}\, e_2$

$\{-\nabla_0(I_3) + \nabla_1(R_{31}) - \nabla_2(R_{23}) - \nabla_4(R_{43}) - \nabla_5(R_{53}) + \nabla_6(R_{36}) + \nabla_7(R_{37})\}\, e_3$
$\{+\nabla_1(R_{57}) - \nabla_5(R_{17}) + \nabla_7(R_{15})\}\, s_2\, e_3$
$\{-\nabla_2(R_{76}) + \nabla_6(R_{72}) + \nabla_7(R_{26})\}\, s_4\, e_3$
$\{+\nabla_1(R_{64}) - \nabla_4(R_{61}) + \nabla_6(R_{41})\}\, s_{10}\, e_3$
$\{-\nabla_2(R_{54}) - \nabla_4(R_{25}) - \nabla_5(R_{42})\}\, s_{12}\, e_3$

$\{-\nabla_0(I_4) + \nabla_1(R_{41}) + \nabla_2(R_{42}) + \nabla_3(R_{43}) - \nabla_5(R_{54}) - \nabla_6(R_{64}) - \nabla_7(R_{74})\}\, e_4$
$\{-\nabla_5(R_{76}) - \nabla_6(R_{57}) - \nabla_7(R_{65})\}\, s_8\, e_4$

$\{+\nabla_1(R_{36}) + \nabla_3(R_{61}) - \nabla_6(R_{31})\}\ s_{10}\ e_4$
$\{+\nabla_2(R_{53}) + \nabla_3(R_{25}) - \nabla_5(R_{23})\}\ s_{12}\ e_4$
$\{+\nabla_1(R_{72}) + \nabla_2(R_{17}) - \nabla_7(R_{12})\}\ s_{14}\ e_4$

$\{-\nabla_0(I_5) - \nabla_1(R_{15}) - \nabla_2(R_{25}) + \nabla_3(R_{53}) + \nabla_4(R_{54}) - \nabla_6(R_{65}) + \nabla_7(R_{57})\}\ e_5$
$\{-\nabla_1(R_{37}) + \nabla_3(R_{17}) + \nabla_7(R_{31})\}\ s_2\ e_5$
$\{-\nabla_1(R_{26}) - \nabla_2(R_{61}) - \nabla_6(R_{12})\}\ s_6\ e_5$
$\{+\nabla_4(R_{76}) - \nabla_6(R_{74}) + \nabla_7(R_{64})\}\ s_8\ e_5$
$\{-\nabla_2(R_{43}) + \nabla_3(R_{42}) + \nabla_4(R_{23})\}\ s_{12}\ e_5$

$\{-\nabla_0(I_6) + \nabla_1(R_{61}) - \nabla_2(R_{26}) - \nabla_3(R_{36}) + \nabla_4(R_{64}) + \nabla_5(R_{65}) - \nabla_7(R_{76})\}\ e_6$
$\{-\nabla_2(R_{37}) - \nabla_3(R_{72}) - \nabla_7(R_{23})\}\ s_4\ e_6$
$\{+\nabla_1(R_{25}) - \nabla_2(R_{15}) + \nabla_5(R_{12})\}\ s_6\ e_6$
$\{+\nabla_4(R_{57}) + \nabla_5(R_{74}) - \nabla_7(R_{54})\}\ s_8\ e_6$
$\{+\nabla_1(R_{43}) - \nabla_3(R_{41}) + \nabla_4(R_{31})\}\ s_{10}\ e_6$

$\{-\nabla_0(I_7) - \nabla_1(R_{17}) + \nabla_2(R_{72}) - \nabla_3(R_{37}) + \nabla_4(R_{74}) - \nabla_5(R_{57}) + \nabla_6(R_{76})\}\ e_7$
$\{-\nabla_1(R_{53}) - \nabla_3(R_{15}) - \nabla_5(R_{31})\}\ s_2\ e_7$
$\{+\nabla_2(R_{36}) - \nabla_3(R_{26}) + \nabla_6(R_{23})\}\ s_4\ e_7$
$\{+\nabla_4(R_{65}) - \nabla_5(R_{64}) + \nabla_6(R_{54})\}\ s_8\ e_7$
$\{-\nabla_1(R_{42}) + \nabla_2(R_{41}) + \nabla_4(R_{12})\}\ s_{14}\ e_7$

When we determined the 8-current above using potential functions, we found it to be an algebraic invariant. We now have variant terms when operating on rotational/irrotational field types as shown, where sets of terms scaling the same basis element and having the same algebraic variance are collected. Reverting each of these variant sets to their potential function form, we find each identically sums to zero as required. Rather than having to go looking for these in the full complement of differentiated product terms with a priori knowledge, by structuring the Octonion math in an algebraic orientation covariant fashion, we need only simply collect (sieve out) sets of terms with common variance and basis element, then examine what Octonion Algebra *tells us to look at*. This is a simple example of the utility and manifest importance of maintaining algebraic orientation covariance.

It is interesting to point out after understanding $R_{ij} = -R_{ji}$ each set is of the form $\{+\nabla_a(R_{bc}) + \nabla_b(R_{ca}) + \nabla_c(R_{ab})\} = 0$, a cyclic shift of indexes. If we desire to simplify the presentation by continuing with singularly stated field types, we must remember each of these identities and additional similar forms that for they will be needed to simplify much of what follows. Removing them, we have

$\mathbf{j} =$
$\{+\nabla_1(I_1) + \nabla_2(I_2) + \nabla_3(I_3) + \nabla_4(I_4) + \nabla_5(I_5) + \nabla_6(I_6) + \nabla_7(I_7)\}\ e_0 \quad = j_0\ e_0$
$\{-\nabla_0(I_1) + \nabla_2(R_{12}) - \nabla_3(R_{31}) - \nabla_4(R_{41}) + \nabla_5(R_{15}) - \nabla_6(R_{61}) + \nabla_7(R_{17})\}\ e_1 \quad = j_1\ e_1$
$\{-\nabla_0(I_2) - \nabla_1(R_{12}) + \nabla_3(R_{23}) - \nabla_4(R_{42}) + \nabla_5(R_{25}) + \nabla_6(R_{26}) - \nabla_7(R_{72})\}\ e_2 \quad = j_2\ e_2$
$\{-\nabla_0(I_3) + \nabla_1(R_{31}) - \nabla_2(R_{23}) - \nabla_4(R_{43}) - \nabla_5(R_{53}) + \nabla_6(R_{36}) + \nabla_7(R_{37})\}\ e_3 \quad = j_3\ e_3$
$\{-\nabla_0(I_4) + \nabla_1(R_{41}) + \nabla_2(R_{42}) + \nabla_3(R_{43}) - \nabla_5(R_{54}) - \nabla_6(R_{64}) - \nabla_7(R_{74})\}\ e_4 \quad = j_4\ e_4$
$\{-\nabla_0(I_5) - \nabla_1(R_{15}) - \nabla_2(R_{25}) + \nabla_3(R_{53}) + \nabla_4(R_{54}) - \nabla_6(R_{65}) + \nabla_7(R_{57})\}\ e_5 \quad = j_5\ e_5$
$\{-\nabla_0(I_6) + \nabla_1(R_{61}) - \nabla_2(R_{26}) - \nabla_3(R_{36}) + \nabla_4(R_{64}) + \nabla_5(R_{65}) - \nabla_7(R_{76})\}\ e_6 \quad = j_6\ e_6$
$\{-\nabla_0(I_7) - \nabla_1(R_{17}) + \nabla_2(R_{72}) - \nabla_3(R_{37}) + \nabla_4(R_{74}) - \nabla_5(R_{57}) + \nabla_6(R_{76})\}\ e_7 \quad = j_7\ e_7$

This is the result form for the two inhomogeneous Octonion Maxwell's Equations. We just need to cast the left side in something more directly applicable to the recognizable 4D Electrodynamics form. The familiar form of Maxwell's Equations are specified in terms of the magnetic (rotational) field and electric (irrotational) field, so it would make sense to split our Octonion fields along those lines. This is easily done with our left and right field definitions:

$\mathbf{F}_{irr} = -\frac{1}{2} \{\mathbf{F}_L + \mathbf{F}_R\}$	$\mathbf{F}_{rot} = \frac{1}{2} \{\mathbf{F}_L - \mathbf{F}_R\}$
$+I_1\, e_1$	$\{+R_{54}\, s_5 + R_{23}\, s_9 + R_{76}\, s_{13}\}\, e_1$
$+I_2\, e_2$	$\{+R_{64}\, s_3 + R_{31}\, s_9 + R_{57}\, s_{11}\}\, e_2$
$+I_3\, e_3$	$\{+R_{74}\, s_7 + R_{12}\, s_9 + R_{65}\, s_{15}\}\, e_3$
$+I_4\, e_4$	$\{+R_{26}\, s_3 + R_{15}\, s_5 + R_{37}\, s_7\}\, e_4$
$+I_5\, e_5$	$\{+R_{41}\, s_5 + R_{72}\, s_{11} + R_{36}\, s_{15}\}\, e_5$
$+I_6\, e_6$	$\{+R_{42}\, s_3 + R_{17}\, s_{13} + R_{53}\, s_{15}\}\, e_6$
$+I_7\, e_7$	$\{+R_{43}\, s_7 + R_{25}\, s_{11} + R_{61}\, s_{13}\}\, e_7$

The Octonion divergence and curl of these are

$$\nabla \cdot \mathbf{F}_{irr} = \sum_{k=1 \text{ to } 7} \nabla_k\, F_{irr\, k}\, e_0$$
$$\nabla \cdot \mathbf{F}_{rot} = \sum_{k=1 \text{ to } 7} \nabla_k\, F_{rot\, k}\, e_0$$
$$\nabla \times \mathbf{F}_{irr} = \sum_{r=1 \text{ to } 6,\ s=r+1 \text{ to } 7} s_{rs(r\wedge s)}\, (\nabla_r\, F_{irr\, s} - \nabla_s\, F_{irr\, r})\, e_{r\wedge s}$$
$$\nabla \times \mathbf{F}_{rot} = \sum_{r=1 \text{ to } 6,\ s=r+1 \text{ to } 7} s_{rs(r\wedge s)}\, (\nabla_r\, F_{rot\, s} - \nabla_s\, F_{rot\, r})\, e_{r\wedge s}$$

We find

$$\nabla \cdot \mathbf{F}_{irr} = \{+\nabla_1(I_1) + \nabla_2(I_2) + \nabla_3(I_3) + \nabla_4(I_4) + \nabla_5(I_5) + \nabla_6(I_6) + \nabla_7(I_7)\}\, e_0$$

This is equal to $j_0\, e_0$ above so we have our first of four Octonion Maxwell's equations $\nabla \cdot \mathbf{F}_{irr} = j_0\, e_0$. Next, we have after grouping by common algebraic variance

$\nabla \times \mathbf{F}_{rot} =$

$$\{+\nabla_2(R_{12}) - \nabla_3(R_{31}) - \nabla_4(R_{41}) + \nabla_5(R_{15}) - \nabla_6(R_{61}) + \nabla_7(R_{17})\}\, e_1$$
$$\{-\nabla_1(R_{12}) + \nabla_3(R_{23}) - \nabla_4(R_{42}) + \nabla_6(R_{26}) + \nabla_5(R_{25}) - \nabla_7(R_{72})\}\, e_2$$
$$\{+\nabla_1(R_{31}) - \nabla_2(R_{23}) - \nabla_4(R_{43}) + \nabla_7(R_{37}) - \nabla_5(R_{53}) + \nabla_6(R_{36})\}\, e_3$$
$$\{+\nabla_1(R_{41}) - \nabla_5(R_{54}) + \nabla_2(R_{42}) - \nabla_6(R_{64}) + \nabla_3(R_{43}) - \nabla_7(R_{74})\}\, e_4$$
$$\{-\nabla_1(R_{15}) + \nabla_4(R_{54}) - \nabla_2(R_{25}) + \nabla_7(R_{57}) + \nabla_3(R_{53}) - \nabla_6(R_{65})\}\, e_5$$
$$\{+\nabla_1(R_{61}) - \nabla_7(R_{76}) - \nabla_2(R_{26}) + \nabla_4(R_{64}) - \nabla_3(R_{36}) + \nabla_5(R_{65})\}\, e_6$$
$$\{-\nabla_1(R_{17}) + \nabla_6(R_{76}) + \nabla_2(R_{72}) - \nabla_5(R_{57}) - \nabla_3(R_{37}) + \nabla_4(R_{74})\}\, e_7$$

$$\{-\nabla_3(R_{57}) + \nabla_5(R_{37}) + \nabla_7(R_{53})\}\, s_2\, e_1$$
$$\{+\nabla_1(R_{57}) + \nabla_7(R_{15}) - \nabla_5(R_{17})\}\, s_2\, e_3$$
$$\{-\nabla_1(R_{37}) + \nabla_7(R_{31}) + \nabla_3(R_{17})\}\, s_2\, e_5$$
$$\{-\nabla_1(R_{53}) - \nabla_5(R_{31}) - \nabla_3(R_{15})\}\, s_2\, e_7$$

$$\{+\nabla_3(R_{76}) + \nabla_6(R_{37}) - \nabla_7(R_{36})\}\, s_4\, e_2$$
$$\{-\nabla_2(R_{76}) + \nabla_7(R_{26}) + \nabla_6(R_{72})\}\, s_4\, e_3$$
$$\{-\nabla_7(R_{23}) - \nabla_2(R_{37}) - \nabla_3(R_{72})\}\, s_4\, e_6$$
$$\{+\nabla_6(R_{23}) + \nabla_2(R_{36}) - \nabla_3(R_{26})\}\, s_4\, e_7$$

$$\{+\nabla_2(R_{65}) + \nabla_5(R_{26}) - \nabla_6(R_{25})\}\, s_6\, e_1$$
$$\{-\nabla_1(R_{65}) + \nabla_6(R_{15}) + \nabla_5(R_{61})\}\, s_6\, e_2$$
$$\{-\nabla_1(R_{26}) - \nabla_2(R_{61}) - \nabla_6(R_{12})\}\, s_6\, e_5$$
$$\{+\nabla_1(R_{25}) - \nabla_2(R_{15}) + \nabla_5(R_{12})\}\, s_6\, e_6$$

$\{-\nabla_5(R_{76}) - \nabla_6(R_{57}) - \nabla_7(R_{65})\}\ s_8\ e_4$
$\{+\nabla_4(R_{76}) + \nabla_7(R_{64}) - \nabla_6(R_{74})\}\ s_8\ e_5$
$\{-\nabla_7(R_{54}) + \nabla_4(R_{57}) + \nabla_5(R_{74})\}\ s_8\ e_6$
$\{+\nabla_6(R_{54}) - \nabla_5(R_{64}) + \nabla_4(R_{65})\}\ s_8\ e_7$

$\{-\nabla_3(R_{64}) - \nabla_4(R_{36}) - \nabla_6(R_{43})\}\ s_{10}\ e_1$
$\{+\nabla_1(R_{64}) - \nabla_4(R_{61}) + \nabla_6(R_{41})\}\ s_{10}\ e_3$
$\{+\nabla_1(R_{36}) - \nabla_6(R_{31}) + \nabla_3(R_{61})\}\ s_{10}\ e_4$
$\{+\nabla_1(R_{43}) + \nabla_4(R_{31}) - \nabla_3(R_{41})\}\ s_{10}\ e_6$

$\{+\nabla_3(R_{54}) - \nabla_4(R_{53}) + \nabla_5(R_{43})\}\ s_{12}\ e_2$
$\{-\nabla_2(R_{54}) - \nabla_4(R_{25}) - \nabla_5(R_{42})\}\ s_{12}\ e_3$
$\{-\nabla_5(R_{23}) + \nabla_2(R_{53}) + \nabla_3(R_{25})\}\ s_{12}\ e_4$
$\{+\nabla_4(R_{23}) - \nabla_2(R_{43}) + \nabla_3(R_{42})\}\ s_{12}\ e_5$

$\{+\nabla_2(R_{74}) - \nabla_4(R_{72}) + \nabla_7(R_{42})\}\ s_{14}\ e_1$
$\{-\nabla_1(R_{74}) - \nabla_4(R_{17}) - \nabla_7(R_{41})\}\ s_{14}\ e_2$
$\{+\nabla_1(R_{72}) + \nabla_2(R_{17}) - \nabla_7(R_{12})\}\ s_{14}\ e_4$
$\{-\nabla_1(R_{42}) + \nabla_2(R_{41}) + \nabla_4(R_{12})\}\ s_{14}\ e_7$

We notice all variant-basis terms are the same as we found above with the vector side of **j**, all of which were seen to be identically zero when written as differentiations of the potential functions. We are left with

$\boldsymbol{\nabla} \times \mathbf{F}_{rot} =$
$\{+\nabla_2(R_{12}) - \nabla_3(R_{31}) - \nabla_4(R_{41}) + \nabla_5(R_{15}) - \nabla_6(R_{61}) + \nabla_7(R_{17})\}\ e_1$
$\{-\nabla_1(R_{12}) + \nabla_3(R_{23}) - \nabla_4(R_{42}) + \nabla_6(R_{26}) + \nabla_5(R_{25}) - \nabla_7(R_{72})\}\ e_2$
$\{+\nabla_1(R_{31}) - \nabla_2(R_{23}) - \nabla_4(R_{43}) + \nabla_7(R_{37}) - \nabla_5(R_{53}) + \nabla_6(R_{36})\}\ e_3$
$\{+\nabla_1(R_{41}) - \nabla_5(R_{54}) + \nabla_2(R_{42}) - \nabla_6(R_{64}) + \nabla_3(R_{43}) - \nabla_7(R_{74})\}\ e_4$
$\{-\nabla_1(R_{15}) + \nabla_4(R_{54}) - \nabla_2(R_{25}) + \nabla_7(R_{57}) + \nabla_3(R_{53}) - \nabla_6(R_{65})\}\ e_5$
$\{+\nabla_1(R_{61}) - \nabla_7(R_{76}) - \nabla_2(R_{26}) + \nabla_4(R_{64}) - \nabla_3(R_{36}) + \nabla_5(R_{65})\}\ e_6$
$\{-\nabla_1(R_{17}) + \nabla_6(R_{76}) + \nabla_2(R_{72}) - \nabla_5(R_{57}) - \nabla_3(R_{37}) + \nabla_4(R_{74})\}\ e_7$

We have for the time differential of $\mathbf{F}_{irr}$

$+\nabla_0(I_1)\ e_1 + \nabla_0(I_2)\ e_2 + \nabla_0(I_3)\ e_3 + \nabla_0(I_4)\ e_4 + \nabla_0(I_5)\ e_5 + \nabla_0(I_6)\ e_6 + \nabla_0(I_7)\ e_7$

Subtracting from $\boldsymbol{\nabla} \times \mathbf{F}_{rot}$ gives our match with the non-scalar portion of our 8-current

$\boldsymbol{\nabla} \times \mathbf{F}_{rot} - \nabla_0(\mathbf{F}_{irr}) = \mathbf{j}$ (non-scalar)

So the two inhomogeneous Octonion Maxwell's Equations are

$\boldsymbol{\nabla} \cdot \mathbf{F}_{irr} = j_0\ e_0$ $\qquad$ $\boldsymbol{\nabla} \times \mathbf{F}_{rot} - \nabla_0(\mathbf{F}_{irr}) = \mathbf{j}$ (non-scalar)

The two homogeneous standard Maxwell's Equations are simple vector identities once the proper potential function connection was made for the electric and magnetic fields. We should then expect the same in the Octonion framework. Writing out the divergence

$\boldsymbol{\nabla} \cdot \mathbf{F}_{rot} =$
$\{+\nabla_2(R_{64}) + \nabla_4(R_{26}) + \nabla_6(R_{42})\}\ s_3\ e_0$
$\{+\nabla_1(R_{54}) + \nabla_4(R_{15}) + \nabla_5(R_{41})\}\ s_5\ e_0$

$\{+\nabla_3(R_{74}) + \nabla_4(R_{37}) + \nabla_7(R_{43})\}\ s_7\ e_0$
$\{+\nabla_1(R_{23}) + \nabla_2(R_{31}) + \nabla_3(R_{12})\}\ s_9\ e_0$
$\{+\nabla_2(R_{57}) + \nabla_5(R_{72}) + \nabla_7(R_{25})\}\ s_{11}\ e_0$
$\{+\nabla_1(R_{76}) + \nabla_6(R_{17}) + \nabla_7(R_{61})\}\ s_{13}\ e_0$
$\{+\nabla_3(R_{65}) + \nabla_5(R_{36}) + \nabla_6(R_{53})\}\ s_{15}\ e_0$

These are similar to the 28 forms we have already encountered that identically sum to zero when expressed in the potential function form, except the three indexes in each are now those of Quaternion subalgebra triplets. These seven also identically sum to zero when represented in potential function form, and complete the 35 possible sets of three triplets of different indexes taken from 1 through 7 that each will sum to zero. Generally, we will have for any $R_{rs} = \nabla_r A_s - \nabla_s A_r$ and $u \neq v \neq w$ cyclically shifting u, v and w

$\nabla_u R_{vw} + \nabla_v R_{wu} + \nabla_w R_{uv} = 0$

So our third Octonion Maxwell's equations gives $\mathbf{\nabla} \cdot \mathbf{F}_{rot} = 0$ as expected, but in seven separate identically 0 segments, each handed to us by utilizing algebraic orientation covariance. For our final Maxwell's equation

$\mathbf{\nabla} \times \mathbf{F}_{irr} =$
$\{-\nabla_4(I_5)s_5 + \nabla_5(I_4)s_5 + \nabla_2(I_3)s_9 - \nabla_3(I_2)s_9 - \nabla_6(I_7)s_{13} + \nabla_7(I_6)s_{13}\}\ e_1$
$\{-\nabla_4(I_6)s_3 + \nabla_6(I_4)s_3 - \nabla_1(I_3)s_9 + \nabla_3(I_1)s_9 + \nabla_5(I_7)s_{11} - \nabla_7(I_5)s_{11}\}\ e_2$
$\{-\nabla_4(I_7)s_7 + \nabla_7(I_4)s_7 + \nabla_1(I_2)s_9 - \nabla_2(I_1)s_9 - \nabla_5(I_6)s_{15} + \nabla_6(I_5)s_{15}\}\ e_3$
$\{+\nabla_2(I_6)s_3 - \nabla_6(I_2)s_3 + \nabla_1(I_5)s_5 - \nabla_5(I_1)s_5 + \nabla_3(I_7)s_7 - \nabla_7(I_3)s_7\}\ e_4$
$\{-\nabla_1(I_4)s_5 + \nabla_4(I_1)s_5 - \nabla_2(I_7)s_{11} + \nabla_7(I_2)s_{11} + \nabla_3(I_6)s_{15} - \nabla_6(I_3)s_{15}\}\ e_5$
$\{-\nabla_2(I_4)s_3 + \nabla_4(I_2)s_3 + \nabla_1(I_7)s_{13} - \nabla_7(I_1)s_{13} - \nabla_3(I_5)s_{15} + \nabla_5(I_3)s_{15}\}\ e_6$
$\{-\nabla_3(I_4)s_7 + \nabla_4(I_3)s_7 + \nabla_2(I_5)s_{11} - \nabla_5(I_2)s_{11} - \nabla_1(I_6)s_{13} + \nabla_6(I_1)s_{13}\}\ e_7$

$\nabla_0(\mathbf{F}_{rot}) =$
$\{+\nabla_0(R_{54})s_5 + \nabla_0(R_{23})s_9 + \nabla_0(R_{76})s_{13}\}\ e_1$
$\{+\nabla_0(R_{64})s_3 + \nabla_0(R_{31})s_9 + \nabla_0(R_{57})s_{11}\}\ e_2$
$\{+\nabla_0(R_{74})s_7 + \nabla_0(R_{12})s_9 + \nabla_0(R_{65})s_{15}\}\ e_3$
$\{+\nabla_0(R_{26})s_3 + \nabla_0(R_{15})s_5 + \nabla_0(R_{37})s_7\}\ e_4$
$\{+\nabla_0(R_{41})s_5 + \nabla_0(R_{72})s_{11} + \nabla_0(R_{36})s_{15}\}\ e_5$
$\{+\nabla_0(R_{42})s_3 + \nabla_0(R_{17})s_{13} + \nabla_0(R_{53})s_{15}\}\ e_6$
$\{+\nabla_0(R_{43})s_7 + \nabla_0(R_{25})s_{11} + \nabla_0(R_{61})s_{13}\}\ e_7$

Adding both of these to form our final Octonion Maxwell's Equation, then grouping by common variance-basis we have

$\mathbf{\nabla} \times \mathbf{F}_{irr} + \nabla_0(\mathbf{F}_{rot}) =$

$\{-\nabla_4(I_6) + \nabla_6(I_4) + \nabla_0(R_{64})\}\ s_3\ e_2$
$\{+\nabla_2(I_6) - \nabla_6(I_2) + \nabla_0(R_{26})\}\ s_3\ e_4$
$\{-\nabla_2(I_4) + \nabla_4(I_2) + \nabla_0(R_{42})\}\ s_3\ e_6$

$\{-\nabla_4(I_5) + \nabla_5(I_4) + \nabla_0(R_{54})\}\ s_5\ e_1$
$\{+\nabla_1(I_5) - \nabla_5(I_1) + \nabla_0(R_{15})\}\ s_5\ e_4$
$\{-\nabla_1(I_4) + \nabla_4(I_1) + \nabla_0(R_{41})\}\ s_5\ e_5$

$\{-\nabla_4(I_7) + \nabla_7(I_4) + \nabla_0(R_{74})\}\ s_7\ e_3$
$\{+\nabla_3(I_7) - \nabla_7(I_3) + \nabla_0(R_{37})\}\ s_7\ e_4$

$\{-\nabla_3(I_4) + \nabla_4(I_3) + \nabla_0(R_{43})\}\ s_7\ e_7$

$\{+\nabla_2(I_3) - \nabla_3(I_2) + \nabla_0(R_{23})\}\ s_9\ e_1$
$\{-\nabla_1(I_3) + \nabla_3(I_1) + \nabla_0(R_{31})\}\ s_9\ e_2$
$\{+\nabla_1(I_2) - \nabla_2(I_1) + \nabla_0(R_{12})\}\ s_9\ e_3$

$\{+\nabla_5(I_7) - \nabla_7(I_5) + \nabla_0(R_{57})\}\ s_{11}\ e_2$
$\{-\nabla_2(I_7) + \nabla_7(I_2) + \nabla_0(R_{72})\}\ s_{11}\ e_5$
$\{+\nabla_2(I_5) - \nabla_5(I_2) + \nabla_0(R_{25})\}\ s_{11}\ e_7$

$\{-\nabla_6(I_7) + \nabla_7(I_6) + \nabla_0(R_{76})\}\ s_{13}\ e_1$
$\{+\nabla_1(I_7) - \nabla_7(I_1) + \nabla_0(R_{17})\}\ s_{13}\ e_6$
$\{-\nabla_1(I_6) + \nabla_6(I_1) + \nabla_0(R_{61})\}\ s_{13}\ e_7$

$\{-\nabla_5(I_6) + \nabla_6(I_5) + \nabla_0(R_{65})\}\ s_{15}\ e_3$
$\{+\nabla_3(I_6) - \nabla_6(I_3) + \nabla_0(R_{36})\}\ s_{15}\ e_5$
$\{-\nabla_3(I_5) + \nabla_5(I_3) + \nabla_0(R_{53})\}\ s_{15}\ e_6$

The result is 21 separate variance-basis combinations, one for each of our 21 rotational R_{uv}, Each is identically 0 once we return to the potential function differentiation form, verifying $\mathbf{\nabla} \times \mathbf{F}_{irr} + \nabla_0(\mathbf{F}_{rot}) = 0$.

Now that we have finished the Octonion Algebra form for Maxwell's Equations, expressing the Octonion equivalent of complex conjugation $e_0 \rightarrow e_0$ and $e_n \rightarrow -e_n$ for $n \neq 0$ with an underscore we can write

$\underline{\mathbf{E}}(\mathbf{A}(\mathbf{v})) = 1/J\ \partial/\partial v_i\ [\ C_{ij}\ \ T_{kl}\ \ A_k\ ]\ \ \underline{e}_j * e_l$ equivalently in the g basis $= \partial/\partial v_i\ [\ A_k\ \underline{g}_i * g_k\ ]$
$\mathbf{E}(\underline{\mathbf{A}}(\mathbf{v})) = 1/J\ \partial/\partial v_i\ [\ C_{ij}\ \ T_{kl}\ \ A_k\ ]\ \ e_j * \underline{e}_l$ equivalently in the g basis $= \partial/\partial v_i\ [\ A_k\ g_i * \underline{g}_k\ ]$
$(\mathbf{A}(\mathbf{v}))\underline{\mathbf{E}} = 1/J\ \partial/\partial v_i\ [\ C_{ij}\ \ T_{kl}\ \ A_k\ ]\ \ e_l * \underline{e}_j$ equivalently in the g basis $= \partial/\partial v_i\ [\ A_k\ g_k * \underline{g}_i\ ]$
$(\underline{\mathbf{A}}(\mathbf{v}))\mathbf{E} = 1/J\ \partial/\partial v_i\ [\ C_{ij}\ \ T_{kl}\ \ A_k\ ]\ \ \underline{e}_l * e_j$ equivalently in the g basis $= \partial/\partial v_i\ [\ A_k\ \underline{g}_k * g_i\ ]$

The proper form in any basis for the continuity equation expressing the conservation of 8-charge can be seen to be

Scalar $\underline{\mathbf{E}}(\mathbf{j}) = 0 =$ scalar $\mathbf{E}(\underline{\mathbf{j}}) =$ scalar $(\mathbf{j})\underline{\mathbf{E}} =$ scalar $(\underline{\mathbf{j}})\mathbf{E}$

For **E** and **j** represented in the intrinsic basis or form invariant global gauge, these equivalent continuity equations hold identically, independent of any particular choice for the potential functions. We must require the continuity equation holds in any basis, it therefore must also be a local algebraic basis gauge invariant.

The physics fields have algebraically variant content, and the 8-current is an algebraic invariant. Products of the two will have variant content. Since force and work are observables, we must limit our 8–work–force expression to the invariant content of such products. The proper covariant form for the Octonion 8–work–force is found to be the content of the following that does not change when the Octonion Algebra orientation is changed up (reference [1] et.al.)

$\mathbf{wf} = -\ \frac{1}{2}\ \{\ \mathbf{j} * \mathbf{F}_R + \mathbf{F}_L * \mathbf{j}\ \}$

Working again within the intrinsic e basis or global algebraic basis gauge transformation g basis, **j** is a simple native algebraic element expressible as $\mathbf{j} = j_n\ e_n$ or $j_n\ g_n$ respectively. **wf** can then be more simply written in the intrinsic e basis as follows:

Invariant$(-\ \frac{1}{2}\ \{\ \mathbf{j} * \mathbf{F}_R + \mathbf{F}_L * \mathbf{j}\ \}) =$

$\{ +j_1 (\nabla_0(A_1) + \nabla_1(A_0)) + j_2 (\nabla_0(A_2) + \nabla_2(A_0)) + j_3 (\nabla_0(A_3) + \nabla_3(A_0)) + j_4 (\nabla_0(A_4) + \nabla_4(A_0))$
$+ j_5 (\nabla_0(A_5) + \nabla_5(A_0)) + j_6 (\nabla_0(A_6) + \nabla_6(A_0)) + j_7 (\nabla_0(A_7) + \nabla_7(A_0)) \} e_0$

$\{$
$+ j_0 (- \nabla_0(A_1) - \nabla_1(A_0))$
$+ j_2 (\nabla_1(A_2) - \nabla_2(A_1)) - j_3 (\nabla_3(A_1) - \nabla_1(A_3))$
$+ j_5 (\nabla_1(A_5) - \nabla_5(A_1)) - j_4 (\nabla_4(A_1) - \nabla_1(A_4))$
$+ j_7 (\nabla_1(A_7) - \nabla_7(A_1)) - j_6 (\nabla_6(A_1) - \nabla_1(A_6))$
$\} e_1$

$\{$
$+ j_0 \{- \nabla_0(A_2) - \nabla_2(A_0))$
$+ j_3 (\nabla_2(A_3) - \nabla_3(A_2)) - j_1 (\nabla_1(A_2) - \nabla_2(A_1))$
$+ j_6 (\nabla_2(A_6) - \nabla_6(A_2)) - j_4 (\nabla_4(A_2) - \nabla_2(A_4))$
$+ j_5 (\nabla_2(A_5) - \nabla_5(A_2)) - j_7 (\nabla_7(A_2) - \nabla_2(A_7))$
$\} e_2$

$\{$
$+ j_0 (-\nabla_0(A_3) - \nabla_3(A_0))$
$+ j_1 (\nabla_3(A_1) - \nabla_1(A_3)) - j_2 (\nabla_2(A_3) - \nabla_3(A_2))$
$+ j_7 (\nabla_3(A_7) - \nabla_7(A_3)) - j_4 (\nabla_4(A_3) - \nabla_3(A_4))$
$+ j_6 (\nabla_3(A_6) - \nabla_6(A_3)) - j_5 (\nabla_5(A_3) - \nabla_3(A_5))$
$\} e_3$

$\{$
$+ j_0 (-\nabla_0(A_4) - \nabla_4(A_0))$
$+ j_1 (\nabla_4(A_1) - \nabla_1(A_4)) - j_5 (\nabla_5(A_4) - \nabla_4(A_5))$
$+ j_2 (\nabla_4(A_2) - \nabla_2(A_4)) - j_6 (\nabla_6(A_4) - \nabla_4(A_6))$
$+ j_3 (\nabla_4(A_3) - \nabla_3(A_4)) - j_7 (\nabla_7(A_4) - \nabla_4(A_7))$
$\} e_4$

$\{$
$+ j_0 (-\nabla_0(A_5) - \nabla_5(A_0))$
$+ j_4 (\nabla_5(A_4) - \nabla_4(A_5)) - j_1 (\nabla_1(A_5) - \nabla_5(A_1))$
$+ j_7 (\nabla_5(A_7) - \nabla_7(A_5)) - j_2 (\nabla_2(A_5) - \nabla_5(A_2))$
$+ j_3 (\nabla_5(A_3) - \nabla_3(A_5)) - j_6 (\nabla_6(A_5) - \nabla_5(A_6))$
$\} e_5$

$\{$
$+ j_0 (-\nabla_0(A_6) - \nabla_6(A_0))$
$+ j_1 (\nabla_6(A_1) - \nabla_1(A_6)) - j_7 (\nabla_7(A_6) - \nabla_6(A_7))$
$+ j_4 (\nabla_6(A_4) - \nabla_4(A_6)) - j_2 (\nabla_2(A_6) - \nabla_6(A_2))$
$+ j_5 (\nabla_6(A_5) - \nabla_5(A_6)) - j_3 (\nabla_3(A_6) - \nabla_6(A_3))$
$\} e_6$

$\{$
$+ j_0 (-\nabla_0(A_7) - \nabla_7(A_0))$
$+ j_6 (\nabla_7(A_6) - \nabla_6(A_7)) - j_1 (\nabla_1(A_7) - \nabla_7(A_1))$
$+ j_2 (\nabla_7(A_2) - \nabla_2(A_7)) - j_5 (\nabla_5(A_7) - \nabla_7(A_5))$
$+ j_4 (\nabla_7(A_4) - \nabla_4(A_7)) - j_3 (\nabla_3(A_7) - \nabla_7(A_3))$

$\} e_7$

If **j** was expressed in terms of its full potential function form, each of these algebraic element coefficients would have 196 product terms, and it would be tough to see the forest for the trees so to speak. Further simplification specifying in terms of the irrotational and rotational field representations used above we have

$\{-j_1I_1 - j_2I_2 - j_3I_3 - j_4I_4 - j_5I_5 - j_6I_6 - j_7I_7\} e_0$
$\{+j_0I_1 + j_2R_{12} - j_3R_{31} + j_5R_{15} - j_4R_{41} + j_7R_{17} - j_6R_{61}\} e_1$
$\{+j_0I_2 + j_3R_{23} - j_1R_{12} + j_6R_{26} - j_4R_{42} + j_5R_{25} - j_7R_{72}\} e_2$
$\{+j_0I_3 + j_1R_{31} - j_2R_{23} + j_7R_{37} - j_4R_{43} + j_6R_{36} - j_5R_{53}\} e_3$
$\{+j_0I_4 + j_1R_{41} - j_5R_{54} + j_2R_{42} - j_6R_{64} + j_3R_{43} - j_7R_{74}\} e_4$
$\{+j_0I_5 + j_4R_{54} - j_1R_{15} + j_3R_{53} - j_6R_{65} + j_7R_{57} - j_2R_{25}\} e_5$
$\{+j_0I_6 + j_1R_{61} - j_7R_{76} + j_4R_{64} - j_2R_{26} + j_5R_{65} - j_3R_{36}\} e_6$
$\{+j_0I_7 + j_6R_{76} - j_1R_{17} + j_2R_{72} - j_5R_{57} + j_4R_{74} - j_3R_{37}\} e_7$

Taking a clue from Electrodynamics, we see the scalar e_0 term is the negated inner product of the non-scalar 8–current and the irrotational 8–field analogous to EM negated scalar product of charge current and electric field, representing work. The non-scalar terms are the analogous Lorentz force; 8–charge density j_0 scaling the irrotational portion of the 8–field, and the Octonion cross product of non-scalar 8–current density and the rotational portion of the 8–field.

The algebraic variant portions of **wf** follow, likewise simplified by using field type components:

wf Variance s_2
$\{-j_3R_{57} + j_5R_{37} + j_7R_{53}\} s_2 e_1$
$\{+j_1R_{57} + j_7R_{15} - j_5R_{17}\} s_2 e_3$
$\{+j_7R_{31} - j_1R_{37} + j_3R_{17}\} s_2 e_5$
$\{-j_5R_{31} - j_3R_{15} - j_1R_{53}\} s_2 e_7$

wf Variance s_4
$\{+j_3R_{76} + j_6R_{37} - j_7R_{36}\} s_4 e_2$
$\{-j_2R_{76} + j_7R_{26} + j_6R_{72}\} s_4 e_3$
$\{-j_7R_{23} - j_2R_{37} - j_3R_{72}\} s_4 e_6$
$\{+j_6R_{23} - j_3R_{26} + j_2R_{36}\} s_4 e_7$

wf Variance s_6
$\{+j_2R_{65} + j_5R_{26} - j_6R_{25}\} s_6 e_1$
$\{-j_1R_{65} + j_6R_{15} + j_5R_{61}\} s_6 e_2$
$\{-j_6R_{12} - j_1R_{26} - j_2R_{61}\} s_6 e_5$
$\{+j_5R_{12} - j_2R_{15} + j_1R_{25}\} s_6 e_6$

wf Variance s_8
$\{-j_5R_{76} - j_6R_{57} - j_7R_{65}\} s_8 e_4$
$\{+j_4R_{76} + j_7R_{64} - j_6R_{74}\} s_8 e_5$
$\{-j_7R_{54} + j_4R_{57} + j_5R_{74}\} s_8 e_6$
$\{+j_6R_{54} - j_5R_{64} + j_4R_{65}\} s_8 e_7$

wf Variance s_{10}
$\{-j_3R_{64} - j_4R_{36} - j_6R_{43}\} s_{10} e_1$
$\{+j_1R_{64} + j_6R_{41} - j_4R_{61}\} s_{10} e_3$
$\{-j_6R_{31} + j_1R_{36} + j_3R_{61}\} s_{10} e_4$
$\{+j_4R_{31} - j_3R_{41} + j_1R_{43}\} s_{10} e_6$

wf Variance s_{12}
$\{+j_3R_{54} - j_4R_{53} + j_5R_{43}\}\ s_{12}\ e_2$
$\{-j_2R_{54} - j_5R_{42} - j_4R_{25}\}\ s_{12}\ e_3$
$\{-j_5R_{23} + j_2R_{53} + j_3R_{25}\}\ s_{12}\ e_4$
$\{+j_4R_{23} + j_3R_{42} - j_2R_{43}\}\ s_{12}\ e_5$

wf Variance s_{14}
$\{+j_2R_{74} - j_4R_{72} + j_7R_{42}\}\ s_{14}\ e_1$
$\{-j_1R_{74} - j_7R_{41} - j_4R_{17}\}\ s_{14}\ e_2$
$\{-j_7R_{12} + j_1R_{72} + j_2R_{17}\}\ s_{14}\ e_4$
$\{+j_4R_{12} + j_2R_{41} - j_1R_{42}\}\ s_{14}\ e_7$

All odd variances are null sets. If we were to keep within the philosophy that all observables are algebraic invariants, the fact that this variant section is a portion of a meaningful observable, we could assert the terms in any given variance set scaling a single basis element sum to zero. I call these homogeneous equations of algebraic constraint. If we take the conclusions found in references [1][2] et.al. that the polar basis the electric field lives in is the set $\{ e_5\ e_6\ e_7 \}$ and axial basis the magnetic field lives in is the set $\{ e_1\ e_2\ e_3 \}$, from **wf** Variance s_8, requiring the sum $\{-j_5R_{76} - j_6R_{57} - j_7R_{65}\}$ to equal zero would be a statement that the magnetic field is orthogonal to the electric charge current, a reasonable restriction.

Enabling us to form conservation equations by rewriting Invariant(**wf**) with an outside differentiation on all terms would be a monumental task if we did not have clues from Electrodynamics, and our requirement the form must be an algebraic invariant to match Invariant(**wf**) as stated. To bring this difficulty home, for grins, and in the interest of full disclosure, it is worth some document space to show the full differentiated potential function form for Invariant(**wf**):

$\{$

$$+\nabla_0(A_1)\nabla^2_0(A_1) +\nabla_0\nabla_1(A_0)\nabla_0(A_1) +\nabla_1(A_0)\nabla^2_0(A_1) +\nabla_1(A_0)\nabla_0\nabla_1(A_0) +\nabla_0(A_2)\nabla^2_0(A_2) +\nabla_0\nabla_2(A_0)\nabla_0(A_2)$$
$$+\nabla_2(A_0)\nabla^2_0(A_2) +\nabla_2(A_0)\nabla_0\nabla_2(A_0) +\nabla_0(A_3)\nabla^2_0(A_3) +\nabla_0\nabla_3(A_0)\nabla_0(A_3) +\nabla_3(A_0)\nabla^2_0(A_3) +\nabla_3(A_0)\nabla_0\nabla_3(A_0)$$
$$+\nabla_0(A_4)\nabla^2_0(A_4) +\nabla_0\nabla_4(A_0)\nabla_0(A_4) +\nabla_4(A_0)\nabla^2_0(A_4) +\nabla_4(A_0)\nabla_0\nabla_4(A_0) +\nabla_0(A_5)\nabla^2_0(A_5) +\nabla_0\nabla_5(A_0)\nabla_0(A_5)$$
$$+\nabla_5(A_0)\nabla^2_0(A_5) +\nabla_5(A_0)\nabla_0\nabla_5(A_0) +\nabla_0(A_6)\nabla^2_0(A_6) +\nabla_0\nabla_6(A_0)\nabla_0(A_6) +\nabla_6(A_0)\nabla^2_0(A_6) +\nabla_6(A_0)\nabla_0\nabla_6(A_0)$$
$$+\nabla_0(A_7)\nabla^2_0(A_7) +\nabla_0\nabla_7(A_0)\nabla_0(A_7) +\nabla_7(A_0)\nabla^2_0(A_7) +\nabla_7(A_0)\nabla_0\nabla_7(A_0) -\nabla_2(A_0)\nabla^2_1(A_2) +\nabla_2(A_0)\nabla_1\nabla_2(A_1)$$
$$-\nabla_0(A_2)\nabla^2_1(A_2) +\nabla_1\nabla_2(A_1)\nabla_0(A_2) -\nabla_3(A_0)\nabla^2_1(A_3) +\nabla_3(A_0)\nabla_1\nabla_3(A_1) -\nabla_0(A_3)\nabla^2_1(A_3) +\nabla_1\nabla_3(A_1)\nabla_0(A_3)$$
$$-\nabla_4(A_0)\nabla^2_1(A_4) +\nabla_4(A_0)\nabla_1\nabla_4(A_1) -\nabla_0(A_4)\nabla^2_1(A_4) +\nabla_1\nabla_4(A_1)\nabla_0(A_4) -\nabla_5(A_0)\nabla^2_1(A_5) +\nabla_5(A_0)\nabla_1\nabla_5(A_1)$$
$$-\nabla_0(A_5)\nabla^2_1(A_5) +\nabla_1\nabla_5(A_1)\nabla_0(A_5) -\nabla_6(A_0)\nabla^2_1(A_6) +\nabla_6(A_0)\nabla_1\nabla_6(A_1) -\nabla_0(A_6)\nabla^2_1(A_6) +\nabla_1\nabla_6(A_1)\nabla_0(A_6)$$
$$-\nabla_7(A_0)\nabla^2_1(A_7) +\nabla_7(A_0)\nabla_1\nabla_7(A_1) -\nabla_0(A_7)\nabla^2_1(A_7) +\nabla_1\nabla_7(A_1)\nabla_0(A_7) -\nabla_1(A_0)\nabla^2_2(A_1) +\nabla_1(A_0)\nabla_1\nabla_2(A_2)$$
$$-\nabla_0(A_1)\nabla^2_2(A_1) +\nabla_0(A_1)\nabla_1\nabla_2(A_2) -\nabla_3(A_0)\nabla^2_2(A_3) +\nabla_3(A_0)\nabla_2\nabla_3(A_2) -\nabla_0(A_3)\nabla^2_2(A_3) +\nabla_2\nabla_3(A_2)\nabla_0(A_3)$$
$$-\nabla_4(A_0)\nabla^2_2(A_4) +\nabla_4(A_0)\nabla_2\nabla_4(A_2) -\nabla_0(A_4)\nabla^2_2(A_4) +\nabla_2\nabla_4(A_2)\nabla_0(A_4) -\nabla_5(A_0)\nabla^2_2(A_5) +\nabla_5(A_0)\nabla_2\nabla_5(A_2)$$
$$-\nabla_0(A_5)\nabla^2_2(A_5) +\nabla_2\nabla_5(A_2)\nabla_0(A_5) -\nabla_6(A_0)\nabla^2_2(A_6) +\nabla_6(A_0)\nabla_2\nabla_6(A_2) -\nabla_0(A_6)\nabla^2_2(A_6) +\nabla_2\nabla_6(A_2)\nabla_0(A_6)$$
$$-\nabla_7(A_0)\nabla^2_2(A_7) +\nabla_7(A_0)\nabla_2\nabla_7(A_2) -\nabla_0(A_7)\nabla^2_2(A_7) +\nabla_2\nabla_7(A_2)\nabla_0(A_7) -\nabla_1(A_0)\nabla^2_3(A_1) +\nabla_1(A_0)\nabla_1\nabla_3(A_3)$$
$$-\nabla_0(A_1)\nabla^2_3(A_1) +\nabla_0(A_1)\nabla_1\nabla_3(A_3) -\nabla_2(A_0)\nabla^2_3(A_2) +\nabla_2(A_0)\nabla_2\nabla_3(A_3) -\nabla_0(A_2)\nabla^2_3(A_2) +\nabla_0(A_2)\nabla_2\nabla_3(A_3)$$
$$-\nabla_4(A_0)\nabla^2_3(A_4) +\nabla_4(A_0)\nabla_3\nabla_4(A_3) -\nabla_0(A_4)\nabla^2_3(A_4) +\nabla_3\nabla_4(A_3)\nabla_0(A_4) -\nabla_5(A_0)\nabla^2_3(A_5) +\nabla_5(A_0)\nabla_3\nabla_5(A_3)$$
$$-\nabla_0(A_5)\nabla^2_3(A_5) +\nabla_3\nabla_5(A_3)\nabla_0(A_5) -\nabla_6(A_0)\nabla^2_3(A_6) +\nabla_6(A_0)\nabla_3\nabla_6(A_3) -\nabla_0(A_6)\nabla^2_3(A_6) +\nabla_3\nabla_6(A_3)\nabla_0(A_6)$$
$$-\nabla_7(A_0)\nabla^2_3(A_7) +\nabla_7(A_0)\nabla_3\nabla_7(A_3) -\nabla_0(A_7)\nabla^2_3(A_7) +\nabla_3\nabla_7(A_3)\nabla_0(A_7) -\nabla_1(A_0)\nabla^2_4(A_1) +\nabla_1(A_0)\nabla_1\nabla_4(A_4)$$
$$-\nabla_0(A_1)\nabla^2_4(A_1) +\nabla_0(A_1)\nabla_1\nabla_4(A_4) -\nabla_2(A_0)\nabla^2_4(A_2) +\nabla_2(A_0)\nabla_2\nabla_4(A_4) -\nabla_0(A_2)\nabla^2_4(A_2) +\nabla_0(A_2)\nabla_2\nabla_4(A_4)$$
$$-\nabla_3(A_0)\nabla^2_4(A_3) +\nabla_3(A_0)\nabla_3\nabla_4(A_4) -\nabla_0(A_3)\nabla^2_4(A_3) +\nabla_0(A_3)\nabla_3\nabla_4(A_4) -\nabla_5(A_0)\nabla^2_4(A_5) +\nabla_5(A_0)\nabla_4\nabla_5(A_4)$$
$$-\nabla_0(A_5)\nabla^2_4(A_5) +\nabla_4\nabla_5(A_4)\nabla_0(A_5) -\nabla_6(A_0)\nabla^2_4(A_6) +\nabla_6(A_0)\nabla_4\nabla_6(A_4) -\nabla_0(A_6)\nabla^2_4(A_6) +\nabla_4\nabla_6(A_4)\nabla_0(A_6)$$
$$-\nabla_7(A_0)\nabla^2_4(A_7) +\nabla_7(A_0)\nabla_4\nabla_7(A_4) -\nabla_0(A_7)\nabla^2_4(A_7) +\nabla_4\nabla_7(A_4)\nabla_0(A_7) -\nabla_1(A_0)\nabla^2_5(A_1) +\nabla_1(A_0)\nabla_1\nabla_5(A_5)$$
$$-\nabla_0(A_1)\nabla^2_5(A_1) +\nabla_0(A_1)\nabla_1\nabla_5(A_5) -\nabla_2(A_0)\nabla^2_5(A_2) +\nabla_2(A_0)\nabla_2\nabla_5(A_5) -\nabla_0(A_2)\nabla^2_5(A_2) +\nabla_0(A_2)\nabla_2\nabla_5(A_5)$$
$$-\nabla_3(A_0)\nabla^2_5(A_3) +\nabla_3(A_0)\nabla_3\nabla_5(A_5) -\nabla_0(A_3)\nabla^2_5(A_3) +\nabla_0(A_3)\nabla_3\nabla_5(A_5) -\nabla_4(A_0)\nabla^2_5(A_4) +\nabla_4(A_0)\nabla_4\nabla_5(A_5)$$

$-\nabla_0(A_4)\nabla^2_5(A_4) +\nabla_0(A_4)\nabla_4\nabla_5(A_5) -\nabla_6(A_0)\nabla^2_5(A_6) +\nabla_6(A_0)\nabla_5\nabla_6(A_5) -\nabla_0(A_6)\nabla^2_5(A_6) +\nabla_5\nabla_6(A_5)\nabla_0(A_6)$
$-\nabla_7(A_0)\nabla^2_5(A_7) +\nabla_7(A_0)\nabla_5\nabla_7(A_5) -\nabla_0(A_7)\nabla^2_5(A_7) +\nabla_5\nabla_7(A_5)\nabla_0(A_7) -\nabla_1(A_0)\nabla^2_6(A_1) +\nabla_1(A_0)\nabla_1\nabla_6(A_6)$
$-\nabla_0(A_1)\nabla^2_6(A_1) +\nabla_0(A_1)\nabla_1\nabla_6(A_6) -\nabla_2(A_0)\nabla^2_6(A_2) +\nabla_2(A_0)\nabla_2\nabla_6(A_6) -\nabla_0(A_2)\nabla^2_6(A_2) +\nabla_0(A_2)\nabla_2\nabla_6(A_6)$
$-\nabla_3(A_0)\nabla^2_6(A_3) +\nabla_3(A_0)\nabla_3\nabla_6(A_6) -\nabla_0(A_3)\nabla^2_6(A_3) +\nabla_0(A_3)\nabla_3\nabla_6(A_6) -\nabla_4(A_0)\nabla^2_6(A_4) +\nabla_4(A_0)\nabla_4\nabla_6(A_6)$
$-\nabla_0(A_4)\nabla^2_6(A_4) +\nabla_0(A_4)\nabla_4\nabla_6(A_6) -\nabla_5(A_0)\nabla^2_6(A_5) +\nabla_5(A_0)\nabla_5\nabla_6(A_6) -\nabla_0(A_5)\nabla^2_6(A_5) +\nabla_0(A_5)\nabla_5\nabla_6(A_6)$
$-\nabla_7(A_0)\nabla^2_6(A_7) +\nabla_7(A_0)\nabla_6\nabla_7(A_6) -\nabla_0(A_7)\nabla^2_6(A_7) +\nabla_6\nabla_7(A_6)\nabla_0(A_7) -\nabla_1(A_0)\nabla^2_7(A_1) +\nabla_1(A_0)\nabla_1\nabla_7(A_7)$
$-\nabla_0(A_1)\nabla^2_7(A_1) +\nabla_0(A_1)\nabla_1\nabla_7(A_7) -\nabla_2(A_0)\nabla^2_7(A_2) +\nabla_2(A_0)\nabla_2\nabla_7(A_7) -\nabla_0(A_2)\nabla^2_7(A_2) +\nabla_0(A_2)\nabla_2\nabla_7(A_7)$
$-\nabla_3(A_0)\nabla^2_7(A_3) +\nabla_3(A_0)\nabla_3\nabla_7(A_7) -\nabla_0(A_3)\nabla^2_7(A_3) +\nabla_0(A_3)\nabla_3\nabla_7(A_7) -\nabla_4(A_0)\nabla^2_7(A_4) +\nabla_4(A_0)\nabla_4\nabla_7(A_7)$
$-\nabla_0(A_4)\nabla^2_7(A_4) +\nabla_0(A_4)\nabla_4\nabla_7(A_7) -\nabla_5(A_0)\nabla^2_7(A_5) +\nabla_5(A_0)\nabla_5\nabla_7(A_7) -\nabla_0(A_5)\nabla^2_7(A_5) +\nabla_0(A_5)\nabla_5\nabla_7(A_7)$
$-\nabla_6(A_0)\nabla^2_7(A_6) +\nabla_6(A_0)\nabla_6\nabla_7(A_7) -\nabla_0(A_6)\nabla^2_7(A_6) +\nabla_0(A_6)\nabla_6\nabla_7(A_7)$
$\} e_0$

$\{$
$+\nabla_0\nabla_2(A_0)\nabla_1(A_2) -\nabla_0\nabla_2(A_0)\nabla_2(A_1) +\nabla_1(A_2)\nabla^2_0(A_2) -\nabla_2(A_1)\nabla^2_0(A_2) +\nabla_0\nabla_3(A_0)\nabla_1(A_3) -\nabla_0\nabla_3(A_0)\nabla_3(A_1)$
$+\nabla_1(A_3)\nabla^2_0(A_3) -\nabla_3(A_1)\nabla^2_0(A_3) +\nabla_0\nabla_4(A_0)\nabla_1(A_4) -\nabla_0\nabla_4(A_0)\nabla_4(A_1) +\nabla_1(A_4)\nabla^2_0(A_4) -\nabla_4(A_1)\nabla^2_0(A_4)$
$+\nabla_0\nabla_5(A_0)\nabla_1(A_5) -\nabla_0\nabla_5(A_0)\nabla_5(A_1) +\nabla_1(A_5)\nabla^2_0(A_5) -\nabla_5(A_1)\nabla^2_0(A_5) +\nabla_0\nabla_6(A_0)\nabla_1(A_6) -\nabla_0\nabla_6(A_0)\nabla_6(A_1)$
$+\nabla_1(A_6)\nabla^2_0(A_6) -\nabla_6(A_1)\nabla^2_0(A_6) +\nabla_0\nabla_7(A_0)\nabla_1(A_7) -\nabla_0\nabla_7(A_0)\nabla_7(A_1) +\nabla_1(A_7)\nabla^2_0(A_7) -\nabla_7(A_1)\nabla^2_0(A_7)$
$-\nabla_1(A_2)\nabla^2_1(A_2) +\nabla_1\nabla_2(A_1)\nabla_1(A_2) +\nabla_2(A_1)\nabla^2_1(A_2) -\nabla_2(A_1)\nabla_1\nabla_2(A_1) -\nabla_1(A_3)\nabla^2_1(A_3) +\nabla_1\nabla_3(A_1)\nabla_1(A_3)$
$+\nabla_3(A_1)\nabla^2_1(A_3) -\nabla_3(A_1)\nabla_1\nabla_3(A_1) -\nabla_1(A_4)\nabla^2_1(A_4) +\nabla_1\nabla_4(A_1)\nabla_1(A_4) +\nabla_4(A_1)\nabla^2_1(A_4) -\nabla_4(A_1)\nabla_1\nabla_4(A_1)$
$-\nabla_1(A_5)\nabla^2_1(A_5) +\nabla_1\nabla_5(A_1)\nabla_1(A_5) +\nabla_5(A_1)\nabla^2_1(A_5) -\nabla_5(A_1)\nabla_1\nabla_5(A_1) -\nabla_1(A_6)\nabla^2_1(A_6) +\nabla_1\nabla_6(A_1)\nabla_1(A_6)$
$+\nabla_6(A_1)\nabla^2_1(A_6) -\nabla_6(A_1)\nabla_1\nabla_6(A_1) -\nabla_1(A_7)\nabla^2_1(A_7) +\nabla_1\nabla_7(A_1)\nabla_1(A_7) +\nabla_7(A_1)\nabla^2_1(A_7) -\nabla_7(A_1)\nabla_1\nabla_7(A_1)$
$+\nabla_0(A_1)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_1) +\nabla_1(A_0)\nabla_0\nabla_1(A_1) +\nabla_1(A_0)\nabla^2_1(A_0) +\nabla_2\nabla_3(A_2)\nabla_1(A_3) -\nabla_1(A_3)\nabla^2_2(A_3)$
$-\nabla_3(A_1)\nabla_2\nabla_3(A_2) +\nabla_3(A_1)\nabla^2_2(A_3) +\nabla_2\nabla_4(A_2)\nabla_1(A_4) -\nabla_1(A_4)\nabla^2_2(A_4) -\nabla_4(A_1)\nabla_2\nabla_4(A_2) +\nabla_4(A_1)\nabla^2_2(A_4)$
$+\nabla_2\nabla_5(A_2)\nabla_1(A_5) -\nabla_1(A_5)\nabla^2_2(A_5) -\nabla_5(A_1)\nabla_2\nabla_5(A_2) +\nabla_5(A_1)\nabla^2_2(A_5) +\nabla_2\nabla_6(A_2)\nabla_1(A_6) -\nabla_1(A_6)\nabla^2_2(A_6)$
$-\nabla_6(A_1)\nabla_2\nabla_6(A_2) +\nabla_6(A_1)\nabla^2_2(A_6) +\nabla_2\nabla_7(A_2)\nabla_1(A_7) -\nabla_1(A_7)\nabla^2_2(A_7) -\nabla_7(A_1)\nabla_2\nabla_7(A_2) +\nabla_7(A_1)\nabla^2_2(A_7)$
$+\nabla_1(A_0)\nabla^2_2(A_0) +\nabla_1(A_0)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_1) +\nabla_0(A_1)\nabla_0\nabla_2(A_2) +\nabla_1(A_2)\nabla_2\nabla_3(A_3) -\nabla_1(A_2)\nabla^2_3(A_2)$
$-\nabla_2(A_1)\nabla_2\nabla_3(A_3) +\nabla_2(A_1)\nabla^2_3(A_2) +\nabla_3\nabla_4(A_3)\nabla_1(A_4) -\nabla_1(A_4)\nabla^2_3(A_4) -\nabla_4(A_1)\nabla_3\nabla_4(A_3) +\nabla_4(A_1)\nabla^2_3(A_4)$
$+\nabla_3\nabla_5(A_3)\nabla_1(A_5) -\nabla_1(A_5)\nabla^2_3(A_5) -\nabla_5(A_1)\nabla_3\nabla_5(A_3) +\nabla_5(A_1)\nabla^2_3(A_5) +\nabla_3\nabla_6(A_3)\nabla_1(A_6) -\nabla_1(A_6)\nabla^2_3(A_6)$
$-\nabla_6(A_1)\nabla_3\nabla_6(A_3) +\nabla_6(A_1)\nabla^2_3(A_6) +\nabla_3\nabla_7(A_3)\nabla_1(A_7) -\nabla_1(A_7)\nabla^2_3(A_7) -\nabla_7(A_1)\nabla_3\nabla_7(A_3) +\nabla_7(A_1)\nabla^2_3(A_7)$
$+\nabla_1(A_0)\nabla^2_3(A_0) +\nabla_1(A_0)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_1) +\nabla_0(A_1)\nabla_0\nabla_3(A_3) +\nabla_1(A_2)\nabla_2\nabla_4(A_4) -\nabla_1(A_2)\nabla^2_4(A_2)$
$-\nabla_2(A_1)\nabla_2\nabla_4(A_4) +\nabla_2(A_1)\nabla^2_4(A_2) +\nabla_1(A_3)\nabla_3\nabla_4(A_4) -\nabla_1(A_3)\nabla^2_4(A_3) -\nabla_3(A_1)\nabla_3\nabla_4(A_4) +\nabla_3(A_1)\nabla^2_4(A_3)$
$+\nabla_4\nabla_5(A_4)\nabla_1(A_5) -\nabla_1(A_5)\nabla^2_4(A_5) -\nabla_5(A_1)\nabla_4\nabla_5(A_4) +\nabla_5(A_1)\nabla^2_4(A_5) +\nabla_4\nabla_6(A_4)\nabla_1(A_6) -\nabla_1(A_6)\nabla^2_4(A_6)$
$-\nabla_6(A_1)\nabla_4\nabla_6(A_4) +\nabla_6(A_1)\nabla^2_4(A_6) +\nabla_4\nabla_7(A_4)\nabla_1(A_7) -\nabla_1(A_7)\nabla^2_4(A_7) -\nabla_7(A_1)\nabla_4\nabla_7(A_4) +\nabla_7(A_1)\nabla^2_4(A_7)$
$+\nabla_1(A_0)\nabla^2_4(A_0) +\nabla_1(A_0)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_1) +\nabla_0(A_1)\nabla_0\nabla_4(A_4) +\nabla_1(A_2)\nabla_2\nabla_5(A_5) -\nabla_1(A_2)\nabla^2_5(A_2)$
$-\nabla_2(A_1)\nabla_2\nabla_5(A_5) +\nabla_2(A_1)\nabla^2_5(A_2) +\nabla_1(A_3)\nabla_3\nabla_5(A_5) -\nabla_1(A_3)\nabla^2_5(A_3) -\nabla_3(A_1)\nabla_3\nabla_5(A_5) +\nabla_3(A_1)\nabla^2_5(A_3)$
$+\nabla_1(A_4)\nabla_4\nabla_5(A_5) -\nabla_1(A_4)\nabla^2_5(A_4) -\nabla_4(A_1)\nabla_4\nabla_5(A_5) +\nabla_4(A_1)\nabla^2_5(A_4) +\nabla_5\nabla_6(A_5)\nabla_1(A_6) -\nabla_1(A_6)\nabla^2_5(A_6)$
$-\nabla_6(A_1)\nabla_5\nabla_6(A_5) +\nabla_6(A_1)\nabla^2_5(A_6) +\nabla_5\nabla_7(A_5)\nabla_1(A_7) -\nabla_1(A_7)\nabla^2_5(A_7) -\nabla_7(A_1)\nabla_5\nabla_7(A_5) +\nabla_7(A_1)\nabla^2_5(A_7)$
$+\nabla_1(A_0)\nabla^2_5(A_0) +\nabla_1(A_0)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_1) +\nabla_0(A_1)\nabla_0\nabla_5(A_5) +\nabla_1(A_2)\nabla_2\nabla_6(A_6) -\nabla_1(A_2)\nabla^2_6(A_2)$
$-\nabla_2(A_1)\nabla_2\nabla_6(A_6) +\nabla_2(A_1)\nabla^2_6(A_2) +\nabla_1(A_3)\nabla_3\nabla_6(A_6) -\nabla_1(A_3)\nabla^2_6(A_3) -\nabla_3(A_1)\nabla_3\nabla_6(A_6) +\nabla_3(A_1)\nabla^2_6(A_3)$
$+\nabla_1(A_4)\nabla_4\nabla_6(A_6) -\nabla_1(A_4)\nabla^2_6(A_4) -\nabla_4(A_1)\nabla_4\nabla_6(A_6) +\nabla_4(A_1)\nabla^2_6(A_4) +\nabla_1(A_5)\nabla_5\nabla_6(A_6) -\nabla_1(A_5)\nabla^2_6(A_5)$
$-\nabla_5(A_1)\nabla_5\nabla_6(A_6) +\nabla_5(A_1)\nabla^2_6(A_5) +\nabla_6\nabla_7(A_6)\nabla_1(A_7) -\nabla_1(A_7)\nabla^2_6(A_7) -\nabla_7(A_1)\nabla_6\nabla_7(A_6) +\nabla_7(A_1)\nabla^2_6(A_7)$
$+\nabla_1(A_0)\nabla^2_6(A_0) +\nabla_1(A_0)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_1) +\nabla_0(A_1)\nabla_0\nabla_6(A_6) +\nabla_1(A_2)\nabla_2\nabla_7(A_7) -\nabla_1(A_2)\nabla^2_7(A_2)$
$-\nabla_2(A_1)\nabla_2\nabla_7(A_7) +\nabla_2(A_1)\nabla^2_7(A_2) +\nabla_1(A_3)\nabla_3\nabla_7(A_7) -\nabla_1(A_3)\nabla^2_7(A_3) -\nabla_3(A_1)\nabla_3\nabla_7(A_7) +\nabla_3(A_1)\nabla^2_7(A_3)$
$+\nabla_1(A_4)\nabla_4\nabla_7(A_7) -\nabla_1(A_4)\nabla^2_7(A_4) -\nabla_4(A_1)\nabla_4\nabla_7(A_7) +\nabla_4(A_1)\nabla^2_7(A_4) +\nabla_1(A_5)\nabla_5\nabla_7(A_7) -\nabla_1(A_5)\nabla^2_7(A_5)$
$-\nabla_5(A_1)\nabla_5\nabla_7(A_7) +\nabla_5(A_1)\nabla^2_7(A_5) +\nabla_1(A_6)\nabla_6\nabla_7(A_7) -\nabla_1(A_6)\nabla^2_7(A_6) -\nabla_6(A_1)\nabla_6\nabla_7(A_7) +\nabla_6(A_1)\nabla^2_7(A_6)$
$+\nabla_1(A_0)\nabla^2_7(A_0) +\nabla_1(A_0)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_1) +\nabla_0(A_1)\nabla_0\nabla_7(A_7)$
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$+\nabla_0\nabla_1(A_0)\nabla_2(A_1) -\nabla_0\nabla_1(A_0)\nabla_1(A_2) +\nabla_2(A_1)\nabla^2_0(A_1) -\nabla^2_0(A_1)\nabla_1(A_2) +\nabla_0\nabla_3(A_0)\nabla_2(A_3) -\nabla_0\nabla_3(A_0)\nabla_3(A_2)$
$+\nabla_2(A_3)\nabla^2_0(A_3) -\nabla_3(A_2)\nabla^2_0(A_3) +\nabla_0\nabla_4(A_0)\nabla_2(A_4) -\nabla_0\nabla_4(A_0)\nabla_4(A_2) +\nabla_2(A_4)\nabla^2_0(A_4) -\nabla_4(A_2)\nabla^2_0(A_4)$
$+\nabla_0\nabla_5(A_0)\nabla_2(A_5) -\nabla_0\nabla_5(A_0)\nabla_5(A_2) +\nabla_2(A_5)\nabla^2_0(A_5) -\nabla_5(A_2)\nabla^2_0(A_5) +\nabla_0\nabla_6(A_0)\nabla_2(A_6) -\nabla_0\nabla_6(A_0)\nabla_6(A_2)$
$+\nabla_2(A_6)\nabla^2_0(A_6) -\nabla_6(A_2)\nabla^2_0(A_6) +\nabla_0\nabla_7(A_0)\nabla_2(A_7) -\nabla_0\nabla_7(A_0)\nabla_7(A_2) +\nabla_2(A_7)\nabla^2_0(A_7) -\nabla_7(A_2)\nabla^2_0(A_7)$
$+\nabla_1\nabla_3(A_1)\nabla_2(A_3) -\nabla_2(A_3)\nabla^2_1(A_3) -\nabla_1\nabla_3(A_1)\nabla_3(A_2) +\nabla_3(A_2)\nabla^2_1(A_3) +\nabla_1\nabla_4(A_1)\nabla_2(A_4) -\nabla_2(A_4)\nabla^2_1(A_4)$
$-\nabla_1\nabla_4(A_1)\nabla_4(A_2) +\nabla_4(A_2)\nabla^2_1(A_4) +\nabla_1\nabla_5(A_1)\nabla_2(A_5) -\nabla_2(A_5)\nabla^2_1(A_5) -\nabla_1\nabla_5(A_1)\nabla_5(A_2) +\nabla_5(A_2)\nabla^2_1(A_5)$
$+\nabla_1\nabla_6(A_1)\nabla_2(A_6) -\nabla_2(A_6)\nabla^2_1(A_6) -\nabla_1\nabla_6(A_1)\nabla_6(A_2) +\nabla_6(A_2)\nabla^2_1(A_6) +\nabla_1\nabla_7(A_1)\nabla_2(A_7) -\nabla_2(A_7)\nabla^2_1(A_7)$
$-\nabla_1\nabla_7(A_1)\nabla_7(A_2) +\nabla_7(A_2)\nabla^2_1(A_7) +\nabla_2(A_0)\nabla^2_1(A_0) +\nabla_2(A_0)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_2) +\nabla_0\nabla_1(A_1)\nabla_0(A_2)$
$-\nabla_1(A_2)\nabla_1\nabla_2(A_2) +\nabla^2_2(A_1)\nabla_1(A_2) +\nabla_2(A_1)\nabla_1\nabla_2(A_2) -\nabla_2(A_1)\nabla^2_2(A_1) -\nabla_2(A_3)\nabla^2_2(A_3) +\nabla_2\nabla_3(A_2)\nabla_2(A_3)$
$+\nabla_3(A_2)\nabla^2_2(A_3) -\nabla_3(A_2)\nabla_2\nabla_3(A_2) -\nabla_2(A_4)\nabla^2_2(A_4) +\nabla_2\nabla_4(A_2)\nabla_2(A_4) +\nabla_4(A_2)\nabla^2_2(A_4) -\nabla_4(A_2)\nabla_2\nabla_4(A_2)$
$-\nabla_2(A_5)\nabla^2_2(A_5) +\nabla_2\nabla_5(A_2)\nabla_2(A_5) +\nabla_5(A_2)\nabla^2_2(A_5) -\nabla_5(A_2)\nabla_2\nabla_5(A_2) -\nabla_2(A_6)\nabla^2_2(A_6) +\nabla_2\nabla_6(A_2)\nabla_2(A_6)$
$+\nabla_6(A_2)\nabla^2_2(A_6) -\nabla_6(A_2)\nabla_2\nabla_6(A_2) -\nabla_2(A_7)\nabla^2_2(A_7) +\nabla_2\nabla_7(A_2)\nabla_2(A_7) +\nabla_7(A_2)\nabla^2_2(A_7) -\nabla_7(A_2)\nabla_2\nabla_7(A_2)$
$+\nabla_0(A_2)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_2) +\nabla_2(A_0)\nabla_0\nabla_2(A_2) +\nabla_2(A_0)\nabla^2_2(A_0) +\nabla_2(A_1)\nabla_1\nabla_3(A_3) -\nabla_2(A_1)\nabla^2_3(A_1)$
$-\nabla_1(A_2)\nabla_1\nabla_3(A_3) +\nabla^2_3(A_1)\nabla_1(A_2) +\nabla_3\nabla_4(A_3)\nabla_2(A_4) -\nabla_2(A_4)\nabla^2_3(A_4) -\nabla_4(A_2)\nabla_3\nabla_4(A_3) +\nabla_4(A_2)\nabla^2_3(A_4)$
$+\nabla_3\nabla_5(A_3)\nabla_2(A_5) -\nabla_2(A_5)\nabla^2_3(A_5) -\nabla_5(A_2)\nabla_3\nabla_5(A_3) +\nabla_5(A_2)\nabla^2_3(A_5) +\nabla_3\nabla_6(A_3)\nabla_2(A_6) -\nabla_2(A_6)\nabla^2_3(A_6)$
$-\nabla_6(A_2)\nabla_3\nabla_6(A_3) +\nabla_6(A_2)\nabla^2_3(A_6) +\nabla_3\nabla_7(A_3)\nabla_2(A_7) -\nabla_2(A_7)\nabla^2_3(A_7) -\nabla_7(A_2)\nabla_3\nabla_7(A_3) +\nabla_7(A_2)\nabla^2_3(A_7)$
$+\nabla_2(A_0)\nabla^2_3(A_0) +\nabla_2(A_0)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_2) +\nabla_0(A_2)\nabla_0\nabla_3(A_3) +\nabla_2(A_1)\nabla_1\nabla_4(A_4) -\nabla_2(A_1)\nabla^2_4(A_1)$
$-\nabla_1(A_2)\nabla_1\nabla_4(A_4) +\nabla^2_4(A_1)\nabla_1(A_2) +\nabla_2(A_3)\nabla_3\nabla_4(A_4) -\nabla_2(A_3)\nabla^2_4(A_3) -\nabla_3(A_2)\nabla_3\nabla_4(A_4) +\nabla_3(A_2)\nabla^2_4(A_3)$
$+\nabla_4\nabla_5(A_4)\nabla_2(A_5) -\nabla_2(A_5)\nabla^2_4(A_5) -\nabla_5(A_2)\nabla_4\nabla_5(A_4) +\nabla_5(A_2)\nabla^2_4(A_5) +\nabla_4\nabla_6(A_4)\nabla_2(A_6) -\nabla_2(A_6)\nabla^2_4(A_6)$
$-\nabla_6(A_2)\nabla_4\nabla_6(A_4) +\nabla_6(A_2)\nabla^2_4(A_6) +\nabla_4\nabla_7(A_4)\nabla_2(A_7) -\nabla_2(A_7)\nabla^2_4(A_7) -\nabla_7(A_2)\nabla_4\nabla_7(A_4) +\nabla_7(A_2)\nabla^2_4(A_7)$
$+\nabla_2(A_0)\nabla^2_4(A_0) +\nabla_2(A_0)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_2) +\nabla_0(A_2)\nabla_0\nabla_4(A_4) +\nabla_2(A_1)\nabla_1\nabla_5(A_5) -\nabla_2(A_1)\nabla^2_5(A_1)$
$-\nabla_1(A_2)\nabla_1\nabla_5(A_5) +\nabla^2_5(A_1)\nabla_1(A_2) +\nabla_2(A_3)\nabla_3\nabla_5(A_5) -\nabla_2(A_3)\nabla^2_5(A_3) -\nabla_3(A_2)\nabla_3\nabla_5(A_5) +\nabla_3(A_2)\nabla^2_5(A_3)$
$+\nabla_2(A_4)\nabla_4\nabla_5(A_5) -\nabla_2(A_4)\nabla^2_5(A_4) -\nabla_4(A_2)\nabla_4\nabla_5(A_5) +\nabla_4(A_2)\nabla^2_5(A_4) +\nabla_5\nabla_6(A_5)\nabla_2(A_6) -\nabla_2(A_6)\nabla^2_5(A_6)$
$-\nabla_6(A_2)\nabla_5\nabla_6(A_5) +\nabla_6(A_2)\nabla^2_5(A_6) +\nabla_5\nabla_7(A_5)\nabla_2(A_7) -\nabla_2(A_7)\nabla^2_5(A_7) -\nabla_7(A_2)\nabla_5\nabla_7(A_5) +\nabla_7(A_2)\nabla^2_5(A_7)$
$+\nabla_2(A_0)\nabla^2_5(A_0) +\nabla_2(A_0)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_2) +\nabla_0(A_2)\nabla_0\nabla_5(A_5) +\nabla_2(A_1)\nabla_1\nabla_6(A_6) -\nabla_2(A_1)\nabla^2_6(A_1)$
$-\nabla_1(A_2)\nabla_1\nabla_6(A_6) +\nabla^2_6(A_1)\nabla_1(A_2) +\nabla_2(A_3)\nabla_3\nabla_6(A_6) -\nabla_2(A_3)\nabla^2_6(A_3) -\nabla_3(A_2)\nabla_3\nabla_6(A_6) +\nabla_3(A_2)\nabla^2_6(A_3)$
$+\nabla_2(A_4)\nabla_4\nabla_6(A_6) -\nabla_2(A_4)\nabla^2_6(A_4) -\nabla_4(A_2)\nabla_4\nabla_6(A_6) +\nabla_4(A_2)\nabla^2_6(A_4) +\nabla_2(A_5)\nabla_5\nabla_6(A_6) -\nabla_2(A_5)\nabla^2_6(A_5)$
$-\nabla_5(A_2)\nabla_5\nabla_6(A_6) +\nabla_5(A_2)\nabla^2_6(A_5) +\nabla_6\nabla_7(A_6)\nabla_2(A_7) -\nabla_2(A_7)\nabla^2_6(A_7) -\nabla_7(A_2)\nabla_6\nabla_7(A_6) +\nabla_7(A_2)\nabla^2_6(A_7)$
$+\nabla_2(A_0)\nabla^2_6(A_0) +\nabla_2(A_0)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_2) +\nabla_0(A_2)\nabla_0\nabla_6(A_6) +\nabla_2(A_1)\nabla_1\nabla_7(A_7) -\nabla_2(A_1)\nabla^2_7(A_1)$
$-\nabla_1(A_2)\nabla_1\nabla_7(A_7) +\nabla^2_7(A_1)\nabla_1(A_2) +\nabla_2(A_3)\nabla_3\nabla_7(A_7) -\nabla_2(A_3)\nabla^2_7(A_3) -\nabla_3(A_2)\nabla_3\nabla_7(A_7) +\nabla_3(A_2)\nabla^2_7(A_3)$
$+\nabla_2(A_4)\nabla_4\nabla_7(A_7) -\nabla_2(A_4)\nabla^2_7(A_4) -\nabla_4(A_2)\nabla_4\nabla_7(A_7) +\nabla_4(A_2)\nabla^2_7(A_4) +\nabla_2(A_5)\nabla_5\nabla_7(A_7) -\nabla_2(A_5)\nabla^2_7(A_5)$
$-\nabla_5(A_2)\nabla_5\nabla_7(A_7) +\nabla_5(A_2)\nabla^2_7(A_5) +\nabla_2(A_6)\nabla_6\nabla_7(A_7) -\nabla_2(A_6)\nabla^2_7(A_6) -\nabla_6(A_2)\nabla_6\nabla_7(A_7) +\nabla_6(A_2)\nabla^2_7(A_6)$
$+\nabla_2(A_0)\nabla^2_7(A_0) +\nabla_2(A_0)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_2) +\nabla_0(A_2)\nabla_0\nabla_7(A_7)$

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$+\nabla_0\nabla_1(A_0)\nabla_3(A_1) -\nabla_0\nabla_1(A_0)\nabla_1(A_3) +\nabla_3(A_1)\nabla^2_0(A_1) -\nabla^2_0(A_1)\nabla_1(A_3) +\nabla_0\nabla_2(A_0)\nabla_3(A_2) -\nabla_0\nabla_2(A_0)\nabla_2(A_3)$
$+\nabla_3(A_2)\nabla^2_0(A_2) -\nabla^2_0(A_2)\nabla_2(A_3) +\nabla_0\nabla_4(A_0)\nabla_3(A_4) -\nabla_0\nabla_4(A_0)\nabla_4(A_3) +\nabla_3(A_4)\nabla^2_0(A_4) -\nabla_4(A_3)\nabla^2_0(A_4)$
$+\nabla_0\nabla_5(A_0)\nabla_3(A_5) -\nabla_0\nabla_5(A_0)\nabla_5(A_3) +\nabla_3(A_5)\nabla^2_0(A_5) -\nabla_5(A_3)\nabla^2_0(A_5) +\nabla_0\nabla_6(A_0)\nabla_3(A_6) -\nabla_0\nabla_6(A_0)\nabla_6(A_3)$
$+\nabla_3(A_6)\nabla^2_0(A_6) -\nabla_6(A_3)\nabla^2_0(A_6) +\nabla_0\nabla_7(A_0)\nabla_3(A_7) -\nabla_0\nabla_7(A_0)\nabla_7(A_3) +\nabla_3(A_7)\nabla^2_0(A_7) -\nabla_7(A_3)\nabla^2_0(A_7)$
$+\nabla_1\nabla_2(A_1)\nabla_3(A_2) -\nabla_3(A_2)\nabla^2_1(A_2) -\nabla_1\nabla_2(A_1)\nabla_2(A_3) +\nabla^2_1(A_2)\nabla_2(A_3) +\nabla_1\nabla_4(A_1)\nabla_3(A_4) -\nabla_3(A_4)\nabla^2_1(A_4)$
$-\nabla_1\nabla_4(A_1)\nabla_4(A_3) +\nabla_4(A_3)\nabla^2_1(A_4) +\nabla_1\nabla_5(A_1)\nabla_3(A_5) -\nabla_3(A_5)\nabla^2_1(A_5) -\nabla_1\nabla_5(A_1)\nabla_5(A_3) +\nabla_5(A_3)\nabla^2_1(A_5)$
$+\nabla_1\nabla_6(A_1)\nabla_3(A_6) -\nabla_3(A_6)\nabla^2_1(A_6) -\nabla_1\nabla_6(A_1)\nabla_6(A_3) +\nabla_6(A_3)\nabla^2_1(A_6) +\nabla_1\nabla_7(A_1)\nabla_3(A_7) -\nabla_3(A_7)\nabla^2_1(A_7)$
$-\nabla_1\nabla_7(A_1)\nabla_7(A_3) +\nabla_7(A_3)\nabla^2_1(A_7) +\nabla_3(A_0)\nabla^2_1(A_0) +\nabla_3(A_0)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_3) +\nabla_0\nabla_1(A_1)\nabla_0(A_3)$
$+\nabla_3(A_1)\nabla_1\nabla_2(A_2) -\nabla_3(A_1)\nabla^2_2(A_1) -\nabla_1\nabla_2(A_2)\nabla_1(A_3) +\nabla^2_2(A_1)\nabla_1(A_3) +\nabla_2\nabla_4(A_2)\nabla_3(A_4) -\nabla_3(A_4)\nabla^2_2(A_4)$
$-\nabla_2\nabla_4(A_2)\nabla_4(A_3) +\nabla_4(A_3)\nabla^2_2(A_4) +\nabla_2\nabla_5(A_2)\nabla_3(A_5) -\nabla_3(A_5)\nabla^2_2(A_5) -\nabla_2\nabla_5(A_2)\nabla_5(A_3) +\nabla_5(A_3)\nabla^2_2(A_5)$
$+\nabla_2\nabla_6(A_2)\nabla_3(A_6) -\nabla_3(A_6)\nabla^2_2(A_6) -\nabla_2\nabla_6(A_2)\nabla_6(A_3) +\nabla_6(A_3)\nabla^2_2(A_6) +\nabla_2\nabla_7(A_2)\nabla_3(A_7) -\nabla_3(A_7)\nabla^2_2(A_7)$

$-\nabla_2\nabla_7(A_2)\nabla_7(A_3) +\nabla_7(A_3)\nabla^2_2(A_7) +\nabla_3(A_0)\nabla^2_2(A_0) +\nabla_3(A_0)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_3) +\nabla_0\nabla_2(A_2)\nabla_0(A_3)$
$-\nabla_1(A_3)\nabla_1\nabla_3(A_3) +\nabla^2_3(A_1)\nabla_1(A_3) +\nabla_3(A_1)\nabla_1\nabla_3(A_3) -\nabla_3(A_1)\nabla^2_3(A_1) -\nabla_2(A_3)\nabla_2\nabla_3(A_3) +\nabla^2_3(A_2)\nabla_2(A_3)$
$+\nabla_3(A_2)\nabla_2\nabla_3(A_3) -\nabla_3(A_2)\nabla^2_3(A_2) -\nabla_3(A_4)\nabla^2_3(A_4) +\nabla_3\nabla_4(A_3)\nabla_3(A_4) +\nabla_4(A_3)\nabla^2_3(A_4) -\nabla_4(A_3)\nabla_3\nabla_4(A_3)$
$-\nabla_3(A_5)\nabla^2_3(A_5) +\nabla_3\nabla_5(A_3)\nabla_3(A_5) +\nabla_5(A_3)\nabla^2_3(A_5) -\nabla_5(A_3)\nabla_3\nabla_5(A_3) -\nabla_3(A_6)\nabla^2_3(A_6) +\nabla_3\nabla_6(A_3)\nabla_3(A_6)$
$+\nabla_6(A_3)\nabla^2_3(A_6) -\nabla_6(A_3)\nabla_3\nabla_6(A_3) -\nabla_3(A_7)\nabla^2_3(A_7) +\nabla_3\nabla_7(A_3)\nabla_3(A_7) +\nabla_7(A_3)\nabla^2_3(A_7) -\nabla_7(A_3)\nabla_3\nabla_7(A_3)$
$+\nabla_0(A_3)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_3) +\nabla_3(A_0)\nabla_0\nabla_3(A_3) +\nabla_3(A_0)\nabla^2_3(A_0) +\nabla_3(A_1)\nabla_1\nabla_4(A_4) -\nabla_3(A_1)\nabla^2_4(A_1)$
$-\nabla_1(A_3)\nabla_1\nabla_4(A_4) +\nabla^2_4(A_1)\nabla_1(A_3) +\nabla_3(A_2)\nabla_2\nabla_4(A_4) -\nabla_3(A_2)\nabla^2_4(A_2) -\nabla_2(A_3)\nabla_2\nabla_4(A_4) +\nabla^2_4(A_2)\nabla_2(A_3)$
$+\nabla_4\nabla_5(A_4)\nabla_3(A_5) -\nabla_3(A_5)\nabla^2_4(A_5) -\nabla_5(A_3)\nabla_4\nabla_5(A_4) +\nabla_5(A_3)\nabla^2_4(A_5) +\nabla_4\nabla_6(A_4)\nabla_3(A_6) -\nabla_3(A_6)\nabla^2_4(A_6)$
$-\nabla_6(A_3)\nabla_4\nabla_6(A_4) +\nabla_6(A_3)\nabla^2_4(A_6) +\nabla_4\nabla_7(A_4)\nabla_3(A_7) -\nabla_3(A_7)\nabla^2_4(A_7) -\nabla_7(A_3)\nabla_4\nabla_7(A_4) +\nabla_7(A_3)\nabla^2_4(A_7)$
$+\nabla_3(A_0)\nabla^2_4(A_0) +\nabla_3(A_0)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_3) +\nabla_0(A_3)\nabla_0\nabla_4(A_4) +\nabla_3(A_1)\nabla_1\nabla_5(A_5) -\nabla_3(A_1)\nabla^2_5(A_1)$
$-\nabla_1(A_3)\nabla_1\nabla_5(A_5) +\nabla^2_5(A_1)\nabla_1(A_3) +\nabla_3(A_2)\nabla_2\nabla_5(A_5) -\nabla_3(A_2)\nabla^2_5(A_2) -\nabla_2(A_3)\nabla_2\nabla_5(A_5) +\nabla^2_5(A_2)\nabla_2(A_3)$
$+\nabla_3(A_4)\nabla_4\nabla_5(A_5) -\nabla_3(A_4)\nabla^2_5(A_4) -\nabla_4(A_3)\nabla_4\nabla_5(A_5) +\nabla_4(A_3)\nabla^2_5(A_4) +\nabla_5\nabla_6(A_5)\nabla_3(A_6) -\nabla_3(A_6)\nabla^2_5(A_6)$
$-\nabla_6(A_3)\nabla_5\nabla_6(A_5) +\nabla_6(A_3)\nabla^2_5(A_6) +\nabla_5\nabla_7(A_5)\nabla_3(A_7) -\nabla_3(A_7)\nabla^2_5(A_7) -\nabla_7(A_3)\nabla_5\nabla_7(A_5) +\nabla_7(A_3)\nabla^2_5(A_7)$
$+\nabla_3(A_0)\nabla^2_5(A_0) +\nabla_3(A_0)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_3) +\nabla_0(A_3)\nabla_0\nabla_5(A_5) +\nabla_3(A_1)\nabla_1\nabla_6(A_6) -\nabla_3(A_1)\nabla^2_6(A_1)$
$-\nabla_1(A_3)\nabla_1\nabla_6(A_6) +\nabla^2_6(A_1)\nabla_1(A_3) +\nabla_3(A_2)\nabla_2\nabla_6(A_6) -\nabla_3(A_2)\nabla^2_6(A_2) -\nabla_2(A_3)\nabla_2\nabla_6(A_6) +\nabla^2_6(A_2)\nabla_2(A_3)$
$+\nabla_3(A_4)\nabla_4\nabla_6(A_6) -\nabla_3(A_4)\nabla^2_6(A_4) -\nabla_4(A_3)\nabla_4\nabla_6(A_6) +\nabla_4(A_3)\nabla^2_6(A_4) +\nabla_3(A_5)\nabla_5\nabla_6(A_6) -\nabla_3(A_5)\nabla^2_6(A_5)$
$-\nabla_5(A_3)\nabla_5\nabla_6(A_6) +\nabla_5(A_3)\nabla^2_6(A_5) +\nabla_6\nabla_7(A_6)\nabla_3(A_7) -\nabla_3(A_7)\nabla^2_6(A_7) -\nabla_7(A_3)\nabla_6\nabla_7(A_6) +\nabla_7(A_3)\nabla^2_6(A_7)$
$+\nabla_3(A_0)\nabla^2_6(A_0) +\nabla_3(A_0)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_3) +\nabla_0(A_3)\nabla_0\nabla_6(A_6) +\nabla_3(A_1)\nabla_1\nabla_7(A_7) -\nabla_3(A_1)\nabla^2_7(A_1)$
$-\nabla_1(A_3)\nabla_1\nabla_7(A_7) +\nabla^2_7(A_1)\nabla_1(A_3) +\nabla_3(A_2)\nabla_2\nabla_7(A_7) -\nabla_3(A_2)\nabla^2_7(A_2) -\nabla_2(A_3)\nabla_2\nabla_7(A_7) +\nabla^2_7(A_2)\nabla_2(A_3)$
$+\nabla_3(A_4)\nabla_4\nabla_7(A_7) -\nabla_3(A_4)\nabla^2_7(A_4) -\nabla_4(A_3)\nabla_4\nabla_7(A_7) +\nabla_4(A_3)\nabla^2_7(A_4) +\nabla_3(A_5)\nabla_5\nabla_7(A_7) -\nabla_3(A_5)\nabla^2_7(A_5)$
$-\nabla_5(A_3)\nabla_5\nabla_7(A_7) +\nabla_5(A_3)\nabla^2_7(A_5) +\nabla_3(A_6)\nabla_6\nabla_7(A_7) -\nabla_3(A_6)\nabla^2_7(A_6) -\nabla_6(A_3)\nabla_6\nabla_7(A_7) +\nabla_6(A_3)\nabla^2_7(A_6)$
$+\nabla_3(A_0)\nabla^2_7(A_0) +\nabla_3(A_0)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_3) +\nabla_0(A_3)\nabla_0\nabla_7(A_7)$
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$+\nabla_0\nabla_1(A_0)\nabla_4(A_1) -\nabla_0\nabla_1(A_0)\nabla_1(A_4) +\nabla_4(A_1)\nabla^2_0(A_1) -\nabla^2_0(A_1)\nabla_1(A_4) +\nabla_0\nabla_2(A_0)\nabla_4(A_2) -\nabla_0\nabla_2(A_0)\nabla_2(A_4)$
$+\nabla_4(A_2)\nabla^2_0(A_2) -\nabla^2_0(A_2)\nabla_2(A_4) +\nabla_0\nabla_3(A_0)\nabla_4(A_3) -\nabla_0\nabla_3(A_0)\nabla_3(A_4) +\nabla_4(A_3)\nabla^2_0(A_3) -\nabla^2_0(A_3)\nabla_3(A_4)$
$+\nabla_0\nabla_5(A_0)\nabla_4(A_5) -\nabla_0\nabla_5(A_0)\nabla_5(A_4) +\nabla_4(A_5)\nabla^2_0(A_5) -\nabla_5(A_4)\nabla^2_0(A_5) +\nabla_0\nabla_6(A_0)\nabla_4(A_6) -\nabla_0\nabla_6(A_0)\nabla_6(A_4)$
$+\nabla_4(A_6)\nabla^2_0(A_6) -\nabla_6(A_4)\nabla^2_0(A_6) +\nabla_0\nabla_7(A_0)\nabla_4(A_7) -\nabla_0\nabla_7(A_0)\nabla_7(A_4) +\nabla_4(A_7)\nabla^2_0(A_7) -\nabla_7(A_4)\nabla^2_0(A_7)$
$+\nabla_1\nabla_2(A_1)\nabla_4(A_2) -\nabla_4(A_2)\nabla^2_1(A_2) -\nabla_1\nabla_2(A_1)\nabla_2(A_4) +\nabla^2_1(A_2)\nabla_2(A_4) +\nabla_1\nabla_3(A_1)\nabla_4(A_3) -\nabla_4(A_3)\nabla^2_1(A_3)$
$-\nabla_1\nabla_3(A_1)\nabla_3(A_4) +\nabla^2_1(A_3)\nabla_3(A_4) +\nabla_1\nabla_5(A_1)\nabla_4(A_5) -\nabla_4(A_5)\nabla^2_1(A_5) -\nabla_1\nabla_5(A_1)\nabla_5(A_4) +\nabla_5(A_4)\nabla^2_1(A_5)$
$+\nabla_1\nabla_6(A_1)\nabla_4(A_6) -\nabla_4(A_6)\nabla^2_1(A_6) -\nabla_1\nabla_6(A_1)\nabla_6(A_4) +\nabla_6(A_4)\nabla^2_1(A_6) +\nabla_1\nabla_7(A_1)\nabla_4(A_7) -\nabla_4(A_7)\nabla^2_1(A_7)$
$-\nabla_1\nabla_7(A_1)\nabla_7(A_4) +\nabla_7(A_4)\nabla^2_1(A_7) +\nabla_4(A_0)\nabla^2_1(A_0) +\nabla_4(A_0)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_4) +\nabla_0\nabla_1(A_1)\nabla_0(A_4)$
$+\nabla_4(A_1)\nabla_1\nabla_2(A_2) -\nabla_4(A_1)\nabla^2_2(A_1) -\nabla_1\nabla_2(A_2)\nabla_1(A_4) +\nabla^2_2(A_1)\nabla_1(A_4) +\nabla_2\nabla_3(A_2)\nabla_4(A_3) -\nabla_4(A_3)\nabla^2_2(A_3)$
$-\nabla_2\nabla_3(A_2)\nabla_3(A_4) +\nabla^2_2(A_3)\nabla_3(A_4) +\nabla_2\nabla_5(A_2)\nabla_4(A_5) -\nabla_4(A_5)\nabla^2_2(A_5) -\nabla_2\nabla_5(A_2)\nabla_5(A_4) +\nabla_5(A_4)\nabla^2_2(A_5)$
$+\nabla_2\nabla_6(A_2)\nabla_4(A_6) -\nabla_4(A_6)\nabla^2_2(A_6) -\nabla_2\nabla_6(A_2)\nabla_6(A_4) +\nabla_6(A_4)\nabla^2_2(A_6) +\nabla_2\nabla_7(A_2)\nabla_4(A_7) -\nabla_4(A_7)\nabla^2_2(A_7)$
$-\nabla_2\nabla_7(A_2)\nabla_7(A_4) +\nabla_7(A_4)\nabla^2_2(A_7) +\nabla_4(A_0)\nabla^2_2(A_0) +\nabla_4(A_0)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_4) +\nabla_0\nabla_2(A_2)\nabla_0(A_4)$
$+\nabla_4(A_1)\nabla_1\nabla_3(A_3) -\nabla_4(A_1)\nabla^2_3(A_1) -\nabla_1\nabla_3(A_3)\nabla_1(A_4) +\nabla^2_3(A_1)\nabla_1(A_4) +\nabla_4(A_2)\nabla_2\nabla_3(A_3) -\nabla_4(A_2)\nabla^2_3(A_2)$
$-\nabla_2\nabla_3(A_3)\nabla_2(A_4) +\nabla^2_3(A_2)\nabla_2(A_4) +\nabla_3\nabla_5(A_3)\nabla_4(A_5) -\nabla_4(A_5)\nabla^2_3(A_5) -\nabla_3\nabla_5(A_3)\nabla_5(A_4) +\nabla_5(A_4)\nabla^2_3(A_5)$
$+\nabla_3\nabla_6(A_3)\nabla_4(A_6) -\nabla_4(A_6)\nabla^2_3(A_6) -\nabla_3\nabla_6(A_3)\nabla_6(A_4) +\nabla_6(A_4)\nabla^2_3(A_6) +\nabla_3\nabla_7(A_3)\nabla_4(A_7) -\nabla_4(A_7)\nabla^2_3(A_7)$
$-\nabla_3\nabla_7(A_3)\nabla_7(A_4) +\nabla_7(A_4)\nabla^2_3(A_7) +\nabla_4(A_0)\nabla^2_3(A_0) +\nabla_4(A_0)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_4) +\nabla_0\nabla_3(A_3)\nabla_0(A_4)$
$-\nabla_1(A_4)\nabla_1\nabla_4(A_4) +\nabla^2_4(A_1)\nabla_1(A_4) +\nabla_4(A_1)\nabla_1\nabla_4(A_4) -\nabla_4(A_1)\nabla^2_4(A_1) -\nabla_2(A_4)\nabla_2\nabla_4(A_4) +\nabla^2_4(A_2)\nabla_2(A_4)$
$+\nabla_4(A_2)\nabla_2\nabla_4(A_4) -\nabla_4(A_2)\nabla^2_4(A_2) -\nabla_3(A_4)\nabla_3\nabla_4(A_4) +\nabla^2_4(A_3)\nabla_3(A_4) +\nabla_4(A_3)\nabla_3\nabla_4(A_4) -\nabla_4(A_3)\nabla^2_4(A_3)$
$-\nabla_4(A_5)\nabla^2_4(A_5) +\nabla_4\nabla_5(A_4)\nabla_4(A_5) +\nabla_5(A_4)\nabla^2_4(A_5) -\nabla_5(A_4)\nabla_4\nabla_5(A_4) -\nabla_4(A_6)\nabla^2_4(A_6) +\nabla_4\nabla_6(A_4)\nabla_4(A_6)$
$+\nabla_6(A_4)\nabla^2_4(A_6) -\nabla_6(A_4)\nabla_4\nabla_6(A_4) -\nabla_4(A_7)\nabla^2_4(A_7) +\nabla_4\nabla_7(A_4)\nabla_4(A_7) +\nabla_7(A_4)\nabla^2_4(A_7) -\nabla_7(A_4)\nabla_4\nabla_7(A_4)$
$+\nabla_0(A_4)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_4) +\nabla_4(A_0)\nabla_0\nabla_4(A_4) +\nabla_4(A_0)\nabla^2_4(A_0) +\nabla_4(A_1)\nabla_1\nabla_5(A_5) -\nabla_4(A_1)\nabla^2_5(A_1)$
$-\nabla_1(A_4)\nabla_1\nabla_5(A_5) +\nabla^2_5(A_1)\nabla_1(A_4) +\nabla_4(A_2)\nabla_2\nabla_5(A_5) -\nabla_4(A_2)\nabla^2_5(A_2) -\nabla_2(A_4)\nabla_2\nabla_5(A_5) +\nabla^2_5(A_2)\nabla_2(A_4)$
$+\nabla_4(A_3)\nabla_3\nabla_5(A_5) -\nabla_4(A_3)\nabla^2_5(A_3) -\nabla_3(A_4)\nabla_3\nabla_5(A_5) +\nabla^2_5(A_3)\nabla_3(A_4) +\nabla_5\nabla_6(A_5)\nabla_4(A_6) -\nabla_4(A_6)\nabla^2_5(A_6)$

$-\nabla_6(A_4)\nabla_5\nabla_6(A_5) +\nabla_6(A_4)\nabla^2_5(A_6) +\nabla_5\nabla_7(A_5)\nabla_4(A_7) -\nabla_4(A_7)\nabla^2_5(A_7) -\nabla_7(A_4)\nabla_5\nabla_7(A_5) +\nabla_7(A_4)\nabla^2_5(A_7)$
$+\nabla_4(A_0)\nabla^2_5(A_0) +\nabla_4(A_0)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_4) +\nabla_0(A_4)\nabla_0\nabla_5(A_5) +\nabla_4(A_1)\nabla_1\nabla_6(A_6) -\nabla_4(A_1)\nabla^2_6(A_1)$
$-\nabla_1(A_4)\nabla_1\nabla_6(A_6) +\nabla^2_6(A_1)\nabla_1(A_4) +\nabla_4(A_2)\nabla_2\nabla_6(A_6) -\nabla_4(A_2)\nabla^2_6(A_2) -\nabla_2(A_4)\nabla_2\nabla_6(A_6) +\nabla^2_6(A_2)\nabla_2(A_4)$
$+\nabla_4(A_3)\nabla_3\nabla_6(A_6) -\nabla_4(A_3)\nabla^2_6(A_3) -\nabla_3(A_4)\nabla_3\nabla_6(A_6) +\nabla^2_6(A_3)\nabla_3(A_4) +\nabla_4(A_5)\nabla_5\nabla_6(A_6) -\nabla_4(A_5)\nabla^2_6(A_5)$
$-\nabla_5(A_4)\nabla_5\nabla_6(A_6) +\nabla_5(A_4)\nabla^2_6(A_5) +\nabla_6\nabla_7(A_6)\nabla_4(A_7) -\nabla_4(A_7)\nabla^2_6(A_7) -\nabla_7(A_4)\nabla_6\nabla_7(A_6) +\nabla_7(A_4)\nabla^2_6(A_7)$
$+\nabla_4(A_0)\nabla^2_6(A_0) +\nabla_4(A_0)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_4) +\nabla_0(A_4)\nabla_0\nabla_6(A_6) +\nabla_4(A_1)\nabla_1\nabla_7(A_7) -\nabla_4(A_1)\nabla^2_7(A_1)$
$-\nabla_1(A_4)\nabla_1\nabla_7(A_7) +\nabla^2_7(A_1)\nabla_1(A_4) +\nabla_4(A_2)\nabla_2\nabla_7(A_7) -\nabla_4(A_2)\nabla^2_7(A_2) -\nabla_2(A_4)\nabla_2\nabla_7(A_7) +\nabla^2_7(A_2)\nabla_2(A_4)$
$+\nabla_4(A_3)\nabla_3\nabla_7(A_7) -\nabla_4(A_3)\nabla^2_7(A_3) -\nabla_3(A_4)\nabla_3\nabla_7(A_7) +\nabla^2_7(A_3)\nabla_3(A_4) +\nabla_4(A_5)\nabla_5\nabla_7(A_7) -\nabla_4(A_5)\nabla^2_7(A_5)$
$-\nabla_5(A_4)\nabla_5\nabla_7(A_7) +\nabla_5(A_4)\nabla^2_7(A_5) +\nabla_4(A_6)\nabla_6\nabla_7(A_7) -\nabla_4(A_6)\nabla^2_7(A_6) -\nabla_6(A_4)\nabla_6\nabla_7(A_7) +\nabla_6(A_4)\nabla^2_7(A_6)$
$+\nabla_4(A_0)\nabla^2_7(A_0) +\nabla_4(A_0)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_4) +\nabla_0(A_4)\nabla_0\nabla_7(A_7)$
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$+\nabla_0\nabla_1(A_0)\nabla_5(A_1) -\nabla_0\nabla_1(A_0)\nabla_1(A_5) +\nabla_5(A_1)\nabla^2_0(A_1) -\nabla^2_0(A_1)\nabla_1(A_5) +\nabla_0\nabla_2(A_0)\nabla_5(A_2) -\nabla_0\nabla_2(A_0)\nabla_2(A_5)$
$+\nabla_5(A_2)\nabla^2_0(A_2) -\nabla^2_0(A_2)\nabla_2(A_5) +\nabla_0\nabla_3(A_0)\nabla_5(A_3) -\nabla_0\nabla_3(A_0)\nabla_3(A_5) +\nabla_5(A_3)\nabla^2_0(A_3) -\nabla^2_0(A_3)\nabla_3(A_5)$
$+\nabla_0\nabla_4(A_0)\nabla_5(A_4) -\nabla_0\nabla_4(A_0)\nabla_4(A_5) +\nabla_5(A_4)\nabla^2_0(A_4) -\nabla^2_0(A_4)\nabla_4(A_5) +\nabla_0\nabla_6(A_0)\nabla_5(A_6) -\nabla_0\nabla_6(A_0)\nabla_6(A_5)$
$+\nabla_5(A_6)\nabla^2_0(A_6) -\nabla_6(A_5)\nabla^2_0(A_6) +\nabla_0\nabla_7(A_0)\nabla_5(A_7) -\nabla_0\nabla_7(A_0)\nabla_7(A_5) +\nabla_5(A_7)\nabla^2_0(A_7) -\nabla_7(A_5)\nabla^2_0(A_7)$
$+\nabla_1\nabla_2(A_1)\nabla_5(A_2) -\nabla_5(A_2)\nabla^2_1(A_2) -\nabla_1\nabla_2(A_1)\nabla_2(A_5) +\nabla^2_1(A_2)\nabla_2(A_5) +\nabla_1\nabla_3(A_1)\nabla_5(A_3) -\nabla_5(A_3)\nabla^2_1(A_3)$
$-\nabla_1\nabla_3(A_1)\nabla_3(A_5) +\nabla^2_1(A_3)\nabla_3(A_5) +\nabla_1\nabla_4(A_1)\nabla_5(A_4) -\nabla_5(A_4)\nabla^2_1(A_4) -\nabla_1\nabla_4(A_1)\nabla_4(A_5) +\nabla^2_1(A_4)\nabla_4(A_5)$
$+\nabla_1\nabla_6(A_1)\nabla_5(A_6) -\nabla_5(A_6)\nabla^2_1(A_6) -\nabla_1\nabla_6(A_1)\nabla_6(A_5) +\nabla_6(A_5)\nabla^2_1(A_6) +\nabla_1\nabla_7(A_1)\nabla_5(A_7) -\nabla_5(A_7)\nabla^2_1(A_7)$
$-\nabla_1\nabla_7(A_1)\nabla_7(A_5) +\nabla_7(A_5)\nabla^2_1(A_7) +\nabla_5(A_0)\nabla^2_1(A_0) +\nabla_5(A_0)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_5) +\nabla_0\nabla_1(A_1)\nabla_0(A_5)$
$+\nabla_5(A_1)\nabla_1\nabla_2(A_2) -\nabla_5(A_1)\nabla^2_2(A_1) -\nabla_1\nabla_2(A_2)\nabla_1(A_5) +\nabla^2_2(A_1)\nabla_1(A_5) +\nabla_2\nabla_3(A_2)\nabla_5(A_3) -\nabla_5(A_3)\nabla^2_2(A_3)$
$-\nabla_2\nabla_3(A_2)\nabla_3(A_5) +\nabla^2_2(A_3)\nabla_3(A_5) +\nabla_2\nabla_4(A_2)\nabla_5(A_4) -\nabla_5(A_4)\nabla^2_2(A_4) -\nabla_2\nabla_4(A_2)\nabla_4(A_5) +\nabla^2_2(A_4)\nabla_4(A_5)$
$+\nabla_2\nabla_6(A_2)\nabla_5(A_6) -\nabla_5(A_6)\nabla^2_2(A_6) -\nabla_2\nabla_6(A_2)\nabla_6(A_5) +\nabla_6(A_5)\nabla^2_2(A_6) +\nabla_2\nabla_7(A_2)\nabla_5(A_7) -\nabla_5(A_7)\nabla^2_2(A_7)$
$-\nabla_2\nabla_7(A_2)\nabla_7(A_5) +\nabla_7(A_5)\nabla^2_2(A_7) +\nabla_5(A_0)\nabla^2_2(A_0) +\nabla_5(A_0)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_5) +\nabla_0\nabla_2(A_2)\nabla_0(A_5)$
$+\nabla_5(A_1)\nabla_1\nabla_3(A_3) -\nabla_5(A_1)\nabla^2_3(A_1) -\nabla_1\nabla_3(A_3)\nabla_1(A_5) +\nabla^2_3(A_1)\nabla_1(A_5) +\nabla_5(A_2)\nabla_2\nabla_3(A_3) -\nabla_5(A_2)\nabla^2_3(A_2)$
$-\nabla_2\nabla_3(A_3)\nabla_2(A_5) +\nabla^2_3(A_2)\nabla_2(A_5) +\nabla_3\nabla_4(A_3)\nabla_5(A_4) -\nabla_5(A_4)\nabla^2_3(A_4) -\nabla_3\nabla_4(A_3)\nabla_4(A_5) +\nabla^2_3(A_4)\nabla_4(A_5)$
$+\nabla_3\nabla_6(A_3)\nabla_5(A_6) -\nabla_5(A_6)\nabla^2_3(A_6) -\nabla_3\nabla_6(A_3)\nabla_6(A_5) +\nabla_6(A_5)\nabla^2_3(A_6) +\nabla_3\nabla_7(A_3)\nabla_5(A_7) -\nabla_5(A_7)\nabla^2_3(A_7)$
$-\nabla_3\nabla_7(A_3)\nabla_7(A_5) +\nabla_7(A_5)\nabla^2_3(A_7) +\nabla_5(A_0)\nabla^2_3(A_0) +\nabla_5(A_0)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_5) +\nabla_0\nabla_3(A_3)\nabla_0(A_5)$
$+\nabla_5(A_1)\nabla_1\nabla_4(A_4) -\nabla_5(A_1)\nabla^2_4(A_1) -\nabla_1\nabla_4(A_4)\nabla_1(A_5) +\nabla^2_4(A_1)\nabla_1(A_5) +\nabla_5(A_2)\nabla_2\nabla_4(A_4) -\nabla_5(A_2)\nabla^2_4(A_2)$
$-\nabla_2\nabla_4(A_4)\nabla_2(A_5) +\nabla^2_4(A_2)\nabla_2(A_5) +\nabla_5(A_3)\nabla_3\nabla_4(A_4) -\nabla_5(A_3)\nabla^2_4(A_3) -\nabla_3\nabla_4(A_4)\nabla_3(A_5) +\nabla^2_4(A_3)\nabla_3(A_5)$
$+\nabla_4\nabla_6(A_4)\nabla_5(A_6) -\nabla_5(A_6)\nabla^2_4(A_6) -\nabla_4\nabla_6(A_4)\nabla_6(A_5) +\nabla_6(A_5)\nabla^2_4(A_6) +\nabla_4\nabla_7(A_4)\nabla_5(A_7) -\nabla_5(A_7)\nabla^2_4(A_7)$
$-\nabla_4\nabla_7(A_4)\nabla_7(A_5) +\nabla_7(A_5)\nabla^2_4(A_7) +\nabla_5(A_0)\nabla^2_4(A_0) +\nabla_5(A_0)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_5) +\nabla_0\nabla_4(A_4)\nabla_0(A_5)$
$-\nabla_1(A_5)\nabla_1\nabla_5(A_5) +\nabla^2_5(A_1)\nabla_1(A_5) +\nabla_5(A_1)\nabla_1\nabla_5(A_5) -\nabla_5(A_1)\nabla^2_5(A_1) -\nabla_2(A_5)\nabla_2\nabla_5(A_5) +\nabla^2_5(A_2)\nabla_2(A_5)$
$+\nabla_5(A_2)\nabla_2\nabla_5(A_5) -\nabla_5(A_2)\nabla^2_5(A_2) -\nabla_3(A_5)\nabla_3\nabla_5(A_5) +\nabla^2_5(A_3)\nabla_3(A_5) +\nabla_5(A_3)\nabla_3\nabla_5(A_5) -\nabla_5(A_3)\nabla^2_5(A_3)$
$-\nabla_4(A_5)\nabla_4\nabla_5(A_5) +\nabla^2_5(A_4)\nabla_4(A_5) +\nabla_5(A_4)\nabla_4\nabla_5(A_5) -\nabla_5(A_4)\nabla^2_5(A_4) -\nabla_5(A_6)\nabla^2_5(A_6) +\nabla_5\nabla_6(A_5)\nabla_5(A_6)$
$+\nabla_6(A_5)\nabla^2_5(A_6) -\nabla_6(A_5)\nabla_5\nabla_6(A_5) -\nabla_5(A_7)\nabla^2_5(A_7) +\nabla_5\nabla_7(A_5)\nabla_5(A_7) +\nabla_7(A_5)\nabla^2_5(A_7) -\nabla_7(A_5)\nabla_5\nabla_7(A_5)$
$+\nabla_0(A_5)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_5) +\nabla_5(A_0)\nabla_0\nabla_5(A_5) +\nabla_5(A_0)\nabla^2_5(A_0) +\nabla_5(A_1)\nabla_1\nabla_6(A_6) -\nabla_5(A_1)\nabla^2_6(A_1)$
$-\nabla_1(A_5)\nabla_1\nabla_6(A_6) +\nabla^2_6(A_1)\nabla_1(A_5) +\nabla_5(A_2)\nabla_2\nabla_6(A_6) -\nabla_5(A_2)\nabla^2_6(A_2) -\nabla_2(A_5)\nabla_2\nabla_6(A_6) +\nabla^2_6(A_2)\nabla_2(A_5)$
$+\nabla_5(A_3)\nabla_3\nabla_6(A_6) -\nabla_5(A_3)\nabla^2_6(A_3) -\nabla_3(A_5)\nabla_3\nabla_6(A_6) +\nabla^2_6(A_3)\nabla_3(A_5) +\nabla_5(A_4)\nabla_4\nabla_6(A_6) -\nabla_5(A_4)\nabla^2_6(A_4)$
$-\nabla_4(A_5)\nabla_4\nabla_6(A_6) +\nabla^2_6(A_4)\nabla_4(A_5) +\nabla_6\nabla_7(A_6)\nabla_5(A_7) -\nabla_5(A_7)\nabla^2_6(A_7) -\nabla_7(A_5)\nabla_6\nabla_7(A_6) +\nabla_7(A_5)\nabla^2_6(A_7)$
$+\nabla_5(A_0)\nabla^2_6(A_0) +\nabla_5(A_0)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_5) +\nabla_0(A_5)\nabla_0\nabla_6(A_6) +\nabla_5(A_1)\nabla_1\nabla_7(A_7) -\nabla_5(A_1)\nabla^2_7(A_1)$
$-\nabla_1(A_5)\nabla_1\nabla_7(A_7) +\nabla^2_7(A_1)\nabla_1(A_5) +\nabla_5(A_2)\nabla_2\nabla_7(A_7) -\nabla_5(A_2)\nabla^2_7(A_2) -\nabla_2(A_5)\nabla_2\nabla_7(A_7) +\nabla^2_7(A_2)\nabla_2(A_5)$
$+\nabla_5(A_3)\nabla_3\nabla_7(A_7) -\nabla_5(A_3)\nabla^2_7(A_3) -\nabla_3(A_5)\nabla_3\nabla_7(A_7) +\nabla^2_7(A_3)\nabla_3(A_5) +\nabla_5(A_4)\nabla_4\nabla_7(A_7) -\nabla_5(A_4)\nabla^2_7(A_4)$
$-\nabla_4(A_5)\nabla_4\nabla_7(A_7) +\nabla^2_7(A_4)\nabla_4(A_5) +\nabla_5(A_6)\nabla_6\nabla_7(A_7) -\nabla_5(A_6)\nabla^2_7(A_6) -\nabla_6(A_5)\nabla_6\nabla_7(A_7) +\nabla_6(A_5)\nabla^2_7(A_6)$
$+\nabla_5(A_0)\nabla^2_7(A_0) +\nabla_5(A_0)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_5) +\nabla_0(A_5)\nabla_0\nabla_7(A_7)$
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$+\nabla_0\nabla_1(A_0)\nabla_6(A_1) -\nabla_0\nabla_1(A_0)\nabla_1(A_6) +\nabla_6(A_1)\nabla^2_0(A_1) -\nabla^2_0(A_1)\nabla_1(A_6) +\nabla_0\nabla_2(A_0)\nabla_6(A_2) -\nabla_0\nabla_2(A_0)\nabla_2(A_6)$
$+\nabla_6(A_2)\nabla^2_0(A_2) -\nabla^2_0(A_2)\nabla_2(A_6) +\nabla_0\nabla_3(A_0)\nabla_6(A_3) -\nabla_0\nabla_3(A_0)\nabla_3(A_6) +\nabla_6(A_3)\nabla^2_0(A_3) -\nabla^2_0(A_3)\nabla_3(A_6)$
$+\nabla_0\nabla_4(A_0)\nabla_6(A_4) -\nabla_0\nabla_4(A_0)\nabla_4(A_6) +\nabla_6(A_4)\nabla^2_0(A_4) -\nabla^2_0(A_4)\nabla_4(A_6) +\nabla_0\nabla_5(A_0)\nabla_6(A_5) -\nabla_0\nabla_5(A_0)\nabla_5(A_6)$
$+\nabla_6(A_5)\nabla^2_0(A_5) -\nabla^2_0(A_5)\nabla_5(A_6) +\nabla_0\nabla_7(A_0)\nabla_6(A_7) -\nabla_0\nabla_7(A_0)\nabla_7(A_6) +\nabla_6(A_7)\nabla^2_0(A_7) -\nabla_7(A_6)\nabla^2_0(A_7)$
$+\nabla_1\nabla_2(A_1)\nabla_6(A_2) -\nabla_6(A_2)\nabla^2_1(A_2) -\nabla_1\nabla_2(A_1)\nabla_2(A_6) +\nabla^2_1(A_2)\nabla_2(A_6) +\nabla_1\nabla_3(A_1)\nabla_6(A_3) -\nabla_6(A_3)\nabla^2_1(A_3)$
$-\nabla_1\nabla_3(A_1)\nabla_3(A_6) +\nabla^2_1(A_3)\nabla_3(A_6) +\nabla_1\nabla_4(A_1)\nabla_6(A_4) -\nabla_6(A_4)\nabla^2_1(A_4) -\nabla_1\nabla_4(A_1)\nabla_4(A_6) +\nabla^2_1(A_4)\nabla_4(A_6)$
$+\nabla_1\nabla_5(A_1)\nabla_6(A_5) -\nabla_6(A_5)\nabla^2_1(A_5) -\nabla_1\nabla_5(A_1)\nabla_5(A_6) +\nabla^2_1(A_5)\nabla_5(A_6) +\nabla_1\nabla_7(A_1)\nabla_6(A_7) -\nabla_6(A_7)\nabla^2_1(A_7)$
$-\nabla_1\nabla_7(A_1)\nabla_7(A_6) +\nabla_7(A_6)\nabla^2_1(A_7) +\nabla_6(A_0)\nabla^2_1(A_0) +\nabla_6(A_0)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_6) +\nabla_0\nabla_1(A_1)\nabla_0(A_6)$
$+\nabla_6(A_1)\nabla_1\nabla_2(A_2) -\nabla_6(A_1)\nabla^2_2(A_1) -\nabla_1\nabla_2(A_2)\nabla_1(A_6) +\nabla^2_2(A_1)\nabla_1(A_6) +\nabla_2\nabla_3(A_2)\nabla_6(A_3) -\nabla_6(A_3)\nabla^2_2(A_3)$
$-\nabla_2\nabla_3(A_2)\nabla_3(A_6) +\nabla^2_2(A_3)\nabla_3(A_6) +\nabla_2\nabla_4(A_2)\nabla_6(A_4) -\nabla_6(A_4)\nabla^2_2(A_4) -\nabla_2\nabla_4(A_2)\nabla_4(A_6) +\nabla^2_2(A_4)\nabla_4(A_6)$
$+\nabla_2\nabla_5(A_2)\nabla_6(A_5) -\nabla_6(A_5)\nabla^2_2(A_5) -\nabla_2\nabla_5(A_2)\nabla_5(A_6) +\nabla^2_2(A_5)\nabla_5(A_6) +\nabla_2\nabla_7(A_2)\nabla_6(A_7) -\nabla_6(A_7)\nabla^2_2(A_7)$
$-\nabla_2\nabla_7(A_2)\nabla_7(A_6) +\nabla_7(A_6)\nabla^2_2(A_7) +\nabla_6(A_0)\nabla^2_2(A_0) +\nabla_6(A_0)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_6) +\nabla_0\nabla_2(A_2)\nabla_0(A_6)$
$+\nabla_6(A_1)\nabla_1\nabla_3(A_3) -\nabla_6(A_1)\nabla^2_3(A_1) -\nabla_1\nabla_3(A_3)\nabla_1(A_6) +\nabla^2_3(A_1)\nabla_1(A_6) +\nabla_6(A_2)\nabla_2\nabla_3(A_3) -\nabla_6(A_2)\nabla^2_3(A_2)$
$-\nabla_2\nabla_3(A_3)\nabla_2(A_6) +\nabla^2_3(A_2)\nabla_2(A_6) +\nabla_3\nabla_4(A_3)\nabla_6(A_4) -\nabla_6(A_4)\nabla^2_3(A_4) -\nabla_3\nabla_4(A_3)\nabla_4(A_6) +\nabla^2_3(A_4)\nabla_4(A_6)$
$+\nabla_3\nabla_5(A_3)\nabla_6(A_5) -\nabla_6(A_5)\nabla^2_3(A_5) -\nabla_3\nabla_5(A_3)\nabla_5(A_6) +\nabla^2_3(A_5)\nabla_5(A_6) +\nabla_3\nabla_7(A_3)\nabla_6(A_7) -\nabla_6(A_7)\nabla^2_3(A_7)$
$-\nabla_3\nabla_7(A_3)\nabla_7(A_6) +\nabla_7(A_6)\nabla^2_3(A_7) +\nabla_6(A_0)\nabla^2_3(A_0) +\nabla_6(A_0)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_6) +\nabla_0\nabla_3(A_3)\nabla_0(A_6)$
$+\nabla_6(A_1)\nabla_1\nabla_4(A_4) -\nabla_6(A_1)\nabla^2_4(A_1) -\nabla_1\nabla_4(A_4)\nabla_1(A_6) +\nabla^2_4(A_1)\nabla_1(A_6) +\nabla_6(A_2)\nabla_2\nabla_4(A_4) -\nabla_6(A_2)\nabla^2_4(A_2)$
$-\nabla_2\nabla_4(A_4)\nabla_2(A_6) +\nabla^2_4(A_2)\nabla_2(A_6) +\nabla_6(A_3)\nabla_3\nabla_4(A_4) -\nabla_6(A_3)\nabla^2_4(A_3) -\nabla_3\nabla_4(A_4)\nabla_3(A_6) +\nabla^2_4(A_3)\nabla_3(A_6)$
$+\nabla_4\nabla_5(A_4)\nabla_6(A_5) -\nabla_6(A_5)\nabla^2_4(A_5) -\nabla_4\nabla_5(A_4)\nabla_5(A_6) +\nabla^2_4(A_5)\nabla_5(A_6) +\nabla_4\nabla_7(A_4)\nabla_6(A_7) -\nabla_6(A_7)\nabla^2_4(A_7)$
$-\nabla_4\nabla_7(A_4)\nabla_7(A_6) +\nabla_7(A_6)\nabla^2_4(A_7) +\nabla_6(A_0)\nabla^2_4(A_0) +\nabla_6(A_0)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_6) +\nabla_0\nabla_4(A_4)\nabla_0(A_6)$
$+\nabla_6(A_1)\nabla_1\nabla_5(A_5) -\nabla_6(A_1)\nabla^2_5(A_1) -\nabla_1\nabla_5(A_5)\nabla_1(A_6) +\nabla^2_5(A_1)\nabla_1(A_6) +\nabla_6(A_2)\nabla_2\nabla_5(A_5) -\nabla_6(A_2)\nabla^2_5(A_2)$
$-\nabla_2\nabla_5(A_5)\nabla_2(A_6) +\nabla^2_5(A_2)\nabla_2(A_6) +\nabla_6(A_3)\nabla_3\nabla_5(A_5) -\nabla_6(A_3)\nabla^2_5(A_3) -\nabla_3\nabla_5(A_5)\nabla_3(A_6) +\nabla^2_5(A_3)\nabla_3(A_6)$
$+\nabla_6(A_4)\nabla_4\nabla_5(A_5) -\nabla_6(A_4)\nabla^2_5(A_4) -\nabla_4\nabla_5(A_5)\nabla_4(A_6) +\nabla^2_5(A_4)\nabla_4(A_6) +\nabla_5\nabla_7(A_5)\nabla_6(A_7) -\nabla_6(A_7)\nabla^2_5(A_7)$
$-\nabla_5\nabla_7(A_5)\nabla_7(A_6) +\nabla_7(A_6)\nabla^2_5(A_7) +\nabla_6(A_0)\nabla^2_5(A_0) +\nabla_6(A_0)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_6) +\nabla_0\nabla_5(A_5)\nabla_0(A_6)$
$-\nabla_1(A_6)\nabla_1\nabla_6(A_6) +\nabla^2_6(A_1)\nabla_1(A_6) +\nabla_6(A_1)\nabla_1\nabla_6(A_6) -\nabla_6(A_1)\nabla^2_6(A_1) -\nabla_2(A_6)\nabla_2\nabla_6(A_6) +\nabla^2_6(A_2)\nabla_2(A_6)$
$+\nabla_6(A_2)\nabla_2\nabla_6(A_6) -\nabla_6(A_2)\nabla^2_6(A_2) -\nabla_3(A_6)\nabla_3\nabla_6(A_6) +\nabla^2_6(A_3)\nabla_3(A_6) +\nabla_6(A_3)\nabla_3\nabla_6(A_6) -\nabla_6(A_3)\nabla^2_6(A_3)$
$-\nabla_4(A_6)\nabla_4\nabla_6(A_6) +\nabla^2_6(A_4)\nabla_4(A_6) +\nabla_6(A_4)\nabla_4\nabla_6(A_6) -\nabla_6(A_4)\nabla^2_6(A_4) -\nabla_5(A_6)\nabla_5\nabla_6(A_6) +\nabla^2_6(A_5)\nabla_5(A_6)$
$+\nabla_6(A_5)\nabla_5\nabla_6(A_6) -\nabla_6(A_5)\nabla^2_6(A_5) -\nabla_6(A_7)\nabla^2_6(A_7) +\nabla_6\nabla_7(A_6)\nabla_6(A_7) +\nabla_7(A_6)\nabla^2_6(A_7) -\nabla_7(A_6)\nabla_6\nabla_7(A_6)$
$+\nabla_0(A_6)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_6) +\nabla_6(A_0)\nabla_0\nabla_6(A_6) +\nabla_6(A_0)\nabla^2_6(A_0) +\nabla_6(A_1)\nabla_1\nabla_7(A_7) -\nabla_6(A_1)\nabla^2_7(A_1)$
$-\nabla_1(A_6)\nabla_1\nabla_7(A_7) +\nabla^2_7(A_1)\nabla_1(A_6) +\nabla_6(A_2)\nabla_2\nabla_7(A_7) -\nabla_6(A_2)\nabla^2_7(A_2) -\nabla_2(A_6)\nabla_2\nabla_7(A_7) +\nabla^2_7(A_2)\nabla_2(A_6)$
$+\nabla_6(A_3)\nabla_3\nabla_7(A_7) -\nabla_6(A_3)\nabla^2_7(A_3) -\nabla_3(A_6)\nabla_3\nabla_7(A_7) +\nabla^2_7(A_3)\nabla_3(A_6) +\nabla_6(A_4)\nabla_4\nabla_7(A_7) -\nabla_6(A_4)\nabla^2_7(A_4)$
$-\nabla_4(A_6)\nabla_4\nabla_7(A_7) +\nabla^2_7(A_4)\nabla_4(A_6) +\nabla_6(A_5)\nabla_5\nabla_7(A_7) -\nabla_6(A_5)\nabla^2_7(A_5) -\nabla_5(A_6)\nabla_5\nabla_7(A_7) +\nabla^2_7(A_5)\nabla_5(A_6)$
$+\nabla_6(A_0)\nabla^2_7(A_0) +\nabla_6(A_0)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_6) +\nabla_0(A_6)\nabla_0\nabla_7(A_7)$

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$+\nabla_0\nabla_1(A_0)\nabla_7(A_1) -\nabla_0\nabla_1(A_0)\nabla_1(A_7) +\nabla_7(A_1)\nabla^2_0(A_1) -\nabla^2_0(A_1)\nabla_1(A_7) +\nabla_0\nabla_2(A_0)\nabla_7(A_2) -\nabla_0\nabla_2(A_0)\nabla_2(A_7)$
$+\nabla_7(A_2)\nabla^2_0(A_2)-\nabla^2_0(A_2)\nabla_2(A_7) +\nabla_0\nabla_3(A_0)\nabla_7(A_3) -\nabla_0\nabla_3(A_0)\nabla_3(A_7) +\nabla_7(A_3)\nabla^2_0(A_3) -\nabla^2_0(A_3)\nabla_3(A_7)$
$+\nabla_0\nabla_4(A_0)\nabla_7(A_4) -\nabla_0\nabla_4(A_0)\nabla_4(A_7) +\nabla_7(A_4)\nabla^2_0(A_4) -\nabla^2_0(A_4)\nabla_4(A_7) +\nabla_0\nabla_5(A_0)\nabla_7(A_5) -\nabla_0\nabla_5(A_0)\nabla_5(A_7)$
$+\nabla_7(A_5)\nabla^2_0(A_5) -\nabla^2_0(A_5)\nabla_5(A_7) +\nabla_0\nabla_6(A_0)\nabla_7(A_6) -\nabla_0\nabla_6(A_0)\nabla_6(A_7) +\nabla_7(A_6)\nabla^2_0(A_6) -\nabla^2_0(A_6)\nabla_6(A_7)$
$+\nabla_1\nabla_2(A_1)\nabla_7(A_2) -\nabla_7(A_2)\nabla^2_1(A_2) -\nabla_1\nabla_2(A_1)\nabla_2(A_7) +\nabla^2_1(A_2)\nabla_2(A_7) +\nabla_1\nabla_3(A_1)\nabla_7(A_3) -\nabla_7(A_3)\nabla^2_1(A_3)$
$-\nabla_1\nabla_3(A_1)\nabla_3(A_7) +\nabla^2_1(A_3)\nabla_3(A_7) +\nabla_1\nabla_4(A_1)\nabla_7(A_4) -\nabla_7(A_4)\nabla^2_1(A_4) -\nabla_1\nabla_4(A_1)\nabla_4(A_7) +\nabla^2_1(A_4)\nabla_4(A_7)$
$+\nabla_1\nabla_5(A_1)\nabla_7(A_5) -\nabla_7(A_5)\nabla^2_1(A_5) -\nabla_1\nabla_5(A_1)\nabla_5(A_7) +\nabla^2_1(A_5)\nabla_5(A_7) +\nabla_1\nabla_6(A_1)\nabla_7(A_6) -\nabla_7(A_6)\nabla^2_1(A_6)$
$-\nabla_1\nabla_6(A_1)\nabla_6(A_7) +\nabla^2_1(A_6)\nabla_6(A_7) +\nabla_7(A_0)\nabla^2_1(A_0) +\nabla_7(A_0)\nabla_0\nabla_1(A_1) +\nabla^2_1(A_0)\nabla_0(A_7) +\nabla_0\nabla_1(A_1)\nabla_0(A_7)$
$+\nabla_7(A_1)\nabla_1\nabla_2(A_2) -\nabla_7(A_1)\nabla^2_2(A_1) -\nabla_1\nabla_2(A_2)\nabla_1(A_7) +\nabla^2_2(A_1)\nabla_1(A_7) +\nabla_2\nabla_3(A_2)\nabla_7(A_3) -\nabla_7(A_3)\nabla^2_2(A_3)$
$-\nabla_2\nabla_3(A_2)\nabla_3(A_7) +\nabla^2_2(A_3)\nabla_3(A_7) +\nabla_2\nabla_4(A_2)\nabla_7(A_4) -\nabla_7(A_4)\nabla^2_2(A_4) -\nabla_2\nabla_4(A_2)\nabla_4(A_7) +\nabla^2_2(A_4)\nabla_4(A_7)$
$+\nabla_2\nabla_5(A_2)\nabla_7(A_5) -\nabla_7(A_5)\nabla^2_2(A_5) -\nabla_2\nabla_5(A_2)\nabla_5(A_7) +\nabla^2_2(A_5)\nabla_5(A_7) +\nabla_2\nabla_6(A_2)\nabla_7(A_6) -\nabla_7(A_6)\nabla^2_2(A_6)$

$-\nabla_2\nabla_6(A_2)\nabla_6(A_7) +\nabla^2_2(A_6)\nabla_6(A_7) +\nabla_7(A_0)\nabla^2_2(A_0) +\nabla_7(A_0)\nabla_0\nabla_2(A_2) +\nabla^2_2(A_0)\nabla_0(A_7) +\nabla_0\nabla_2(A_2)\nabla_0(A_7)$
$+\nabla_7(A_1)\nabla_1\nabla_3(A_3) -\nabla_7(A_1)\nabla^2_3(A_1) -\nabla_1\nabla_3(A_3)\nabla_1(A_7) +\nabla^2_3(A_1)\nabla_1(A_7) +\nabla_7(A_2)\nabla_2\nabla_3(A_3) -\nabla_7(A_2)\nabla^2_3(A_2)$
$-\nabla_2\nabla_3(A_3)\nabla_2(A_7) +\nabla^2_3(A_2)\nabla_2(A_7) +\nabla_3\nabla_4(A_3)\nabla_7(A_4) -\nabla_7(A_4)\nabla^2_3(A_4) -\nabla_3\nabla_4(A_3)\nabla_4(A_7) +\nabla^2_3(A_4)\nabla_4(A_7)$
$+\nabla_3\nabla_5(A_3)\nabla_7(A_5) -\nabla_7(A_5)\nabla^2_3(A_5) -\nabla_3\nabla_5(A_3)\nabla_5(A_7) +\nabla^2_3(A_5)\nabla_5(A_7) +\nabla_3\nabla_6(A_3)\nabla_7(A_6) -\nabla_7(A_6)\nabla^2_3(A_6)$
$-\nabla_3\nabla_6(A_3)\nabla_6(A_7) +\nabla^2_3(A_6)\nabla_6(A_7) +\nabla_7(A_0)\nabla^2_3(A_0) +\nabla_7(A_0)\nabla_0\nabla_3(A_3) +\nabla^2_3(A_0)\nabla_0(A_7) +\nabla_0\nabla_3(A_3)\nabla_0(A_7)$
$+\nabla_7(A_1)\nabla_1\nabla_4(A_4) -\nabla_7(A_1)\nabla^2_4(A_1) -\nabla_1\nabla_4(A_4)\nabla_1(A_7) +\nabla^2_4(A_1)\nabla_1(A_7) +\nabla_7(A_2)\nabla_2\nabla_4(A_4) -\nabla_7(A_2)\nabla^2_4(A_2)$
$-\nabla_2\nabla_4(A_4)\nabla_2(A_7) +\nabla^2_4(A_2)\nabla_2(A_7) +\nabla_7(A_3)\nabla_3\nabla_4(A_4) -\nabla_7(A_3)\nabla^2_4(A_3) -\nabla_3\nabla_4(A_4)\nabla_3(A_7) +\nabla^2_4(A_3)\nabla_3(A_7)$
$+\nabla_4\nabla_5(A_4)\nabla_7(A_5) -\nabla_7(A_5)\nabla^2_4(A_5) -\nabla_4\nabla_5(A_4)\nabla_5(A_7) +\nabla^2_4(A_5)\nabla_5(A_7) +\nabla_4\nabla_6(A_4)\nabla_7(A_6) -\nabla_7(A_6)\nabla^2_4(A_6)$
$-\nabla_4\nabla_6(A_4)\nabla_6(A_7) +\nabla^2_4(A_6)\nabla_6(A_7) +\nabla_7(A_0)\nabla^2_4(A_0) +\nabla_7(A_0)\nabla_0\nabla_4(A_4) +\nabla^2_4(A_0)\nabla_0(A_7) +\nabla_0\nabla_4(A_4)\nabla_0(A_7)$
$+\nabla_7(A_1)\nabla_1\nabla_5(A_5) -\nabla_7(A_1)\nabla^2_5(A_1) -\nabla_1\nabla_5(A_5)\nabla_1(A_7) +\nabla^2_5(A_1)\nabla_1(A_7) +\nabla_7(A_2)\nabla_2\nabla_5(A_5) -\nabla_7(A_2)\nabla^2_5(A_2)$
$-\nabla_2\nabla_5(A_5)\nabla_2(A_7) +\nabla^2_5(A_2)\nabla_2(A_7) +\nabla_7(A_3)\nabla_3\nabla_5(A_5) -\nabla_7(A_3)\nabla^2_5(A_3) -\nabla_3\nabla_5(A_5)\nabla_3(A_7) +\nabla^2_5(A_3)\nabla_3(A_7)$
$+\nabla_7(A_4)\nabla_4\nabla_5(A_5) -\nabla_7(A_4)\nabla^2_5(A_4) -\nabla_4\nabla_5(A_5)\nabla_4(A_7) +\nabla^2_5(A_4)\nabla_4(A_7) +\nabla_5\nabla_6(A_5)\nabla_7(A_6) -\nabla_7(A_6)\nabla^2_5(A_6)$
$-\nabla_5\nabla_6(A_5)\nabla_6(A_7) +\nabla^2_5(A_6)\nabla_6(A_7) +\nabla_7(A_0)\nabla^2_5(A_0) +\nabla_7(A_0)\nabla_0\nabla_5(A_5) +\nabla^2_5(A_0)\nabla_0(A_7) +\nabla_0\nabla_5(A_5)\nabla_0(A_7)$
$+\nabla_7(A_1)\nabla_1\nabla_6(A_6) -\nabla_7(A_1)\nabla^2_6(A_1) -\nabla_1\nabla_6(A_6)\nabla_1(A_7) +\nabla^2_6(A_1)\nabla_1(A_7) +\nabla_7(A_2)\nabla_2\nabla_6(A_6) -\nabla_7(A_2)\nabla^2_6(A_2)$
$-\nabla_2\nabla_6(A_6)\nabla_2(A_7) +\nabla^2_6(A_2)\nabla_2(A_7) +\nabla_7(A_3)\nabla_3\nabla_6(A_6) -\nabla_7(A_3)\nabla^2_6(A_3) -\nabla_3\nabla_6(A_6)\nabla_3(A_7) +\nabla^2_6(A_3)\nabla_3(A_7)$
$+\nabla_7(A_4)\nabla_4\nabla_6(A_6) -\nabla_7(A_4)\nabla^2_6(A_4) -\nabla_4\nabla_6(A_6)\nabla_4(A_7) +\nabla^2_6(A_4)\nabla_4(A_7) +\nabla_7(A_5)\nabla_5\nabla_6(A_6) -\nabla_7(A_5)\nabla^2_6(A_5)$
$-\nabla_5\nabla_6(A_6)\nabla_5(A_7) +\nabla^2_6(A_5)\nabla_5(A_7) +\nabla_7(A_0)\nabla^2_6(A_0) +\nabla_7(A_0)\nabla_0\nabla_6(A_6) +\nabla^2_6(A_0)\nabla_0(A_7) +\nabla_0\nabla_6(A_6)\nabla_0(A_7)$
$-\nabla_1(A_7)\nabla_1\nabla_7(A_7) +\nabla^2_7(A_1)\nabla_1(A_7) +\nabla_7(A_1)\nabla_1\nabla_7(A_7) -\nabla_7(A_1)\nabla^2_7(A_1) -\nabla_2(A_7)\nabla_2\nabla_7(A_7) +\nabla^2_7(A_2)\nabla_2(A_7)$
$+\nabla_7(A_2)\nabla_2\nabla_7(A_7) -\nabla_7(A_2)\nabla^2_7(A_2) -\nabla_3(A_7)\nabla_3\nabla_7(A_7) +\nabla^2_7(A_3)\nabla_3(A_7) +\nabla_7(A_3)\nabla_3\nabla_7(A_7) -\nabla_7(A_3)\nabla^2_7(A_3)$
$-\nabla_4(A_7)\nabla_4\nabla_7(A_7) +\nabla^2_7(A_4)\nabla_4(A_7) +\nabla_7(A_4)\nabla_4\nabla_7(A_7) -\nabla_7(A_4)\nabla^2_7(A_4) -\nabla_5(A_7)\nabla_5\nabla_7(A_7) +\nabla^2_7(A_5)\nabla_5(A_7)$
$+\nabla_7(A_5)\nabla_5\nabla_7(A_7) -\nabla_7(A_5)\nabla^2_7(A_5) -\nabla_6(A_7)\nabla_6\nabla_7(A_7) +\nabla^2_7(A_6)\nabla_6(A_7) +\nabla_7(A_6)\nabla_6\nabla_7(A_7) -\nabla_7(A_6)\nabla^2_7(A_6)$
$+\nabla_0(A_7)\nabla_0\nabla_7(A_7) +\nabla^2_7(A_0)\nabla_0(A_7) +\nabla_7(A_0)\nabla_0\nabla_7(A_7) +\nabla_7(A_0)\nabla^2_7(A_0)$
$\} e_7$

We expect the foundation for Octonion conservation of energy and momentum to be based on a form comparable to the Lorentz covariant 4D space-time differential contraction (divergence) of a stress–energy–momentum tensor, composed from our now Octonion physics field components. Leaving no stone unturned, and for simplicity staying within the intrinsic e basis, it makes sense to try all algebraic invariant products of the form (reference [1] et.al.)

$n [\nabla_i e_i] * [\{\nabla_j A_k (e_j * e_k)\} * \{\nabla_l A_m) (e_l * e_m)\}]$

The constant n is anticipated by our knowledge of the 4D classical stress–energy–momentum tensor where some terms will need scaling by a factor of ±1 or ±½. The Octonion stress–energy–momentum equivalent components, call **S** are then represented by select Octonion algebraic elements in the form of the product of two physics field components:

$\mathbf{S}(j, k, l, m) = [\{\nabla_j A_k (e_j * e_k) + \nabla_k A_j (e_k * e_j)\} * \{\nabla_l A_m (e_l * e_m) + \nabla_m A_l (e_m * e_l)\}]$

We will be selecting all invariant forms for the ordered products $e_i * [(e_j * e_k) * (e_l * e_m)]$ even though the form **S**(j, k, l, m) also includes commuted selections. These additional forms have identical algebraic orientation variance since it is the ordered permutation product rule involved and not the order of any two basis elements in a product that determines the variance/invariance.

Our 8–work–force equivalence outside differentiation algebraically invariant components will then be of the form

$[\nabla_i e_i] * n\, \mathbf{S}(j, k, l, m)$

Algebraic invariant results with this structure partition into nine separate forms which have parallels with

Electrodynamics. When summed, the result will match the invariant portion of **wf**. They are as follows:

Invariant Form 1; for i: 1 to 7

$- \frac{1}{2} [\nabla_0 e_0] * \mathbf{S}(i, 0, i, 0)$

We should recognize this as representing the scalar time rate of change of the energy density maintained within the irrotational fields, and we observe both their energy density and scalar time derivatives are both Octonion algebraic invariants as we might expect.

Invariant Form 2; for i,j: 1 to 7, $i \neq j$

$+ [\nabla_0 e_0] * \mathbf{S}(i, 0, j, i)$

Looking at indexes representative of our fields from classical Electrodynamics, we find the differential expression for the negated scalar time rate of change of the cross product of the electric and magnetic fields, known as the Poynting Vector. So **S**(i, 0, j, i) must represent the negated Octonion equivalent of the Poynting Vector. Since energy flux is an observable, the Octonion Poynting Vector must be and is an algebraic invariant as is its scalar time derivative.

Invariant Form 3; for i: 1 to 6, j: i+1 to 7, $i \neq j$

$- \frac{1}{2} [\nabla_0 e_0] * \mathbf{S}(i, j, i, j)$

We can easily see the differential expression for this algebraic invariant represents the scalar time rate of change for the energy density maintained within the rotational fields. Once again, their energy density and scalar time derivatives are both algebraic invariants.

Invariant Form 4; for i,j: 1 to 7

$+ \frac{1}{2} s [\nabla_j e_j] * \mathbf{S}(i, 0, i, 0) \qquad s = +1 \text{ if } i \neq j \text{ else } s = -1$

We can recognize this as representing selectively + or – the gradient of the energy density maintained within the irrotational fields. The selection of $s = -1$ when the outside differential index is the same as the index of the irrotational field is necessary to match the 8–work–force without outside differentiation.

Invariant Form 5; for i,j: 1 to 7, $i \neq j$

$+ [\nabla_j e_j] * \mathbf{S}(i, 0, j, 0)$

Examining selected differential results for inside **S**(i, 0, j, 0), we see the dyadic products of different electric field components (e.g. $E_x E_y$) just as found in the classical Electrodynamics 4D stress–energy–momentum tensor. But now we have additional irrotational field types. This algebraically invariant form represents the differential contraction of the dyadic products of all irrotational field components.

Invariant Form 6; for i,j: 1 to 7, $i \neq j$

$+ [\nabla_j e_j] * \mathbf{S}(i, 0, j, i)$

The resultant differential expression for this combination of indexes represents the divergence of the Octonion Poynting Vector.

Invariant Form 7; for i: 1 to 6, j: i+1 to 7, $i \neq j$ and k: 1 to 7

$+ \frac{1}{2} s [\nabla_k e_k] * \mathbf{S}(i, j, i, j) \qquad s = -1 \text{ if } k \neq i \text{ and } k \neq j \text{ else } s = +1$

We can recognize this as representing selective negation of the gradient of the energy density maintained within the rotational fields. The selection of s = +1 when the outside differential index is the same as one of the indexes of the rotational field components is necessary to match the 8–work–force without outside differentiation.

Invariant Form 8; for i,j,k: 1 to 7, $i \neq j \neq k$, with i^j^k ≠ 0 implying $\{e_i\ e_j\ e_k\}$ is not a Quaternion triplet

$+ [\nabla_k e_k] * \mathbf{S}(i, j, j, k)$

Here again, "^" is the binary bit-wise exclusive or (xor) logical operator. It only works for us here because the Quaternion subalgebra triplets were partitioned such that the xor of all three indexes is zero.

The final Invariant Form is like Invariant Form 8 but requires rather than excludes i^j^k = 0, implying $\{e_i\ e_j\ e_k\}$ is a Quaternion subalgebra triplet.

Invariant Form 9; for i,j,k: 1 to 7, $i \neq j \neq k$, with i^j^k = 0

$- [\nabla_k e_k] * \mathbf{S}(i, j, j, k)$

Invariant Forms 8 and 9 contain properly signed rotational field dyadic terms.

If we sum all algebraic elements n **S**(j, k, l, m) for each separate outside differentiation index i in our set of nine forms specified above, the result will be our analogous stress–energy–momentum form, call $\mathbf{\Omega}_i$. Since it is a set of Octonion algebraic elements, this could be written generally in the intrinsic e basis as $\Omega_{uv}\ e_v$, and our Invariant(**wf**) matching differential "contraction" written as

Invariant(**wf**) $= [\nabla_u e_u] * \mathbf{\Omega}_u = [\nabla_u e_u] * \Omega_{uv}\ e_v$

The scalar coefficient matrix Ω_{uv} is the analogous Octonion stress–energy–momentum "tensor". Keep in mind we are not doing associative tensor (matrix) algebra here, we are doing non-associative Octonion Algebra. The row-column structure for Ω_{uv} is set up for the Octonion product * in $[\nabla_u e_u] * \mathbf{\Omega}_u$ to give the correct result. Now we could manipulate things by first doing the basis products making the del operator then "look" like a scalar differential contraction ala tensor differential contraction, and make things look more recognizable. We will do this further down to explicitly show the match with EM expectations.

The equality Invariant(**wf**) $= [\nabla_u e_u] * \mathbf{\Omega}_u = [\nabla_u e_u] * \Omega_{uv}\ e_v$ is appropriately form invariant for a global algebraic basis gauge transformation by replacing the e basis references with the gauge g basis and understanding the del operator is a partial on the g basis v position.

As above, we can greatly simplify the presentation if we again singularly represent the irrotational and rotational field types, and define singular representations for each component of the Octonion Poynting vector, the irrotational energy density EI and rotational energy density ER.

Before dipping in, an important aside. The irrotational energy density EI and rotational energy density ER are scalar squared magnitudes. The magnitude of an Octonion algebraic element **A** is called its norm, defined as $[\mathbf{A}*\underline{\mathbf{A}}]^{1/2}$, so we should look at half of the norm squared of the full left side field to represent the total energy density: $ET = EI + ER = \frac{1}{2}\, \mathbf{F}_L * \underline{\mathbf{F}}_L$. The result has both algebraic invariant and variant content. Energy density must be an algebraic invariant. The variant portion is:

$\{+R_{31}R_{57} + R_{15}R_{37} + R_{17}R_{53}\}\ s_2\ e_0$
$\{-I_2R_{64} - I_4R_{26} - I_6R_{42}\}\ s_3\ e_0$
$\{+R_{23}R_{76} + R_{26}R_{37} + R_{36}R_{72}\}\ s_4\ e_0$

$\{-I_1R_{54} - I_4R_{15} - I_5R_{41}\}\ s_5\ e_0$
$\{+R_{12}R_{65} + R_{15}R_{26} + R_{25}R_{61}\}\ s_6\ e_0$
$\{-I_3R_{74} - I_4R_{37} - I_7R_{43}\}\ s_7\ e_0$
$\{+R_{54}R_{76} + R_{57}R_{64} + R_{65}R_{74}\}\ s_8\ e_0$
$\{-I_1R_{23} - I_2R_{31} - I_3R_{12}\}\ s_9\ e_0$
$\{+R_{31}R_{64} + R_{36}R_{41} + R_{43}R_{61}\}\ s_{10}\ e_0$
$\{-I_2R_{57} - I_5R_{72} - I_7R_{25}\}\ s_{11}\ e_0$
$\{+R_{23}R_{54} + R_{42}R_{53} + R_{25}R_{43}\}\ s_{12}\ e_0$
$\{-I_1R_{76} - I_6R_{17} - I_7R_{61}\}\ s_{13}\ e_0$
$\{+R_{12}R_{74} + R_{41}R_{72} + R_{17}R_{42}\}\ s_{14}\ e_0$
$\{-I_3R_{65} - I_5R_{36} - I_6R_{53}\}\ s_{15}\ e_0$

If we were to have used $\mathbf{F}_R$ instead, all odd variances would change sign, unimportant for what follows. Notice the indexes on the rotational field components in the even parity variances are select permutations of the basic quad indexes for the Quaternion subalgebra associated with the structure constant s_n. If we were to require each variance to individually sum to zero, ER = ½ $\mathbf{F}_{rot}*\underline{\mathbf{F}}_{rot}$ becomes an algebraic invariant as required. Each is seen to be an inner product of rotational components living within the Quaternion subalgebra associated with the structure constant. Zero sums would express orthogonality requirements on select rotational field components.

The odd variances are seen to be index permutations of the associated Quaternion subalgebra triplet indexes, now the negated inner product of the three irrotational field components living in that subalgebra with the three remaining rotational components also living in the subalgebra but not in the even variance just above. By also assigning zero values to each sum here, we will form select orthogonality requirements that will make ET = ½ $\mathbf{F}_L*\underline{\mathbf{F}}_L$ an algebraic invariant. Incorporating these orthogonality restrictions, the total field energy density will then be the algebraic invariant sum of rotational and irrotational field energy densities given by ER = ½ $\mathbf{F}_{rot}*\underline{\mathbf{F}}_{rot}$ and EI = ½ $\mathbf{F}_{irr}*\underline{\mathbf{F}}_{irr}$ and each of these will be equivalent to the definitions just below.

When we move on to local algebraic basis gauge transformations, we will find both ½ $\mathbf{F}_{irr}*\underline{\mathbf{F}}_{irr}$ and ½ $\mathbf{F}_{rot}*\underline{\mathbf{F}}_{rot}$ will have variant content, whereas for intrinsic basis and global algebraic gauge basis forms ½ $\mathbf{F}_{irr}*\underline{\mathbf{F}}_{irr}$ will have no variant content.

Define:

$P_j = \sum_{k=1 \text{ to } 7} (-\nabla_k A_0 - \nabla_0 A_k)(\nabla_j A_k - \nabla_k A_j)\ e_j$ 7 components of the Octonion Poynting vector
$EI = ½ \sum_{k=1 \text{ to } 7} (\nabla_k A_0 + \nabla_0 A_k)^2\ e_0$ irrotational field energy
$ER = ½ \sum_{r=1 \text{ to } 6,\ s=2 \text{ to } 7,\ s>r} (\nabla_r A_s - \nabla_s A_r)^2\ e_0$ rotational field energy

Using the intrinsic e basis representation for simplicity, we have for $\mathbf{\Omega}$

$\mathbf{\Omega}_0$ =
$(+EI + ER)\ e_0$
$-P_1\ e_1$
$-P_2\ e_2$
$-P_3\ e_3$
$-P_4\ e_4$
$-P_5\ e_5$
$-P_6\ e_6$
$-P_7\ e_7$

$\mathbf{\Omega}_1$ =
$\{+ER - R_{12}^2 - R_{31}^2 - R_{41}^2 - R_{15}^2 - R_{61}^2 - R_{17}^2 - EI + I_1^2\}\ e_0$
$-P_1\ e_1$

$\{+R_{12}\ R_{23} - R_{41}\ R_{43} + R_{15}\ R_{53} + R_{36}\ R_{61} - R_{17}\ R_{37} + I_1\ I_3\}\ s_9\ e_2$
$\{-R_{23}\ R_{31} + R_{41}\ R_{42} + R_{15}\ R_{25} - R_{26}\ R_{61} - R_{17}\ R_{72} - I_1\ I_2\}\ s_9\ e_3$
$\{-R_{12}\ R_{25} - R_{31}\ R_{53} - R_{41}\ R_{54} + R_{61}\ R_{65} + R_{17}\ R_{57} - I_1\ I_5\}\ s_5\ e_4$
$\{-R_{12}\ R_{42} + R_{31}\ R_{43} + R_{15}\ R_{54} - R_{61}\ R_{64} + R_{17}\ R_{74} + I_1\ I_4\}\ s_5\ e_5$
$\{+R_{12}\ R_{72} + R_{31}\ R_{37} - R_{41}\ R_{74} - R_{15}\ R_{57} - R_{61}\ R_{76} - I_1\ I_7\}\ s_{13}\ e_6$
$\{+R_{12}\ R_{26} - R_{31}\ R_{36} + R_{41}\ R_{64} - R_{15}\ R_{65} + R_{17}\ R_{76} + I_1\ I_6\}\ s_{13}\ e_7$

$\mathbf{\Omega_2}$ =
$\{+ER - R_{12}^2 - R_{23}^2 - R_{42}^2 - R_{25}^2 - R_{26}^2 - R_{72}^2 - EI + I_2^2\}\ e_0$
$\{-R_{12}\ R_{31} + R_{42}\ R_{43} - R_{25}\ R_{53} + R_{26}\ R_{36} - R_{37}\ R_{72} - I_2\ I_3\}\ s_9\ e_1$
$-P_2\ e_2$
$\{+R_{23}\ R_{31} - R_{41}\ R_{42} - R_{15}\ R_{25} + R_{26}\ R_{61} + R_{17}\ R_{72} + I_1\ I_2\}\ s_9\ e_3$
$\{-R_{12}\ R_{61} - R_{23}\ R_{36} - R_{42}\ R_{64} + R_{25}\ R_{65} + R_{72}\ R_{76} - I_2\ I_6\}\ s_3\ e_4$
$\{-R_{12}\ R_{17} + R_{23}\ R_{37} + R_{42}\ R_{74} + R_{25}\ R_{57} - R_{26}\ R_{76} + I_2\ I_7\}\ s_{11}\ e_5$
$\{+R_{12}\ R_{41} - R_{23}\ R_{43} + R_{25}\ R_{54} + R_{26}\ R_{64} - R_{72}\ R_{74} + I_2\ I_4\}\ s_3\ e_6$
$\{+R_{12}\ R_{15} + R_{23}\ R_{53} - R_{42}\ R_{54} - R_{26}\ R_{65} - R_{57}\ R_{72} - I_2\ I_5\}\ s_{11}\ e_7$

$\mathbf{\Omega_3}$ =
$\{+ER - R_{31}^2 - R_{23}^2 - R_{43}^2 - R_{53}^2 - R_{36}^2 - R_{37}^2 - EI + I_3^2\}\ e_0$
$\{+R_{12}\ R_{31} - R_{42}\ R_{43} + R_{25}\ R_{53} - R_{26}\ R_{36} + R_{37}\ R_{72} + I_2\ I_3\}\ s_9\ e_1$
$\{-R_{12}\ R_{23} + R_{41}\ R_{43} - R_{15}\ R_{53} - R_{36}\ R_{61} + R_{17}\ R_{37} - I_1\ I_3\}\ s_9\ e_2$
$-P_3\ e_3$
$\{-R_{17}\ R_{31} - R_{23}\ R_{72} - R_{43}\ R_{74} + R_{53}\ R_{57} + R_{36}\ R_{76} - I_3\ I_7\}\ s_7\ e_4$
$\{+R_{31}\ R_{61} + R_{23}\ R_{26} - R_{43}\ R_{64} - R_{53}\ R_{65} - R_{37}\ R_{76} - I_3\ I_6\}\ s_{15}\ e_5$
$\{+R_{15}\ R_{31} - R_{23}\ R_{25} + R_{43}\ R_{54} + R_{36}\ R_{65} - R_{37}\ R_{57} + I_3\ I_5\}\ s_{15}\ e_6$
$\{-R_{31}\ R_{41} + R_{23}\ R_{42} - R_{53}\ R_{54} + R_{36}\ R_{64} + R_{37}\ R_{74} + I_3\ I_4\}\ s_7\ e_7$

$\mathbf{\Omega_4}$ =
$\{+ER - R_{41}^2 - R_{42}^2 - R_{43}^2 - R_{54}^2 - R_{64}^2 - R_{74}^2 - EI + I_4^2\}\ e_0$
$\{+R_{15}\ R_{41} + R_{25}\ R_{42} - R_{43}\ R_{53} - R_{64}\ R_{65} + R_{57}\ R_{74} + I_4\ I_5\}\ s_5\ e_1$
$\{-R_{41}\ R_{61} + R_{26}\ R_{42} + R_{36}\ R_{43} + R_{54}\ R_{65} - R_{74}\ R_{76} + I_4\ I_6\}\ s_3\ e_2$
$\{+R_{17}\ R_{41} - R_{42}\ R_{72} + R_{37}\ R_{43} - R_{54}\ R_{57} + R_{64}\ R_{76} + I_4\ I_7\}\ s_7\ e_3$
$-P_4\ e_4$
$\{+R_{12}\ R_{42} - R_{31}\ R_{43} - R_{15}\ R_{54} + R_{61}\ R_{64} - R_{17}\ R_{74} - I_1\ I_4\}\ s_5\ e_5$
$\{-R_{12}\ R_{41} + R_{23}\ R_{43} - R_{25}\ R_{54} - R_{26}\ R_{64} + R_{72}\ R_{74} - I_2\ I_4\}\ s_3\ e_6$
$\{+R_{31}\ R_{41} - R_{23}\ R_{42} + R_{53}\ R_{54} - R_{36}\ R_{64} - R_{37}\ R_{74} - I_3\ I_4\}\ s_7\ e_7$

$\mathbf{\Omega_5}$ =
$\{+ER - R_{15}^2 - R_{25}^2 - R_{53}^2 - R_{54}^2 - R_{65}^2 - R_{57}^2 - EI + I_5^2\}\ e_0$
$\{-R_{15}\ R_{41} - R_{25}\ R_{42} + R_{43}\ R_{53} + R_{64}\ R_{65} - R_{57}\ R_{74} - I_4\ I_5\}\ s_5\ e_1$
$\{+R_{15}\ R_{17} - R_{25}\ R_{72} - R_{37}\ R_{53} + R_{54}\ R_{74} - R_{65}\ R_{76} - I_5\ I_7\}\ s_{11}\ e_2$
$\{+R_{15}\ R_{61} - R_{25}\ R_{26} + R_{36}\ R_{53} - R_{54}\ R_{64} + R_{57}\ R_{76} + I_5\ I_6\}\ s_{15}\ e_3$
$\{+R_{12}\ R_{25} + R_{31}\ R_{53} + R_{41}\ R_{54} - R_{61}\ R_{65} - R_{17}\ R_{57} + I_1\ I_5\}\ s_5\ e_4$
$-P_5\ e_5$
$\{-R_{15}\ R_{31} + R_{23}\ R_{25} - R_{43}\ R_{54} - R_{36}\ R_{65} + R_{37}\ R_{57} - I_3\ I_5\}\ s_{15}\ e_6$
$\{-R_{12}\ R_{15} - R_{23}\ R_{53} + R_{42}\ R_{54} + R_{26}\ R_{65} + R_{57}\ R_{72} + I_2\ I_5\}\ s_{11}\ e_7$

$\mathbf{\Omega_6}$ =
$\{+ER - R_{61}^2 - R_{26}^2 - R_{36}^2 - R_{64}^2 - R_{65}^2 - R_{76}^2 - EI + I_6^2\}\ e_0$
$\{+R_{17}R_{61} + R_{26}\ R_{72} - R_{36}\ R_{37} - R_{64}\ R_{74} + R_{57}\ R_{65} + I_6\ I_7\}\ s_{13}\ e_1$

$\{+R_{41}\ R_{61} - R_{26}\ R_{42} - R_{36}\ R_{43} - R_{54}\ R_{65} + R_{74}\ R_{76} - I_4\ I_6\ \}\ s_3\ e_2$
$\{-R_{15}\ R_{61} + R_{25}\ R_{26} - R_{36}\ R_{53} + R_{54}\ R_{64} - R_{57}\ R_{76} - I_5\ I_6\ \}\ s_{15}\ e_3$
$\{+R_{12}\ R_{61} + R_{23}\ R_{36} + R_{42}\ R_{64} - R_{25}\ R_{65} - R_{72}\ R_{76} + I_2\ I_6\ \}\ s_3\ e_4$
$\{-R_{31}\ R_{61} - R_{23}\ R_{26} + R_{43}\ R_{64} + R_{53}\ R_{65} + R_{37}\ R_{76} + I_3\ I_6\ \}\ s_{15}\ e_5$
$-P_6\ e_6$
$\{-R_{12}\ R_{26} + R_{31}\ R_{36} - R_{41}\ R_{64} + R_{15}\ R_{65} - R_{17}\ R_{76} - I_1\ I_6\ \}\ s_{13}\ e_7$

$\mathbf{\Omega}_7 =$
$\{+ER - R_{17}^2 - R_{72}^2 - R_{37}^2 - R_{74}^2 - R_{57}^2 - R_{76}^2 - EI + I_7^2\ \}\ e_0$
$\{-R_{17}R_{61} - R_{26}\ R_{72} + R_{36}\ R_{37} + R_{64}\ R_{74} - R_{57}\ R_{65} - I_6\ I_7\ \}\ s_{13}\ e_1$
$\{-R_{15}\ R_{17} + R_{25}\ R_{72} + R_{37}\ R_{53} - R_{54}\ R_{74} + R_{65}\ R_{76} + I_5\ I_7\ \}\ s_{11}\ e_2$
$\{-R_{17}\ R_{41} + R_{42}\ R_{72} - R_{37}\ R_{43} + R_{54}\ R_{57} - R_{64}\ R_{76} - I_4\ I_7\ \}\ s_7\ e_3$
$\{+R_{17}\ R_{31} + R_{23}\ R_{72} + R_{43}\ R_{74} - R_{53}\ R_{57} - R_{36}\ R_{76} + I_3\ I_7\ \}\ s_7\ e_4$
$\{+R_{12}\ R_{17} - R_{23}\ R_{37} - R_{42}\ R_{74} - R_{25}\ R_{57} + R_{26}\ R_{76} - I_2\ I_7\ \}\ s_{11}\ e_5$
$\{-R_{12}\ R_{72} - R_{31}\ R_{37} + R_{41}\ R_{74} + R_{15}\ R_{57} + R_{61}\ R_{76} + I_1\ I_7\ \}\ s_{13}\ e_6$
$-P_7\ e_7$

We are looking for this methodology to properly represent the conservation of Electrodynamics energy and momentum as a subset of the presentation. In references [1][2] et.al. it was shown a proper home for the electric field is the basis triplet $\{e_5\ e_6\ e_7\}$ and for the magnetic field and additionally the gravitational field is the triplet $\{e_1\ e_2\ e_3\}$, both in rectangular coordinate {x y z} order. Limit the potential functions to $A_0\ e_0$, $A_5\ e_5$, $A_6\ e_6$ and $A_7\ e_7$ with all others 0 which will zero out the gravitational field. We must reduce the I, R, P, ER and EI definitions as follows

$I_k = -\nabla_k A_0 - \nabla_0 A_k$ 3 irrotational field component index k= 5 to 7 : $I_5\ I_6\ I_7$ all others 0

$R_{jk} = \nabla_j A_k - \nabla_k A_j$ 3 rotational field component index where $e_j * e_k = +e_{j\wedge k}$: $R_{76}\ R_{57}\ R_{65}$ only

$P_5 = I_6\ R_{65} - I_7\ R_{57}$ $P_6 = I_7\ R_{76} - I_5\ R_{65}$ $P_7 = I_5\ R_{57} - I_6\ R_{76}$ all others 0

$EI = ½\ \{I^2{}_5 + I^2{}_6 + I^2{}_7\}$

$ER = ½\ \{R^2{}_{76} + R^2{}_{57} + R^2{}_{65}\}$

Doing the reductions on $\mathbf{\Omega}_0\ \mathbf{\Omega}_5\ \mathbf{\Omega}_6\ \mathbf{\Omega}_7$ we are left with

$\mathbf{\Omega}_0 =$
$(+EI + ER)\ e_0$
$-P_5\ e_5$
$-P_6\ e_6$
$-P_7\ e_7$

$\mathbf{\Omega}_5 =$
$\{+ER - R_{65}^2 - R_{57}^2 - EI + I_5^2\ \}\ e_0$
$\{-R_{65}\ R_{76} - I_5\ I_7\ \}\ s_{11}\ e_2$
$\{+R_{57}\ R_{76} + I_5\ I_6\ \}\ s_{15}\ e_3$
$-P_5\ e_5$

$\mathbf{\Omega}_6 =$
$\{+ER - R_{65}^2 - R_{76}^2 - EI + I_6^2\ \}\ e_0$

$\{+ R_{57} R_{65} + I_6 I_7 \}\ s_{13}\ e_1$
$\{- R_{57} R_{76} - I_5 I_6 \}\ s_{15}\ e_3$
$-P_6\ e_6$

$\mathbf{\Omega}_7 =$
$\{+ER - R_{57}^2 - R_{76}^2 - EI + I_7^2 \}\ e_0$
$\{- R_{57} R_{65} - I_6 I_7 \}\ s_{13}\ e_1$
$\{+ R_{65} R_{76} + I_5 I_7 \}\ s_{11}\ e_2$
$-P_7\ e_7$

Now for easier comparison, replace magnetic field components $R_{76} = B_x$, $R_{57} = B_y$, $R_{65} = B_z$ and electric field components $I_5 = E_x$, $I_6 = E_y$, $I_7 = E_z$. Then do the * by e_u product in Invariant(**wf**) = $[\nabla_u\ e_u\ ] * \mathbf{\Omega}_u$ to make the differential contraction with ∇_u look more matrix like. The result after some rearrangement follows. Notice the result is fully an algebraic invariant. This should be expected since our result needs to be an algebraic invariant and we have cast things here such that the analogous "tensor differential contraction" is a scalar operation on a matrix form.

$\mathbf{\Omega}_0 =$
$(EI+ER)\ e_0$
$-P_5\ e_5$
$-P_6\ e_6$
$-P_7\ e_7$

$\mathbf{\Omega}_5 =$
$+P_5\ e_0$
$\{B_x^2-ER+E_x^2-EI\}\ e_5$
$\{B_xB_y+E_xE_y\}\ e_6$
$\{B_xB_z+E_x\ E_z\}\ e_7$

$\mathbf{\Omega}_6 =$
$+P_6\ e_0$
$\{B_yB_x+E_yE_x\}\ e_5$
$\{B_y^2-ER+E_y^2-EI\}\ e_6$
$\{B_yB_z+E_yE_z\}\ e_7$

$\mathbf{\Omega}_7 =$
$+P_7\ e_0$
$\{B_zB_x+E_xE_z\}\ e_5$
$\{B_zB_y+E_zE_y\}\ e_6$
$\{B_z^2-ER+E_z^2-EI\}\ e_7$

Using the $\mathbf{\Omega}$ index for the row and the e index for the column, suppressing the e basis gives the following

$$\begin{bmatrix} EI+ER & -P_5 & -P_6 & -P_7 \\ +P_5 & B_x^2-ER+E_x^2-EI & B_xB_y+E_xE_y & B_xB_z+E_x\ E_z \\ +P_6 & B_yB_x+E_yE_x & B_y^2-ER+E_y^2-EI & B_yB_z+E_yE_z \\ +P_7 & B_zB_x+E_xE_z & B_zB_y+E_zE_y & B_z^2-ER+E_z^2-EI \end{bmatrix}$$

But for missing π and speed of light c coefficient scalings that can be absorbed by a choice of units, unnecessary imaginary unit scaling, and the asymmetry on the Poynting vector components, this is precisely the relativistic 4D Electrodynamics stress–energy–momentum tensor. Doing the math here fully using scalar differential contractions on real components gives the correct result set by the expectations of relativistic Electrodynamics. Its stress–energy–momentum tensor is only symmetric because it includes the imaginary unit scaling not utilized here, where we actually have seven "imaginaries" that have properly been used on the path leading to here.

This result speaks volumes. First, our choice of where to park the electric and magnetic fields in an Octonion framework is legitimized by the fact that the same scope limitations just done on **Ω** shows the **wf** side Electrodynamics work and Lorentz force forms are properly reproduced from the Octonion EM portion SEM "contraction" just covered. The equality of the space-time covariant work–force with the differential contraction of the stress–energy–momentum tensor is the foundation for space-time EM conservation of energy and momentum. We should expect then that our full blown **Ω** algebraic product formulation analogous to SEM tensor contraction, which restates the full 8-D Octonion work–force with an outside differentiation * product, properly frames the forces, work, conservation of energy and momentum for Octonion Dynamics. From the derivation of the Ensemble Derivative, it is apparent that Gauss's Law for integration equating the integral of a vector divergence over an arbitrary volume and the scalar product of the vector and the surface normal over the enclosing surface holds for the Ensemble Derivative. In our Octonion setting, we can thus also integrate over arbitrary volumes, convert divergences into fluxes entering or leaving through the surface enclosing the volume, and balance this out with a change in volume content. This the essence of our conservation of momentum and energy from a physics field perspective.

Drilling down a bit more on conservation, the algebraically invariant scalar result forms are Invariant Forms 1, 3, and 6. They are respectively the time rate of change in EI, the time rate of change in ER, and the divergence of **P**. These three forms sum to an equivalent representation for the scalar (work) portion of the Octonion 8–work–force expression. Thus, for j, k = 1 to 7 we have

$$[\ \nabla_j P_j + \nabla_0 (ER + EI)\ ]\ e_0 \ = \ -\ j_k\ (-\nabla_0 A_k \ - \nabla_k A_0)\ e_0$$

This equation provides the mathematical statement of the conservation of energy after both sides are integrated over any arbitrary volume. The volume integral over the divergence of **P** is converted to an integral of the scalar product of **P** and the outward pointing surface normal vector over the enclosing surface. This represents the net flux of energy leaving the prescribed volume. The volume integral over the time rate of change in field energy density represents the time rate change in the total field energy within the prescribed volume. The volume integral over the Octonion work expression on the right-hand side is the negative rate of increase in mechanical energy within the volume. Moving over the expression on the right side of the equality, all three sum to zero, stating an increase in one necessarily requires a decrease in one or both of the other two. Total energy is neither created or destroyed, it can only move from place to place or change form.

The algebraically invariant non–scalar result forms are Invariant Forms 2, 4, 7, 5, 8 and 9. They are respectively minus the time rate of change for **P**, the modified gradient of EI, the modified gradient of ER, the differential contraction of the irrotational field dyadic products, and the differential contraction of the rotational field dyadic products.

The selectively sign modified gradients on field energy density components summing Invariant Forms 4 and 7 is equivalent to

$$\nabla_j\ [\ (\ ER - EI) + (\nabla_j A_0 + \nabla_0 A_j)^2 \ -\ (\nabla_j A_k - \nabla_k A_j)^2\ ]\ e_j \qquad \text{for } j,k = 1 \text{ to } 7$$

Notice that the sum over j,k of $[\ (\ ER - EI) + (\nabla_j A_0 + \nabla_0 A_j)^2 \ -\ (\nabla_j A_k - \nabla_k A_j)^2\ ]$ is zero.

Thus we have the following for s, j , k, n = 1 to 7 with $j \neq k \neq s$ and $j \neq n$, $j \neq m$

$$- \nabla_0 P_j e_j + \nabla_j [(ER - EI) + (\nabla_j A_0 + \nabla_0 A_j)^2 - (\nabla_j A_n - \nabla_n A_j)^2] e_j$$
$$+ \nabla_k [(- \nabla_0 A_j - \nabla_j A_0) (- \nabla_0 A_k - \nabla_k A_0) + (\nabla_j A_s - \nabla_s A_j)(\nabla_s A_k - \nabla_k A_s)] e_j$$
$$= j_0 (- \nabla_0 A_j - \nabla_j A_0) e_j + j_m (\nabla_j A_m - \nabla_m A_j) e_j$$

This equation is a statement of the conservation of momentum when integrated over an arbitrary volume. The first term on the left side of = represents the negative time rate of change for field momentum within the prescribed volume. The remainder of the left side is converted to an integral over the enclosing surface, representing the net flow of momentum into the volume. The Octonion Lorentz force on the right side of the equation integrates to the time rate of change for mechanical momentum within the volume. Thus, the change in total momentum within the prescribed volume is balanced by the flux of momentum across its enclosing surface, equivalent to the force applied on the prescribed volume by the outside world.

While this has nicely shown the reconstruction of the work-force equation with an outside differentiation on all product terms, it suffers by being a bit "by hand", cherry picking algebraic invariant basis product combinations. It was constructed quite rectilinearly, potentially not adequately producing a fully covariant presentation. Once again, we can look to the proper representation of relativistic Electrodynamics for a better path forward.

At this point, it would be remiss of me not to give thanks to the person whose insight put me on the path I have followed getting to this point. I had the distinct privilege to have a year-long course in Electrodynamics taught by Professor Melvin Schwartz (prior to his Nobel Prize award) while I was an undergraduate Physics major at Stanford University (gack!) a half century ago. His presentation was strictly relativistic, as covered in his book (ref.[10]) which we used in manuscript form since it was not printed until the third quarter of that school year. The book and his lectures stressed the importance of structure and covariance. I smile every time I hark back to his pat answer on why such structure was present, he always said "because it is beautiful". Indeed, it was, and I immediately adopted beautiful structure as a requirement for mathematical physics. Being left fully unsatisfied with Einstein's use of intrinsic curvature to explain Gravitation, I was left thinking relativistic Electrodynamics was only half-correct. Additional structure was missing but had to be consistent with the intrinsic beauty of this cover of Electrodynamics. It became immediately clear to me so many years ago the answer had to come from an increase in dimensions, doubling to twice 4D space-time seemed the way to go. I knew nothing about Octonion Algebra at the time, and spent too many years trying my own "by hand" 8D structures with only a modicum of success. When I finally stumbled across the Kantor and Solodovnikov book on Hypercomplex Numbers (ref.[12]) in the Stanford Bookstore, and saw Octonion Algebra for the first time, I said to myself, "Well, there it is". Thirty-five years later, I am still amazed by the beautiful and directive nature of Octonion Algebra structure.

The relativistic cover of Electrodynamics beautifully presented by Schwartz uses two 4D second-rank field tensors. The divergence of one, **F** provides the inhomogeneous pair of Maxwell's Equations yielding the relativistic 4-current. The divergence of the other, **G** yields the two homogeneous Maxwell's Equations. Both are required to form the covariant stress–energy–momentum tensor Schwartz called **T**, given by

$$T_{\rho\sigma} = 1/8\pi \{ F_{\rho\upsilon} F_{\upsilon\sigma} + G_{\rho\upsilon} G_{\upsilon\sigma} \}$$

It is quite reasonable to presume a more covariant Octonion Algebra cover should pattern off of this, and indeed it is possible. Once again, we will absorb the π leaving it out of the presentation. I hazard to call our Octonion **F** and **G** forms "tensors" for any other reason except familiarity. They are matrices, now with each element a full 8D Octonion algebraic element, but do not transform as tensors do. Covariance is provided by the proper application of a segment of the Ensemble Derivative used to form each matrix element. This is analogous to what I presented in the paper on the derivation of this generally covariant derivative form (reference [6]), where e.g., the spherical-polar divergence was provided by limiting the intrinsic basis index pair to summed like values. The Octonion algebraic structure is ever-present, and intrinsically directive for forming proper covariance.

In typical fashion, we must respect the fact the field components can be formed with left and right applications of the Ensemble Derivative. From the 8-current form above, the analogous left application $F_{\rho\upsilon}$ form $LF_{\rho\upsilon}$ is

$LF_{\rho\upsilon} =$

$$\begin{bmatrix} 0 & -I_1e_1 & -I_2e_2 & -I_3e_3 & -I_4e_4 & -I_5e_5 & -I_6e_6 & -I_7e_7 \\ -I_1e_1 & 0 & +R_{12}s_9e_3 & +R_{31}s_9e_2 & +R_{41}s_5e_5 & +R_{15}s_5e_4 & +R_{61}s_{13}e_7 & +R_{17}s_{13}e_6 \\ -I_2e_2 & +R_{12}s_9e_3 & 0 & +R_{23}s_9e_1 & +R_{42}s_3e_6 & +R_{25}s_{11}e_7 & +R_{26}s_3e_4 & +R_{72}s_{11}e_5 \\ -I_3e_3 & +R_{31}s_9e_2 & +R_{23}s_9e_1 & 0 & +R_{43}s_7e_7 & +R_{53}s_{15}e_6 & +R_{36}s_{15}e_5 & +R_{37}s_7e_4 \\ -I_4e_4 & +R_{41}s_5e_5 & +R_{42}s_3e_6 & +R_{43}s_7e_7 & 0 & +R_{54}s_5e_1 & +R_{64}s_3e_2 & +R_{74}s_7e_3 \\ -I_5e_5 & +R_{15}s_5e_4 & +R_{25}s_{11}e_7 & +R_{53}s_{15}e_6 & +R_{54}s_5e_1 & 0 & +R_{65}s_{15}e_3 & +R_{57}s_{11}e_2 \\ -I_6e_6 & +R_{61}s_{13}e_7 & +R_{26}s_3e_4 & +R_{36}s_{15}e_5 & +R_{64}s_3e_2 & +R_{65}s_{15}e_3 & 0 & +R_{76}s_{13}e_1 \\ -I_7e_7 & +R_{17}s_{13}e_6 & +R_{72}s_{11}e_5 & +R_{37}s_7e_4 & +R_{74}s_7e_3 & +R_{57}s_{11}e_2 & +R_{76}s_{13}e_1 & 0 \end{bmatrix}$$

The required right side Ensemble Derivative application form $RF_{\rho\upsilon}$ is $LF_{\rho\upsilon}$ with all rotational field components R negated. The differential contraction or divergence of $RF_{\rho\upsilon}$ is differentiation from the right, yielding the same result as the differential contraction of $LF_{\rho\upsilon}$ from the left. Both are representations of the Octonion 8-current, where each component is its own algebraic element with non-zero basis element scaling only in the appropriate index.

Unlike the Electrodynamics form, these are fully symmetric instead of fully anti-symmetric. This is necessary because the elements are Octonion and the divergence will include products of basis elements which by design provide the proper result. If instead of forming the divergence, we contract the matrix with a "vector" representation for the 8-current, the result is seen to properly represent the invariant work-force as presented above. The j contraction product is from the left for $LF_{\rho\upsilon}$ and from the right for $RF_{\rho\upsilon}$, yielding identical results.

We found above the homogeneous Octonion Maxwell's Equations were indeed vector identities, but they cleanly partitioned into multiple forms clearly separated by doing the representation in an orientation covariant fashion. We cannot sum all, and then form a matrix product of the sum and properly account for this separate covariance. We must matrix product them individually to properly form their contribution to the Octonion stress–energy–momentum "tensor". Their left application forms $\mathbf{LG}_i$ are as follows with right application forms similarly negating just the rotational field components. Their construction matricizes each partition of the two homogeneous Maxwell's Equations relevant to Quaternion subalgebra triplet Q_i within $\mathbf{G}_i$. As with standard Electrodynamics, the two homogeneous Maxwell's Equations are left side differential contractions on $\mathbf{LG}_i$ and right side differential contractions on $\mathbf{RG}_i$.

$LG_{1\rho\upsilon} =$

$$\begin{bmatrix} 0 & 0 & +R_{64}s_3e_2 & 0 & +R_{26}s_3e_4 & 0 & +R_{42}s_3e_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{64}s_3e_2 & 0 & 0 & 0 & +I_6e_6 & 0 & +I_4e_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{26}s_3e_4 & 0 & +I_6e_6 & 0 & 0 & 0 & +I_2e_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{42}s_3e_6 & 0 & +I_4e_4 & 0 & +I_2e_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$LG_{2\rho\upsilon} =$

$$\begin{bmatrix} 0 & +R_{54}s_5e_1 & 0 & 0 & +R_{15}s_5e_4 & +R_{41}s_5e_5 & 0 & 0 \\ +R_{54}s_5e_1 & 0 & 0 & 0 & +I_5e_5 & +I_4e_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{15}s_5e_4 & +I_5e_5 & 0 & 0 & 0 & +I_1e_1 & 0 & 0 \\ +R_{41}s_5e_5 & +I_4e_4 & 0 & 0 & +I_1e_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$LG_{3\rho\upsilon} =$

$$\begin{bmatrix} 0 & 0 & 0 & +R_{74}s_7e_3 & +R_{37}s_7e_4 & 0 & 0 & +R_{43}s_7e_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{74}s_7e_3 & 0 & 0 & 0 & +I_7e_7 & 0 & 0 & +I_4e_4 \\ +R_{37}s_7e_4 & 0 & 0 & +I_7e_7 & 0 & 0 & 0 & +I_3e_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{43}s_7e_7 & 0 & 0 & +I_4e_4 & +I_3e_3 & 0 & 0 & 0 \end{bmatrix}$$

$LG_{4\rho\upsilon} =$

$$\begin{bmatrix} 0 & +R_{23}s_9e_1 & +R_{31}s_9e_2 & +R_{12}s_9e_3 & 0 & 0 & 0 & 0 \\ +R_{23}s_9e_1 & 0 & +I_3e_3 & +I_2e_2 & 0 & 0 & 0 & 0 \\ +R_{31}s_9e_2 & +I_3e_3 & 0 & +I_1e_1 & 0 & 0 & 0 & 0 \\ +R_{12}s_9e_3 & +I_2e_2 & +I_1e_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$LG_{5\rho\upsilon} =$

$$\begin{bmatrix} 0 & 0 & +R_{57}s_{11}e_2 & 0 & 0 & +R_{72}s_{11}e_5 & 0 & +R_{25}s_{11}e_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{57}s_{11}e_2 & 0 & 0 & 0 & 0 & +I_7e_7 & 0 & +I_5e_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{72}s_{11}e_5 & 0 & +I_7e_7 & 0 & 0 & 0 & 0 & +I_2e_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{25}s_{11}e_7 & 0 & +I_5e_5 & 0 & 0 & +I_2e_2 & 0 & 0 \end{bmatrix}$$

$LG_{6\rho\upsilon} =$

$$\begin{bmatrix} 0 & +R_{76}s_{13}e_1 & 0 & 0 & 0 & 0 & +R_{17}s_{13}e_6 & +R_{61}s_{13}e_7 \end{bmatrix}$$

$$\begin{bmatrix} +R_{76}s_{13}e_1 & 0 & 0 & 0 & 0 & 0 & +I_7e_7 & +I_6e_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{17}s_{13}e_6 & +I_7e_7 & 0 & 0 & 0 & 0 & 0 & +I_1e_1 \\ +R_{61}s_{13}e_7 & +I_6e_6 & 0 & 0 & 0 & 0 & +I_1e_1 & 0 \end{bmatrix}$$

$LG_{7\rho\upsilon} =$

$$\begin{bmatrix} 0 & 0 & 0 & +R_{65}s_{15}e_3 & 0 & +R_{36}s_{15}e_5 & +R_{53}s_{15}e_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{65}s_{15}e_3 & 0 & 0 & 0 & 0 & +I_6e_6 & +I_5e_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +R_{36}s_{15}e_5 & 0 & 0 & +I_6e_6 & 0 & 0 & +I_3e_3 & 0 \\ +R_{53}s_{15}e_6 & 0 & 0 & +I_5e_5 & 0 & +I_3e_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

There are additional homogeneous equations we will need that we discovered above, when we made $\nabla \times \mathbf{F}_{rot}$ an algebraic invariant by forcing sums of like variance results to zero results. As they are zero divergence like the $\mathbf{G}_i$ just covered, we will continue indexing them as follows, requiring the right application forms as always, negating all rotational field components. These are matricized within each $\mathbf{G}_i$ by including rotational field elements within the four non-zero indexes not found in Quaternion subalgebra triplet Q_{i-7}.

$LG_{8\rho\upsilon} =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +R_{57}s_{11}e_2 & 0 & +R_{37}s_7e_4 & 0 & +R_{53}s_{15}e_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +R_{57}s_{11}e_2 & 0 & 0 & 0 & +R_{17}s_{13}e_6 & 0 & +R_{15}s_5e_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +R_{37}s_7e_4 & 0 & +R_{17}s_{13}e_6 & 0 & 0 & 0 & +R_{31}s_9e_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +R_{53}s_{15}e_6 & 0 & +R_{15}s_5e_4 & 0 & +R_{31}s_9e_2 & 0 & 0 \end{bmatrix}$$

$LG_{9\rho\upsilon} =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +R_{76}s_{13}e_1 & 0 & 0 & +R_{37}s_7e_4 & +R_{36}s_{15}e_5 \\ 0 & 0 & +R_{76}s_{13}e_1 & 0 & 0 & 0 & +R_{72}s_{11}e_5 & +R_{26}s_3e_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +R_{37}s_7e_4 & +R_{72}s_{11}e_5 & 0 & 0 & 0 & +R_{23}s_9e_1 \\ 0 & 0 & +R_{36}s_{15}e_5 & +R_{26}s_3e_4 & 0 & 0 & +R_{23}s_9e_1 & 0 \end{bmatrix}$$

$LG_{10\rho\upsilon} =$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & +R_{65}s_{15}e_3 & 0 & 0 & +R_{26}s_3e_4 & +R_{25}s_{11}e_7 & 0 \\
0 & +R_{65}s_{15}e_3 & 0 & 0 & 0 & +R_{61}s_{13}e_7 & +R_{15}s_5e_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +R_{26}s_3e_4 & +R_{61}s_{13}e_7 & 0 & 0 & 0 & +R_{12}s_9e_3 & 0 \\
0 & +R_{25}s_{11}e_7 & +R_{15}s_5e_4 & 0 & 0 & +R_{12}s_9e_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$LG_{11\rho\upsilon} =$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & +R_{76}s_{13}e_1 & +R_{57}s_{11}e_2 & +R_{65}s_{15}e_3 \\
0 & 0 & 0 & 0 & +R_{76}s_{13}e_1 & 0 & +R_{74}s_7e_3 & +R_{64}s_3e_2 \\
0 & 0 & 0 & 0 & +R_{57}s_{11}e_2 & +R_{74}s_7e_3 & 0 & +R_{54}s_5e_1 \\
0 & 0 & 0 & 0 & +R_{65}s_{15}e_3 & +R_{64}s_3e_2 & +R_{54}s_5e_1 & 0
\end{bmatrix}$$

$LG_{12\rho\upsilon} =$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & +R_{64}s_3e_2 & +R_{36}s_{15}e_5 & 0 & +R_{43}s_7e_7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +R_{64}s_3e_2 & 0 & 0 & +R_{61}s_{13}e_7 & 0 & +R_{41}s_5e_5 & 0 \\
0 & +R_{36}s_{15}e_5 & 0 & +R_{61}s_{13}e_7 & 0 & 0 & +R_{31}s_9e_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +R_{43}s_7e_7 & 0 & +R_{41}s_5e_5 & +R_{31}s_9e_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$LG_{13\rho\upsilon} =$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & +R_{54}s_5e_1 & +R_{53}s_{15}e_6 & +R_{43}s_7e_7 & 0 & 0 \\
0 & 0 & +R_{54}s_5e_1 & 0 & +R_{25}s_{11}e_7 & +R_{42}s_3e_6 & 0 & 0 \\
0 & 0 & +R_{53}s_{15}e_6 & +R_{25}s_{11}e_7 & 0 & +R_{23}s_9e_1 & 0 & 0 \\
0 & 0 & +R_{43}s_7e_7 & +R_{42}s_3e_6 & +R_{23}s_9e_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$LG_{14\rho\upsilon} =$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$
\begin{bmatrix}
0 & 0 & +R_{74}s_7e_3 & 0 & +R_{72}s_{11}e_5 & 0 & 0 & +R_{42}s_3e_6 \\
0 & +R_{74}s_7e_3 & 0 & 0 & +R_{17}s_{13}e_6 & 0 & 0 & +R_{41}s_5e_5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +R_{72}s_{11}e_5 & +R_{17}s_{13}e_6 & 0 & 0 & 0 & 0 & +R_{12}s_9e_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & +R_{42}s_3e_6 & +R_{41}s_5e_5 & 0 & +R_{12}s_9e_3 & 0 & 0 & 0
\end{bmatrix}
$$

We will require the matrix product of the left side application field matrix with the right-side application field matrix for each of these individually. Define

$LFRF_{\rho\sigma} = LF_{\rho\upsilon}\, RF_{\upsilon\sigma}$ $\qquad$ $LGRG_{i\rho\sigma} = LG_{i\rho\upsilon}\, RG_{i\upsilon\sigma}$

The stress–energy–momentum matrix of Octonion algebraic elements is found to be built from 15 separate tensor analogous Octonion matrices instead of the two required for the more simplistic 4D EM stress–energy–momentum tensor

$SEM_{\rho\sigma} = -\tfrac{1}{2}\{ LFRF_{\rho\sigma} - \Sigma_{i=1 \text{ to } 7} LGRG_{i\rho\sigma} - \Sigma_{\rho=\sigma, i=8 \text{ to } 14} LGRG_{i\rho\sigma} + 3\, \Sigma_{\rho\neq\sigma, i=8 \text{ to } 14} LGRG_{i\rho\sigma} \}$

The stress–energy–momentum "tensor" produced by the select products method above is found to be

$\mathbf{\Omega}_\rho = \Sigma_\sigma SEM_{\rho\sigma}$

You might be wondering where the scale by 3 comes from in the last term above. It turns out for particular $\rho\neq\sigma$ in $LFRF_{\rho\sigma}$, the wrong sign is produced on terms requiring non-fractional magnitudes after the indicated division by two. Basically, we have forms analogous to $\tfrac{1}{2}(-a + 3a) = +a$ accomplishing this sign change, nicely accommodated by all $LGRG_{i\rho\sigma}$ as indicated.

Once again, our algebraic invariant work–force equivalence with an outside differentiation on all product terms is given by

Invariant(**wf**) = $[\nabla_u\, e_u\,] * \mathbf{\Omega}_u$

Moving to a local algebraic basis gauge transformation requires us to account for the additional terms related to the covariant derivative connection. Reviewing what was covered in some detail in reference [11], the covariant derivative **E** takes the form

$\mathbf{E}(\mathbf{A}(\mathbf{v})) = s_{ik(i\wedge k)}\ \{ g_{(i\wedge k)}\, \partial/\partial v_i\, [A_k] + g_p\, \Gamma^p{}_{ik}\, A_k \}$ where the transformation specific connection takes the form

$\Gamma^p{}_{ik} = \partial/\partial v_i\, [T_{(i\wedge k)n}]\, T_{pn}$

The first part of the sum gives the results we just produced in the intrinsic basis, form invariant with any global algebraic basis transformation representation, where $\Gamma^p{}_{ik} = 0$ for all i,k,p. The covariant Ensemble Derivative takes the shape of the del operator algebraic element * product used above.

For the 8–work–force side of the conservation equations, we have full applications of the covariant derivative. They are fully covariant representations, needing no form modification for any transformed basis whether or not we have a non-zero connection. If the transformation is an algebraic basis gauge transformation yielding a basis system isomorphism with the Octonion intrinsic e basis, our analogous SEM differential contraction is also form appropriate as stated above. Each individual i,k: $(\nabla_i\, e_i) * (A_k\, e_k) = \nabla_i\, A_k\, s_{ik(i\wedge k)}\, e_{(i\wedge k)}$ used above must be replaced with the following

$\mathbf{E}_i (A_k) = s_{ik(i\hat{}k)} \{ g_{(i\hat{}k)} \partial/\partial v_i [A_k] + g_p \Gamma^p{}_{ik} A_k \}$ where only the sum over p remains.

Our Octonion analogy to stress–energy–momentum tensor differential contraction for local algebraic basis gauge transformations now is written as

Invariant(**wf**) = $\mathbf{E}_u * \mathbf{\Omega}_u$

The SEM component general form S[i,j,k,l] becomes $[\{\mathbf{E}_i (A_j) + \mathbf{E}_j (A_i)\} * \{ \mathbf{E}_k (A_l) + \mathbf{E}_l (A_k)\}]$

Expanding

$\{\mathbf{E}_i (A_j) + \mathbf{E}_j (A_i)\} = g_{(i\hat{}j)} \{ s_{ij(i\hat{}j)} \partial/\partial v_i [A_j] + s_{ji(i\hat{}j)} \partial/\partial v_j [A_i] \} + g_p \{ s_{ij(i\hat{}j)} \Gamma^p{}_{ij} A_j + s_{ji(i\hat{}j)} \Gamma^p{}_{ji} A_i \}$

$\{\mathbf{E}_k (A_l) + \mathbf{E}_l (A_k)\} = g_{(k\hat{}l)} \{ s_{kl(k\hat{}l)} \partial/\partial v_k [A_l] + s_{lk(k\hat{}l)} \partial/\partial v_l [A_k] \} + g_q \{ s_{kl(k\hat{}l)} \Gamma^q{}_{kl} A_l + s_{lk(k\hat{}l)} \Gamma^q{}_{lk} A_k \}$

If we generally have $\{ s_{ij(i\hat{}j)} \Gamma^r{}_{st} A_t + s_{ji(i\hat{}j)} \Gamma^r{}_{ts} A_s \} = 0$ for each r, s and t, then our local algebraic basis gauge SEM matrix Ω_{uv} will be form invariant with the intrinsic/global basis form. At this point it is informative to point out that since the exclusive-or logic function commutes, the differentiated components of transformation matrix T are the same for $\Gamma^r{}_{st}$ as they are for $\Gamma^r{}_{ts}$. The only difference between $\Gamma^r{}_{st}$ and $\Gamma^r{}_{ts}$ is the differentiation index changes from s to t.

Assume the local algebraic basis gauge parametrization of T is the set $[\alpha_m]$ where generally all have full position dependency. We can apply the chain rule for differentiation to each scalar piece of the connection:

$\Gamma^p{}_{ij} = \partial/\partial v_i [T_{(i\hat{}j)n}(\alpha_m)] T_{pn} = \partial(\alpha_m)/\partial v_i \, \partial/\partial\alpha_m [T_{(i\hat{}j)n}(\alpha_m)] T_{pn}$ thus, we have

$\{ s_{ij(i\hat{}j)} \Gamma^p{}_{ij} A_j + s_{ji(i\hat{}j)} \Gamma^p{}_{ji} A_i \} = \{ s_{ij(i\hat{}j)} \partial(\alpha_m)/\partial v_i A_j + s_{ji(i\hat{}j)} \partial(\alpha_m)/\partial v_j A_i \} \partial/\partial\alpha_m [T_{(i\hat{}j)n}(\alpha_m)] T_{pn}$

From this, we will have form invariance between the local algebraic basis gauge form and the intrinsic/global form for both the SEM matrix Ω_{uv} and all physics field components if we restrict the potential function solutions with $\{ s_{st(s\hat{}t)} \partial(\alpha_r)/\partial v_s A_t + s_{ts(s\hat{}t)} \partial(\alpha_r)/\partial v_t A_s \} = 0$ for each r, s and t.

There is great parsimonious utility in doing this, but is it physically meaningful? On the Octonion SEM “contraction” side, if **Ω** is a local gauge invariant, since the outside differentiation applied is the sum of the intrinsic/global basis differentiation and the non-differentiation connection, the result will be the sum of the intrinsic/global gauge invariant differentiated form and a new product of the connection form and **Ω**. On the **wf** side, cutting off terms added to the physics fields early keeps them from propagating into the next order level of differentiations, which will have their own additions that we may want to or need to also dispatch with further restrictions on the potential functions to remove them.

The end of the compounded differentiation road is the 8–current continuity equation, which is a third application. This is a conservation equation we should expect to be valid in any basis. This differentiation is a scalar divergence, where the differentiating basis index matches the basis index of the functional. Reviewing the scalar divergence connection forms for a local algebraic basis gauge transformation we have

$\Gamma^p{}_{ik} = \partial/\partial v_i [T_{(i\hat{}k)n}] T_{pn} \rightarrow \Gamma^p{}_{ii} = \partial/\partial v_i [T_{0n}] T_{pn}$

The only non-zero term for T_{0n} is T_{00} and it is a constant. The scalar divergence connection terms for each index are $\Gamma^p{}_{ii} = 0$, making the divergence always index by index a local algebraic basis gauge invariant, independent of any T parametrization. If the 8–current is made a local algebraic basis gauge invariant, then the continuity equation will end up identically the required zero value as it is for the intrinsic and global gauge basis. If the 8–current is not a local gauge invariant, the continuity equation will carry forward the 8–current added connection terms. This would likely take the result to a non-zero value which would need to be offset by added requirements on the potential functions.

Furthermore, we need the equality Invariant(**wf**) = $\mathbf{E}_u * \mathbf{\Omega}_u$ to hold. This mandate may be rewritten as equations of constraint identifying a zero result for each algebraic variance-basis element combination in

Invariant(**wf**) – $\mathbf{E}_u * \mathbf{\Omega}_u = \mathbf{0}$ where **0** is the null Octonion algebraic element

Opening up to local algebraic basis gauge transformations leaves us many paths to follow. The continuity equation zero result and Invariant(**wf**) – $\mathbf{E}_u * \mathbf{\Omega}_u = \mathbf{0}$ are absolutes. We could carry the complexity of unrestricted potential functions all the way to these two requirements, then dig in. The alternative is to nip things in the bud at earlier, that is lower order differentiation constructions, forming restrictions on the potential functions. At this point, we are given no clear-cut way to proceed other than not violating the two aforementioned requirements. We also have a bit of a "chicken or the egg" thing. Do we pick a transformation parametrization and force restrictions on potential functions, or do we pick the potential functions and force a compatible parametrization, or maybe a bit of both?

Rather than continuing on with the complexities of even a simple local algebraic basis gauge transformation, I think I will close out this document with some concluding remarks.

Is the full mathematical cover of nature intrinsically associative? Or does it require an algebraic structure with some degree of non-associative product structure? If the latter, it might be suboptimal to follow a discovery path steeped in associative group theory, or tensor calculus and the like. One cannot argue with the success in advancing knowledge these fundamental constructs have provided over the years, but issues finding something that could reasonably be called Grand Unification in some sense, seems to be saying something is missing. Will the missing link be found by hammering harder on the Standard Model group theoretical structure? If something more is required, will the associative nature group concepts of the Standard Model be directive enough? These are all important questions without definitive answers.

In contrast, Octonion Algebra involves commutative, non-commutative, associative and non-associative product content with algebraic structure defining how all of these play well together. A betting person might hedge their position by betting on a path that covers all of the bases. So why then have Octonions gotten so little attention?

The answer is mostly two-fold. The complexity of the algebra makes pencil on paper work impractical. One cannot do meaningful work without the aid of a computer *and* suitable software. Personal computers are readily available, but the application software for the most part is not. Even if effective software would be available, its use would require significant computer programming skills on top of math and physics ability, cutting off people whose specialization path limited one or more of these skill sets. But it is somewhat worse than this. Purchased software is what it is, and does only as much as the software developers knew to put in it. As knowledge through use progresses, so progresses demands for enhanced software capabilities. Software for sale requires a big enough market to profitably offset the development and maintenance costs, a fundamental obstacle for Octonions in the first place with such a small number of interested people that would be willing to buy. Enhancements must also be justified on a profit basis. So even if front line mathematical physicist users desire enhanced functionality, there may be no business justification for it.

Then there is the paradigm the professional academic must live within. Academia is structured for incremental improvements to dogmatic ideological practices. A maverick with new ideas and approaches would have a tough sell, and the risks involved not demonstrating significant and consistent progress could make such a decision a career ender.

I embarked on my chosen career path when microprocessors were first becoming available. Besides making personal computers possible, they created a paradigm shift in electronics, where today the use of microprocessors has become ubiquitous. Designing microprocessor-based product hardware, firmware and support software provided a good career as well as an opportunity for me to develop my programming skills to the benefit of my long-standing love of mathematical physics. Recognizing the need years ago for a good symbolic algebra software tool to do the heavy lifting applying Octonion Algebra to mathematical physics, I started development of my software tool which is now on its third major revision. The beauty of having developed it myself is that when I found it beneficial to add functionality, I could make the changes myself, rather than being dependent on business concerns and mathematical physics understanding of career software engineers. With thousands of hours of development/use/debug time, my software has become quite useful. Without it the conclusions I have come to in the several papers I have put into the public domain would not have been possible.

For now, this software is private to me. The market for it is too small at this time to monetize it. I would be surprised if I could recoup 10 cents for every hour of development time invested, and this would come at lost opportunity cost not being able to adequately continue my own creative work due to necessary support efforts. I fully appreciate difficulties people may have absorbing the information I have provided to the point of not being able to do an adequate peer review that might enhance acceptance, leading to increased interest in Octonion Algebra, stimulating growth of a suitable market size where today it is inadequate.

I hold out hope readers will appreciate the structural beauty Octonion Algebra provides us. Its generally non-associative nature scares off some people who look to necessarily associative group theory for answers. This is short-sighted since there are many connections to be made between Octonion Algebra and group theory. At the bedrock level, what might be called pre-algebraic, there is a very nice group connection for every order 2^n hypercomplex algebra and the Boolean logic exclusive-or operator put to good use above. Assigning basis element indexes zero for scalar and non-scalar bases 1 through $2^n - 1$, all basis element products may be expressed within sign as $e_a * e_b = \pm e_{a\wedge b}$. Binary integers 0 through $2^n - 1$ form Dedekind groups under group operation ^. These groups and all of their (always normal) subgroups provide the basis element product structure of each hypercomplex algebra and all of its subalgebras one-for-one if basis element product negative signage is dropped (hence the notion pre-algebraic). This is covered in detail in reference [4].

Orientation choices for all division algebras are found to be given by Hadamard Matrices, whose rows or columns form a group under positional product composition. This is covered in detail in references [1], [5] et.al. These matrix compositions also give us the outline for the proof of the product of variance indicators presented above and covered in detail within reference [2] et.al.

Then of course there is the group PSL(2,7), the automorphism group of the Fano Plane and hence for all Octonion Algebra orientations respecting Quaternion subalgebra triplet partitioning. It gives us the full group of basis element permutations providing the full set of equivalent basis element multiplication tables. This is discussed in nearly all of my public works.

The two algebraic basis gauge transformations discussed in detail within reference [11] were shown to each form groups under composition.

With so many connections to group theory, fear of the non-associative nature of Octonion Algebra should be set aside. Previous work forming matrices from left and right actions of basis products to achieve an associative framework suitable for direct injection to group theory (reference [13]) to me cuts the beautiful Octonion Algebra and its directive nature off at the knees.

I find it hard to believe Octonion Algebra Dynamics gives plausible potential function mathematical physics by accident or coincidence. The cover of Electrodynamics as a subset of the presentation of its dynamics is complete, and the additional structure provides a home for a compatible integrated potential function cover of Gravitation (reference [1] et.al.). As so titled in this reference, Octonion Algebra truly is *The Algebra of Everything*.

References

[1] Richard D. Lockyer, 2012 FQXi Essay *The Algebra of "Everything"*, August 31, 2012
https://fqxi.org/data/essay-contest-files/Lockyer_fqxi_essay_RickLock.pdf

[2] Richard D. Lockyer, 2018 FQXi Essay *Truth*, February 5, 2018
https://fqxi.org/data/essay-contest-files/Lockyer_Truth_RickLockyer_F.pdf

[3] Richard D. Lockyer, December 2020 *An Algebraic proof Sedenions are not a division algebra and other consequences of Cayley-Dickson Algebra definition variation*
https://vixra.org/pdf/2010.0086v3.pdf

[4] Richard D. Lockyer, January 2022 *The Exclusive Or Group X(n) Correspondence With Cayley-Dickson Algebras*
https://vixra.org/pdf/2201.0095v1.pdf

[5] Richard D. Lockyer, February 2022 *Hadamard Matrices And Division Algebras Only*
https://vixra.org/pdf/2202.0072v1.pdf

[6] Richard D. Lockyer, February 2022 *Division Algebra Covariant Derivative*
https://vixra.org/pdf/2202.0154v1.pdf

[7] Richard D. Lockyer, March 2022 *Octonions, Broctonions and Sedenions*
https://vixra.org/pdf/2203.0098v1.pdf

[8] J. Baez, *The Octonions*. Bulletin of the American Mathematical Society, 39:145-205, 2002

[9] Richard D. Lockyer, FQXi Forum post *Conservation of Octonion Energy and Momentum*, March 30, 2018
https://fqxi.org/data/forum-attachments/chapter_10.pdf

[10] Melvin Schwartz, *Principles of Electrodynamics*, McGraw-Hill 1972

[11] Richard D. Lockyer, October 2022 *Octonion Automorphisms as Algebraic Basis Gauge Transformations*
https://vixra.org/pdf/2210.0014v2.pdf

[12] I.L. Kantor and A.S. Solodovnikov, *Hypercomplex Numbers*, Springer Verlag, 1989

[13] Geoffrey Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics* Kluwer Academic Publishers, 1994

