Effective Field Theory as Asymptotic Regime of Chaotic Dynamics

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Abstract

We argue here that the onset of classical chaos above the Fermi scale underlies the construction of Effective Field Theory (EFT). According to this view, particle physics and gravitational dynamics are low-energy manifestations of chaotic behavior and multifractal geometry.

Key words: classical chaos, multifractals, critical phenomena, decoherence, normal form equations, effective field theory.

1. Introduction and Motivation

For far too long, the mainstream research in high-energy theory has overlooked two key facts concerning the dynamics of fundamental fields. In particular,

a) Quantum Field Theory (QFT) is a manifestation of *critical phenomena*, which in turn, can be shown to be equivalent to classical *deterministic chaos* [1-3]. This observation is reinforced by the Renormalization Group (RG) program, which describes both critical phenomena and the transition to *classical chaos* within a unified framework. It is important to set a clear distinction between classical chaos and *quantum chaos*, the latter dealing with the quantum dynamics of systems that are classically chaotic [4].

b) *Universality* of nonlinear dynamics: large N-body gravitational systems, many Hamiltonian systems, and ensembles of unstable fields exhibit similar phase-space behavior in their transition to chaos [5]. There is growing evidence that, under certain conditions, the RG flow also evolves towards fully developed chaos, which is relevant to both condensed matter applications and the Standard Model of particle physics (SM) [6-7].

It is our view that, to a large extent, many of today's challenges confronting high-energy theory are the result of these oversights. The goal of this work is to suggest that, accounting for a) – b), opens an unexplored path to the underlying physics of EFT, including the SM and General Relativity (GR).

A key ingredient of this viewpoint is the *decoherence* process, which is likely to develop far above the Fermi scale of electroweak interactions. Decoherence amounts to the destruction of quantum interference in open systems, systems exposed to persistent noise or ensembles evolving outside thermodynamic equilibrium. A reasonable expectation is that, if decoherence sets in somewhere above the Fermi scale, unstable systems of interacting quantum fields are prone to turn classical and flow towards chaos in a universal way. This observation is consistent with the tendency of Hamiltonian systems to become *nonintegrable* under time-dependent perturbations or in the long-term limit [8-10].

An upfront distinction must also be drawn between the ideas discussed here and the formalism of *non-equilibrium QFT*, as the latter ignores the transition to classical chaos driven by decoherence [11].

The paper is organized as follows: next section details our perspective on the interplay between critical behavior and classical chaos. Section 3 covers critical behavior in continuous dimension and brings into focus the link between multifractal geometry and EFT. The Appendix surveys the topic of normal form equations and their relation to the universal approach to chaos in the dynamics of complex systems.

2. Critical behavior as chaotic dynamics

Characterization of classical dynamical systems edging towards chaos can be done through several related concepts. Two familiar concepts are the Lyapunov exponents and the Kolmogorov (K-) entropy, as generic measures of *dynamical instability*. Let λ_L^{Σ} denote the sum of positive Lyapunov exponents $\lambda_{L,i}$, that is,

$$\lambda_L^{\Sigma} = \sum_i \lambda_{L,i}\,; \quad \lambda_{L,i} > 0 \tag{1}$$

Under certain conditions, K-entropy is defined as integral of (1) over the phase-space, i.e.

$$S_k = \int_{\Omega} \lambda_L^{\Sigma}\, d\rho \tag{2}$$

in which $d\rho$ denotes the differential phase-space measure.

It is known that critical phenomena describe second-order phase transitions in which the scale of correlations becomes unbounded. At the transition point, as the control parameter of the system nears a critical value ($\lambda \to \lambda_c$), strong fluctuations develop and the relevant physical parameters either diverge or vanish. The correlation function of any fluctuating quantity $a(x)$ defined at two separate points decays according to

$$\langle \delta a(x)\delta a(0)\rangle \propto \exp(-x/\xi)\,; \quad |x| \to \infty \tag{3}$$

and the correlation length scales as

$$\xi \propto (\lambda - \lambda_c)^{-\nu} \tag{4}$$

The approach to criticality means the onset of a *conformal state* characterized by vanishing masses, i.e.

$$m \propto \xi^{-1} \propto (\lambda - \lambda_c)^{\nu} \tag{5}$$

As pointed out in the Introduction, critical behavior and classical chaos are complementary formulations of the underlying dynamics. It follows that a close relationship must exist between (1)-(2) and (3)-(5) as detailed below:

1) First off, the *relaxation time* to equilibrium (τ) may be defined as

$$\tau \propto \frac{1}{\lambda_L^{\Sigma}} = (\frac{dS_k}{d\rho})^{-1}; \quad S_K < \infty \tag{6}$$

Relation (6) states that a system having a vanishing K-entropy rate relaxes infinitely slow to thermodynamic equilibrium. The converse is that singular K-entropy rates correspond to systems instantaneously reaching equilibrium. Note that a zero K-entropy rate ($dS_K/d\rho \to 0$) *does* *not preclude* a thermalized state described by a singular and stationary K-entropy ($S_K \to \infty$).

2) Secondly, the correlation length (4) and the relaxation time (6) are taken to be commensurate in magnitude, i.e. [1-2].

$$\xi \propto \tau \propto (\frac{dS_k}{d\rho})^{-1}; \quad S_K < \infty \tag{7}$$

On account of (4) and near criticality, the free energy F (the analog of Lagrangian for a field theory in d spacetime dimensions), the field average over a lattice of points and the external current can be written as [12-13]

$$F \propto \xi^{-d} \propto (\lambda - \lambda_c)^{\nu d} \tag{8}$$

$$\langle \varphi \rangle \propto \xi^{-\frac{1}{2}(d-2+\eta)} \propto (\lambda - \lambda_c)^{\frac{\nu}{2}(d-2+\eta)} \tag{9}$$

$$J \propto \xi^{-\frac{1}{2\delta}(d-2+\eta)} \propto (\lambda - \lambda_c)^{\frac{\nu}{2\delta}(d-2+\eta)} \tag{10}$$

In (9) and (10), η and δ represent two critical exponents controlling, respectively, the behavior of field correlations and of the external current. It is apparent from (5), (8)-(10) that the spectrum of masses, free energy, field average and external current are *self-similar functions* with respect to the deviation from criticality ($\lambda - \lambda_c$).

Analysis of far-from-equilibrium and multi-variable systems reveals that their long-time evolution ($t \to \infty$) reduces to a lower dimensional set of *normal-form equations* with a single emerging variable z playing the role of an effective order parameter [Appendix]. The structure of these universal equations depends on whether external perturbations are stationary (independent of t) or oscillatory (periodic in t). The Appendix section indicates that two scenarios are possible, namely,

a) When perturbations are stationary, the normal form equations describe one-dimensional *parametric bifurcations* [saddle-node (A6), pitchfork (A7) or transcritical (A8), respectively]. A basic model of pitchfork bifurcations can be shown to generate the electroweak sector of the SM [14, 19]. Bifurcations may also lie at the root of the spin-statistics theorem of Quantum Mechanics [21].

b) When perturbations are oscillatory with frequency ω_0, the resulting normal form equation represents a Stuart-Landau oscillator. This is a particular embodiment of the complex Ginzburg-Landau equation (CGLE), which is a *universal model* for the long-wavelength dynamics of complex systems. CGLE sets up a non-perturbative framework likely to play a key role in the physics of high-energy interactions, beyond the methodology of Feynman diagrams [15].

In addition, when the normal form equations (A6-A8) are turned into iterated maps, the control parameter follows a Feigenbaum-like series of the form $\lambda_n - \lambda_c \propto \delta^{-n}$, where n is the iteration index. The transition to chaos in (A6-A8) involves a *hierarchical pattern* of timescales. The impact of these observations on the physics of EFT is briefly surveyed in the next section.

3. Critical behavior in continuous dimensions

As phase transitions abound in Nature, the control parameter λ can take on many forms, from temperature to pressure, density, chemical potential, number of occupied lattice sites, excitation threshold, characteristic system size and so on. An important (and yet insufficiently appreciated) control parameter of field theory is the continuous *dimensional deviation* from four space dimensions defined as [6]

$$\varepsilon = 4 - D = O(m^2 / \Lambda_{UV}^2) << 1 \tag{11}$$

where Λ_{UV} stands for the large ultraviolet cutoff of the theory. The expectation is that (11) becomes relevant in *far-from-equilibrium* conditions, prone to develop well above the Fermi scale. In particular, (11) arises from several premises, namely,

1) Dimensional Regularization of QFT,

2) Emergence of nontrivial fixed points of the RG equations in statistical physics and the ε-expansion evaluation of critical exponents,

3) Emergence of fractal spacetime from the fragmented structure of phase-space in Hamiltonian chaos [16].

There are few straightforward consequences resulting from the onset of (11) above the Fermi scale, as discussed below.

a) By (5) and (11), taking $\lambda = \varepsilon$ and $\varepsilon_c = 0$ at $D = 4$, recovers the critical exponent of the correlation length in the mean-field approximation of Landau theory ($\nu = 1/2$).

b) By (7) and (11), taking again $\lambda = \varepsilon$ and $\varepsilon_c = 0$ at $D = 4$, one obtains

$$\frac{dS_K}{d\rho} \propto \varepsilon^{\nu} \tag{12}$$

The K-entropy can be alternatively defined using the concept of *information dimension* $D_1(\mu)$, i.e.

$$D_1(\mu) = -\frac{S_K(\mu)}{\log \varepsilon(\mu)} \tag{13}$$

Here, μ is the sliding scale of the RG flow associated with a time-like parameter as in

$$dt = d(\log \mu) = \frac{d\mu}{\mu} \tag{14}$$

Relation (13) leads to

$$\frac{dS_K(\mu)}{d\rho} = -\frac{D_1(\mu)}{\varepsilon(\mu)} \frac{\beta_\varepsilon(\varepsilon)}{\beta_\rho(\rho)} \tag{15}$$

in which the "beta-functions" of the dimensional deviation and the phase space measure are respectively given by

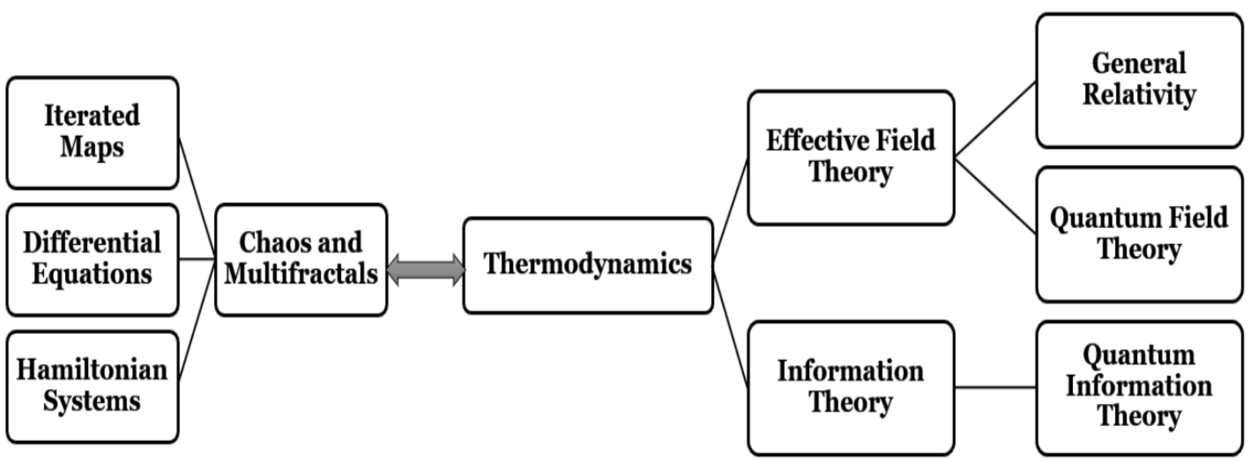
$$\beta_\varepsilon(\varepsilon) = \frac{d\varepsilon}{d\mu} \tag{16a}$$

$$\beta_\rho(\rho) = \frac{d\rho}{d\mu} \tag{16b}$$

Combined use of (12), (13) and (15) yields the following constraint between (16a) and (16b)

$$\beta_\varepsilon(\varepsilon) \propto -\frac{\varepsilon^{1+\nu}}{D_1} \beta_\rho(\rho) \approx -\frac{\varepsilon^{3/2}}{D_1} \beta_\rho(\rho) \tag{17}$$

Invoking again (5) and (8)-(10) shows that observables of interest are self-similar functions of the dimensional deviation $\varepsilon(\mu)$, which in turn, embodies the *information content* of the K-entropy. Our research reveals that various aspects of $\varepsilon(\mu)$ lie behind the multifractal geometry of EFT, the repetitive architecture of SM parameters, the Cantor Dust structure of Dark Matter and the thermodynamic interpretation of GR [6, 20]. Inspired by these findings, the diagram below highlights the connections between classical chaos, multifractal geometry, EFT and Information Theory.



APPENDIX:

<u>Reduction of complex dynamics to normal-form equations</u>

The emergence paradigm of complex dynamics hints that all field theories evaluated at low-energy scales arise from an underlying system of high-energy entities called *primary variables*. Let the high-energy sector of field theory be described by a large set of such variables $x \equiv \{x_i\}$, $i = 1, 2, ..., n$, $n >> 1$ whose mutual coupling and dynamics is far-from-equilibrium. The specific nature of the high-energy variables is irrelevant to our context, as they can take the form of irreducible objects such as, but not limited to, spinors, quaternions, twistors, octonions, strings, branes, loops, knots, information bits and so on.

The downward flow of $x \equiv \{x_i\}$ may be mapped to a system of ordinary differential equations having the universal form

$$x_t' = f(x(t), \lambda(t), D(t)) \tag{A1}$$

Here, λ, t, D denote, respectively, the control parameters vector $\lambda = \{\lambda_k\}, k = 1, 2, \ldots m$, the evolution parameter and the dimension of the embedding space. If the dimension of the embedding space is taken to be an independent variable or control parameter, the system (A1) further reduces to

$$x_t' = f(x(t), \lambda(t)) \tag{A2}$$

It is sensible to assume that the flow (A1) or (A2) occurs in the presence of perturbations induced by far-from-equilibrium conditions. These may surface, for example, from primordial density fluctuations in the early Universe or from unbalanced vacuum fluctuations in the high-energy regime of QFT.

To make explicit the effect of perturbations, we resolve $x(t)$ into a reference stable state $x_s(t)$ and a deviation generated by perturbations, i.e.,

$$x(t) = x_s(t) + y(t) \tag{A3}$$

Direct substitution in (A2) yields the set of homogeneous equations

$$y_t' = f(\{x_s + y\}, \lambda) - f(\{x_s\}, \lambda) \tag{A4}$$

Further expanding around the reference state leads to

$$y_t' = \sum_j L_{ij}(x_s, \lambda) y_j + h_i(\{y_j\}, \lambda) \tag{A5}$$

where L_{ij} and h_i denote, respectively, the coefficients of the linear and nonlinear contributions induced by departures from the reference state. Here, L_{ij} represents a $n \times n$ matrix dependent on the reference state and on the control parameters vector. Under the assumption that parameters λ stay close to their critical values $(\lambda = \lambda_c)$, it can be shown that (A5) undergoes bifurcations and its behavior can be mapped to a closed set of universal equations referred to as normal forms [17-18] . If, at $\lambda = \lambda_c$ perturbations are non-oscillatory (steady state), the *normal form equations* are

$$z_t' = (\lambda - \lambda_c) - u z^2 \tag{A6}$$

$$z_t' = (\lambda - \lambda_c) z - u z^3 \tag{A7}$$

$$z'_t = (\lambda - \lambda_c)z - uz^2 \qquad \text{(A8)}$$

Instead, if perturbations are oscillatory at $\lambda = \lambda_c$, the normal form equation models the dynamics of a Stuart-Landau oscillator

$$\boxed{z'_t = [(\lambda - \lambda_c) + i\omega_0]z - uz|z|^2} \qquad \text{(A9)}$$

where ω_0 is the frequency of perturbations at the bifurcation point and both u and z are complex-valued. (A9) is a particular embodiment of the *complex Ginzburg-Landau equation* (CGLE), a universal model that holds for all pattern forming systems undergoing a Hopf bifurcation. Furthermore, if $\omega_0 << 1$ and u assumes real values, (A9) reduces to the more familiar *real Ginzburg-Landau equation* (RGLE), a benchmark model of statistical physics and condensed matter theory. Note that the effective order parameter z can relate either to a *field* or a *collective property* of the system such as magnetization, polarization, conductivity, density, number of occupied lattice sites and so on.

References

1. https://www.lptmc.jussieu.fr/user/lesne/rg-IHP-2.pdf

2. Lesne, A. “Regularization, Renormalization, and Renormalization Groups: Relationships and Epistemological Aspects”, in Vision of Oneness, Aracne Editrice, pp.121 – 153, 2011.

3. Maggiore, M. M., A Modern Introduction to Quantum Field Theory, Oxford Univ. Press, 2005.

4. Gutzwiller M. C., Chaos in Classical and Quantum Mechanics, Springer-Verlag, 1990.

5. https://arxiv.org/pdf/2202.09846.pdf

6. https://www.researchgate.net/publication/278849474

7. https://arxiv-export3.library.cornell.edu/pdf/2212.06632v1

8. https://www.researchgate.net/publication/361644297

9. https://www.researchgate.net/publication/359137152

10. https://www.researchgate.net/publication/349476440

11. https://www.cambridge.org/core/books/nonequilibrium-quantum-field-theory/8C6D7E585852342B43C9C9EF778EDA74

12. Goldenfeld N., Lectures on Phase Transitions and the Renormalization Group, CRC Press, Taylor and Francis, 1992.

13. Bruce A. and Wallace D., Critical point phenomena: universal physics at large length scales, in New Physics, in New Physics, Cambridge Univ. Press, pp. 237 – 267, 1992.

14. https://www.researchgate.net/publication/357093456

15. https://iopscience.iop.org/article/10.1209/0295-5075/82/11001/pdf

16. Zaslavsky G. M., Hamiltonian Chaos and Fractional Dynamics, Oxford Univ. Press, 2006.

17. Nicolis, G., "Physics of far-from-equilibrium systems and self-organization", in "New Physics" pp. 316-347, Ed. Paul Davies, Cambridge Univ. Press, 1992.

18. Guckenheimer, J. and Holmes, P. "Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields", Springer, Berlin (1983).

19. https://www.researchgate.net/publication/357093467

20. Relevant references can be found at https://www.researchgate.net/profile/Ervin-Goldfain/research

21. Goldfain, E. "The Spin-Statistics Theorem from the Stuart-Landau Equation", (*in progress*).