Knot, refractive index and phase singularity

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We treat the geometrical optics as an Abelian $U(1)$ local gauge theory the same as the Abelian $U(1)$ Maxwell's gauge theory. We propose there exists a knot in a 3-dimensional Euclidean (flat) space of the geometrical optics (the eikonal equation) as a consequence there exists a knot in the Maxwell's theory in a vacuum. We formulate the Chern-Simons integral using an eikonal. We obtain the relation between the knot (the geometric optical helicity, an integer number) and the refractive index. We propose that the nature of the singularities of the phase is determined by the fact that the gauge potential is a smooth single-valued function of its variables.

Keywords: *knot, geometrical optics, eikonal equation, Maxwell's theory, Abelian $U(1)$ local gauge theory, Chern-Simons integral, helicity, refractive index, phase singularity.*

In our previous work[1], in the case of the 3-dimensional space, we obtain the Abelian Chern-Simons integral expressed in the refractive index related to *the geometric optical helicity*, h_{go}, as follow

$$\int_{\mathbb{R}^3} \varepsilon^{\alpha\mu\nu}\ \vec{a}_\alpha\ e^{iX\left(\int_{x_1}^{x_2} n\ d^3x - ct\right)} \left\{\partial_\mu \left[\vec{a}_\nu\ e^{iX\left(\int_{x_1}^{x_2} n\ d^3x - ct\right)}\right] - \partial_\nu \left[\vec{a}_\mu\ e^{iX\left(\int_{x_1}^{x_2} n\ d^3x - ct\right)}\right]\right\} d^3x = \ h_{\mathrm{go}} \tag{1}$$

where we replaced the electromagnetic helicity to the geometric optical helicity or *the geometric optical knot*. Both, h_{em} and h_{go}, are *integer numbers*.

Eq.(1) shows explicitly the relation between the geometric optical knot and the refractive index. The refractive index is typically supplied as *known input, given*, and we seek *the solution, the phase*, ψ_1[2]. It means that the geometric optical knot as an integer *restricts* the choice of the value of refractive index, so it makes the phase becomes singular. This phase singularity[3] where the phase is undefined[3] is the geometric optical knot solution. In our case, the geometric optical knot could exist in the weak field only.

In this article, we would like to explore deeper the meaning of the geometric optical knot as an integer restricts the choice of the value of the refractive index and how the restricted value of the real scalar function of the refractive index could give rise to the phase singularity. We propose that the nature of the singularities of the phase is determined by the fact that the gauge potential is a smooth single-valued function of its variables.

Let us discuss some basic concepts. Wavefronts are defined as the contour surfaces of phase[4]. We see from eq.(1), the phase is

$$\psi = X\left(\int_{x_1}^{x_2} n\ d^3x - ct\right) \tag{2}$$

where the related gauge potential can be written as[2]

$$\vec{A}_\mu = \vec{a}_\mu\ e^{i\psi}, \quad \psi = X(\psi_1 - ct) \tag{3}$$

$\vec{a}_\mu$ is the amplitude. The most important features of wavefronts are their singularities, which correspond to singularities of the phase function, $\psi(\vec{r},t)$[4].

Inspired by Berry[4], we consider that the nature of the singularities of the phase is determined by the fact that *the gauge potential is a smooth single-valued function of its variables.*

Single-valuedness of the gauge potential (due to the integer value of the geometic optical helicity) implies that during a circuit C in space-time, the phase may change by $2m\pi$, where m is an integer. Suppose m is not zero, and *let C be shrunk to a small loop* in such a way that m does not change. *Then C encloses a singularity, because the phase is varying infinitely fast*[4].

The smoothness of the gauge potential now *implies* that this (the phase is varying infinitely fast) *can happen only* when the gauge potential is so weak, $\vec{A}_\mu \to 0$.

These phase singularities are *lines* in space, or *points* in the plane[4].

The work is still in progress.

Thank to Richard Tao Roni Hutagalung, Idham Syah Alam for fruitful discussions. Thank to Reviewers for reviewing the manuscript. Special thank to beloved Juwita Armilia and Aliya Syauqina Hadi for much love. To Ibunda and Ayahanda, may Allah bless them with the Jannatul Firdaus. This research is fully supported by self-funding.

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