Plithogenic Probability & Statistics are generalizations of MultiVariate Probability & Statistics

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Abstract: In this paper we exemplify the types of Plithogenic Probability and respectively Plithogenic Statistics. Several applications are given.
The Plithogenic Probability of an event to occur is composed from the chances that the event occurs with respect to all random variables (parameters) that determine it. Each such a variable is described by a Probability Distribution (Density) Function, which may be a classical, (T,I,F)-neutrosophic, I-neutrosophic, (T,F)-intuitionistic fuzzy, (T,N,F)-picture fuzzy, (T,N,F)-spherical fuzzy, or (other fuzzy extension) distribution function.
The Plithogenic Probability is a generalization of the classical MultiVariate Probability.
The analysis of the events described by the plithogenic probability is the Plithogenic Statistics.

Keywords: MultiVariate Probability; MultiVariate Statistics; Plithogenic Probability; Plithogenic Refined Probability; Plithogenic Statistics; Plithogenic Refined Statistics; Neutrosophic Data; Neutrosophic Sample, Neutrosophic Population, Neutrosophic Random Variables; Plithogenic Variate Data

1. Introduction

The Plithogeny, as generalization of Dialectics and Neutrosophy, and then its applications to Plithogenic Set/Logic/Probability/Statistics (as generalization of fuzzy, intuitionistic fuzzy, neutrosophic set/logic/probability/statistics) [1, 2] were introduced by Smarandache in 2017.

Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities.

Plithogenic means what is pertaining to plithogeny. Etymologically, plitho-geny comes from: (Gr.) πλήθος (plithos) = crowd, many while -geny < (Gr.) -γενιά (-geniá) = generation, the production of something

2. Neutrosophic (or Indeterminate) Data

Neutrosophic (or Indeterminate) Data is a vague, unclear, incomplete, partially unknown, conflicting indeterminate data. The neutrosophic data can be metrical, or categorical, or both. Plithogenic Variate Data summarizes the associations (or inter-relationships) between Neutrosophic variables. While Neutrosophic Variable is a variable (or function, operator), that deals with neutrosophic data) either in its arguments or in its values, or in both. The problem to solve may have many dimensions, therefore multiple measurements and observations are needed since there are many sides to the problem, not only one. Neutrosophic variables may be: dependent; independent; or partially dependent, partially independent, and partially indeterminate.

(i.e. unknown if dependent or independent). The data's attributes (features, functions etc.) are investigated by survey-based techniques within the frame of Neutrosophic Conjoint Analysis (which includes the choice based conjoint and the adaptive choice-based conjoint.) Indeterminacy may occur at the level of attributes as well. We may thus deal with neutrosophic (indeterminate, unclear, partially known etc.) attributes [3-14].

3. Classical MultiVariate Analysis vs. Plithogenic Variate Analysis

The **Classical MultiVariate Analysis** (MVA) studies a system, which is characterized by many variables, or one may call it a system-of-systems. The variables, i.e. the subsystems, and the system as a whole are also classical (i.e. they do not deal with indeterminacy). Many classical measurements are needed, and the classical relations between variables to be determined. This system-of-systems is generally represented by a surrogate approximate model.

The **Plithogenic Variate Analysis** (PVA) is an extension of of the classical MultiVariate Analysis, where indeterminate data or procedures, that are called neutrosophic data and respectively neutrosophic procedures, are allowed. Therefore PVA deals with neutrosophic/ indeterminate variables, neutrosophic/indeterminate subsystems, and neutrosophic/indeterminate system-of-systems as a whole.

Therefore the Plithogenic Variate Analysis studies a neutrosophic/indeterminate system as a whole, characterized by many neutrosophic/indeterminate variables (i.e. neutrosophic/indeterminate sub-systems), and many neutrosophic/indeterminate relationships. Hence many neutrosophic measurements and observations are needed.

The Plithogenic Variate Analysis requires complex computations, hence it is more complicated than the Classical MultiVariate Analysis due to the neutrosophic (indeterminate) data it deals with; nonetheless the PVA better reflects our world, giving results nearer to real-life situation. With the dramatic increase of computers power this complexity is overcome.

The Plithogenic Variate Analysis elucidates each attribute of the data, using various methods, such as: regression/factor/cluster/path/discriminant/latent (trait or profile)/multilevel analysis / structural equation/recursive partition/redundancy/ constrained correspondence/ artificial neural networks, multidimensional scaling, and so on.

The **Plithogenic UniVariate Analysis** (PUVA) comprises the procedures for analysis of neutrosophic/indeterminate data that contains only one neutrosophic/indeterminate variable.

4. Plithogenic Probability

The Plithogenic Probability of an event to occur is composed from the chances that the event occurs with respect to all random variables (parameters) that determine it.

The Plithogenic Probability, based on Plithogenic Variate Analysis, is a multi-dimensional probability ("plitho" means "many", synonym with "multi"). We may say that it is a probability of sub-probabilities, where each sub-probability describes the behavior of one variable. We assume that the event we study is produced by one or more variables.

Each variable is represented by a **Probability Distribution (Density) Function** (PDF).

5. Subclasses of Plithogenic Probability are:

(i) If all PDFs are classical, then we have a classical **MultiVariate Probability**.

(ii) If all PDFs are in the neutrosophic style, i.e. of the form (T, I, F),
where T is the chance that the event occurs,
I is the indeterminate-chance of the event to occur or not,
and F is the chance that the event do not occur, with T, I, F $\in [0, 1]$, $0 \leq T+I+F \leq 3$,
then we have a **Plithogenic Neutrosophic Probability**.

(iii) If all PDFs are indeterminate functions (i.e. functions that have indeterminate data in their arguments, or in their values, or in both), then we have a **Plithogenic Indeterminate Probability**.

(iv) If all PDFs are Intuitionistic Fuzzy in the form of (T, F),
where T is the chance that the event occurs,
and F is the chance that the event do not occur, with $T, F \in [0, 1]$, $0 \leq T+ F \leq 1$,
then we have a **Plithogenic Intuitionistic Fuzzy Probability**.

(v) If all PDFs are in the Picture Fuzzy Set style, i.e. of the form (T, N, F),
where T is the chance that the event occurs,
N is the neutral-chance of the event to occur or not,
and F is the chance that the event do not occur, with $T, N, F \in [0, 1]$, $0 \leq T+N+F \leq 1$,
then we have a **Plithogenic Picture Fuzzy Probability**.

(vi) If all PDFs are in the Spherical Fuzzy Set style, i.e. of the form (T, H, F),
where T is the chance that the event occurs,
H is the neutral-chance of the event to occur or not,
and F is the chance that the event do not occur, with $T, H, F \in [0,1]$, $0 \leq T^2+H^2+F^2 \leq 1$,
then we have a **Plithogenic Spherical Fuzzy Probability**.

(vii) In general, if all PDFs are in any (fuzzy-extension set) style,
then we have a **Plithogenic (*fuzzy-extension*) Probability**.

(viii) If some PDFs are in one of the above styles, while others are in different styles,
then we have a **Plithogenic Hybrid Probability**.

6. Plithogenic Refined Probability

The most general form of probability is **Plithogenic Refined Probability**, when the components of T (Truth = Occurrence), I (Indeterminate-Occurrence), and F (Falsehood-NonOccurrence) are refined/split into sub-components: $T_1, T_2, \ldots, T_p$ (sub-truths = sub-occurrences) and $I_1, I_2, \ldots, I_r$ (sub-indeterminate-occurrences), and $F_1, F_2, \ldots, F_s$ (sub-falsehoods = sub-nonoccurrences), where p, r, $s \geq 0$ are integers, and $p + r + s \geq 1$.

All the above sub-classes of plithogenic probability may be refined this way.

7. Convergence from MultiVariate to UniVariate Analysis

In order to be able to make a decision, we need to convert from Plithogenic (*MultiVariate*) Probability and Statistics to Plithogenic *UniVariate* Probability and Statistics. Actually we need to fusion (combine) all variables and obtain a single cumulative variable.

The **Classical Probability Space** is complete, i.e. all possible event that may occur are known.

For example, let's consider a soccer game between teams A and B. The classical probability space is CPS = {A wins, tie game, B wins}.

The **Neutrosophic Probability Space** is in general incomplete, i.e. not all possible events are known, and there also are events that are only partially known. In our world, most real probability spaces are neutrosophic.

Example. Considering the same soccer game, the neutrosophic probability space NPS = {A wins, tie game, B wins, interrupted game, etc.}, "interrupted" means that due to some unexpected weather conditions, or to a surprising terrorist attack on the stadium, etc. the game is interrupted and rescheduled (this has happened in our world many times).

Let's assume an event E in a given (classical or neutrosophic) probability space is determined by $n \geq 2$ variables $v_1, v_2, \ldots, v_n$, and we denote it as $E(v_1, v_2, \ldots, v_n)$. The multi-variate probability of the event E to occur, denoted by MVP(E), depends on many probabilities, i.e. on the probability that the event E occurs with respect to variable v_1, denoted by $P_1(E(v_1))$, on the probability that the event E occurs with respect to variable v_2, denoted by $P_2(E(v_2))$, and so on.
Therefore, $MVP(E(v_1, v_2, \ldots, v_n)) = (P_1(E(v_1)), P_2(E(v_2)), \ldots, P_n(E(v_n)))$.
The variables $v_1, v_2, \ldots, v_n$, and the probabilities $P_1, P_2, \ldots, P_n$, may be classical, or having some degree of indeterminacy.

In order *to convert from multi-probability to uni-probability*, we apply various logical operators (conjunctions, disjunctions, negations, implications, etc. and their combinations, depending on the application to do and on the expert) on the multi-probability.

Such applications are presented towards the end of the paper.

7. Plithogenic Statistics

Plithogenic Statistics (PS) encompasses the analysis and observations of the events studied by the Plithogenic Probability.

Plithogenic Statistics is a generalization of classical MultiVariate Statistics, and it is a simultaneous analysis of many outcome neutrosophic/indeterminate variables, and it as well is a multi-indeterminate statistics.

8. Subclasses of Plithogenic Statistics are:

- **MultiVariate Statistics**
- **Plithogenic Neutrosophic Statistics**
- **Plithogenic Indeterminate Statistics**
- **Plithogenic Intuitionistic Fuzzy Statistics**
- **Plithogenic Picture Fuzzy Statistics**
- **Plithogenic Spherical Fuzzy Statistics**
- and in general: **Plithogenic (*fuzzy-extension*) Statistics**
- and **Plithogenic Hybrid Statistics**.

9. Plithogenic Refined Statistics are, similarly, the most general form of statistics that studies the analysis and observations of the events described by the Plithogenic Refined Probability.

10. Applications of Plithogenic Probability

We retour our 2017 example [1] and pass it through all sub-classes of Plithogenic Probability.

In the Spring 2021 semester, at The University of New Mexico, United States, in a program of Electrical Engineering, Jenifer needs to pass four courses in order to graduate at the end of the semester: two courses of Mathematics (Second Order Differential Equations, and Stochastic Analysis), and two courses of Mechanics (Fluid Mechanics and Solid Mechanics). What is the Plithogenic Probability that Jenifer will graduate?

Her chances of graduating are estimated by the university's advisors.

There are four variables (courses), v_1, v_2, v_3, v_4 respectively, that generate four probability distributions. We consider the discrete probability distribution functions.
[For the continuous ones, it will be similar.]

10.1. Classical MultiVariate Probability (CMVP)

The advisers have estimated that CMVP(Jenifer) = (0.5, 0.6, 0.8, 0.4),

which means that Jenifer has 50% chance to pass the Second Order Differential Equations class, 60% chance of passing the Stochastic Analysis class (both as part of Mathematics), and 80% chance of passing the Fluid Mechanics class, and 40% chance of passing the Solid Mechanics class (both as part of Mechanics).

Since she has to pass all four classes, Jenifer's chance of graduating is min{0.5, 0.6, 0.8, 0.4} = 0.4 or 40% chance.

10.2 Plithogenic Neutrosophic Probability

PNP(Jenifer) = ((0.5, 0.9, 0.2), (0.6, 0.7, 0.4); (0.8, 0.2, 0.1), (0.4, 0.3, 0.5)),

which similarly means that :

Jenifer's chance to pass the Second Order Differential Equations class is 50%, and the indeterminate chance is 90%, and the chance to fail it is 20%. Similarly for the other three classes.

In conclusion: (min{0.5, 0.6, 0.8, 0.4}, max{0.9, 0.7, 0.2, 0.3}, max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.9, 0.5).

10.3. Plithogenic Indeterminate Probability

PIP(Jenifer) = ([0.4, 0.5], 0.2 or 0.4, 0.1 or unknown)
which unclear information, i.e. chance of graduating is between [40%, 50%], 20% or 40% is indeterminate-chance of graduating, and chance of not graduating is 10% or unknown (i.e. the advisors were not able to estimate it well).

10.4. Plithogenic Intuitionistic Fuzzy Probability (PIFP) provides more information

PIFP(Jenifer) = ((0.5, 0.2), (0.6, 0.4); (0.8, 0.1), (0.4, 0.5)),

which means that Jenifer has 50% chance to pass the Second Order Differential Equations class, and 20% chance to fail it. And similarly for the other three classes.

In conclusion: ((min{0.5, 0.6, 0.8, 0.4), max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.5).

10.5. Plithogenic Picture Fuzzy Probability (PPFP) brings even more information

PPFP(Jenifer) = ((0.5,0.1, 0.2), (0.6, 0.0, 0.4); (0.8, 0.1, 0.1), (0.4, 0.0, 0.5)),

which means that Jenifer's chance to pass the Second Order Differential Equations class is 50%, and 10% neutral chance, and 20% chance to fail it. Similarly for the other three classes.

In conclusion: PSFP(Jenifer) = ((min{0.5, 0.6, 0.8, 0.4), max{0.1, 0.0, 0.1, 0.0}, max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.1, 0.5).

10.6. Plithogenic Spherical Fuzzy Probability (PSFP) enlarges the value spectrum of the previous one

PSFP(Jenifer) = ((0.5, 0.3, 0.2), (0.6, 0.5, 0.4); (0.8, 0.3, 0.1), (0.4, 0.6, 0.5)),

with the same meaning as the previous one.

In conclusion: PSFP(Jenifer) = ((min{0.5, 0.6, 0.8, 0.4), max{0.3, 0.5, 0.3, 0.6}, max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.6, 0.5).

10.7. Plithogenic Hybrid Probability (PHP)

PHP(Jenifer) = (0.5; (0.7, 0.1, 0.4); (0.1, 0.2); 0.4 or 0.3),

which means that Jenifer has 50% chance to pass the Second Order Differential Equations class; 70% chance of passing and 10% indeterminate-chance and 40% chance of failing the Stochastic Analysis class (both as part of Mathematics); and 10% chance of passing and 20% of failing the Fluid Mechanics class; and 40% or 30% chance of passing the Solid Mechanics class (both as part of Mechanics).

We have mixed herein: the fuzzy, neutrosophic, intuitionistic fuzzy, and indeterminate above cases.

Or PHP(Jenifer) = ((0.5, 0.0, 0.0); (0.7, 0.1, 0.4); (0.1, 0.7, 0.2); (0.4 or 0.3, 0.0, 0.0)), since for the intuitionistic fuzzy the hesitancy is : $1 – 0.1 – 0.2 = 0.7$.

In conclusion: PHP(Jenifer) = ((min{0.5, 0.7, 0.1, 0.4 or 0.3), max{0.0, 0.1, 0.7, 0.0}, max{0.0, 0.4, 0.2, 0.0}) = (0.1, 0.7, 0.4).

10.8. Plithogenic Refined Probability (PRP)

Let's assume that for each class, Jenifer has to pass an oral test and a written test. Therefore T,I,F are refined/split into:

T1(oral test), T2(written test);

I1(oral test), I2(written test);

F1(oral test), F2(written test).

Then, we may have, as an example:

PRP(Jenifer) = (((0.5, 0.6), (0.4, 0.7), (0.1, 0.2)),
((0.6, 0.8), (0.0, 0.7), (0.3, 0.4)),
((0.8, 0.8), (0.1, 0.2), (0.1, 0.0)),
((0.3, 0.7), (0.2, 0.3), (0.5, 0.4))),

which means :

with respect to the first class,

Jenifer's chance to pass the oral test is 50% and the written test is 60%;

indeterminate-chance to pass the oral test is 40% and the written test is 70%;

and chance not to pass the oral test is 10% and the written test 20%.

Similarly for the other classes.

11. Converging/Transforming from MultiVariate to UniVariate Analysis

11.1. From Classical MultiVariate Probability (CMVP) to UniVariate Probability

(i) Since Jenifer has to pass *all* four classes, we use the conjunction operator: $v_1 \wedge v_2 \wedge v_3 \wedge v_4$. In this case it is fuzzy conjunction (t-norm).

Therefore, Jenifer's chance of graduating is CMVP(Jenifer) = min{0.5, 0.6, 0.8, 0.4} = 0.4, or 40% chance.

(ii) Let's change the example and assume that for Jenifer to graduate she needs to pass at least one class among the four. Now we use the disjunction operator: $v_1 \vee v_2 \vee v_3 \vee v_4$ or fuzzy disjunction (t-conorm). Therefore, Jenifer's chance of graduating is CMVP(Jenifer) = max{0.5, 0.6, 0.8, 0.4} = 0.8, or 80% chance.

(iii) Let's change again the example and assume that for Jenifer to graduate she needs to pass at least one class of Mathematics and at least one class of Mechanics. Then, we use a mixture of conjunctions and disjunctions: CMVP(Jenifer) = $(v_1 \vee v_2) \wedge (v_3 \vee v_4)$ = min{max{v_1, v_2}, max{v_3, v_4}} = min{max{0.5, 0.6}, max{0.8, 0.4}} = min{0.6, 0.8} = 0.6, or 60% chance.

11.2 From Plithogenic Neutrosophic Probability (PNP) to UniVariate Neutrosophic Probability

In conclusion: PNP(Jenifer) = ((min{0.5, 0.6, 0.8, 0.4), max{0.9, 0.7, 0.2, 0.3}, max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.9, 0.5).

11.3. This Plithogenic Indeterminate Probability happens to be a UniVariate Indeterminate Probability

Therefore, no converting (or transformation) needed.

PIP(Jenifer) = ([0.4, 0.5], 0.2 or 0.4, 0.1 or unknown)

with unclear information, i.e. chance of graduating is between [40%, 50%], 20% or 40% indeterminate-chance of graduating, and chance of not graduating is (10% or unknown – i.e. the advisors were not able to estimate it well).

11.4. From Plithogenic Intuitionistic Fuzzy Probability (PIFP) to UniVariate Intuitionistic Fuzzy Probability

In conclusion: PIPF(Jenifer) = ((min{0.5, 0.6, 0.8, 0.4), max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.5), which means that chance of graduating is 40%, and chance of not graduating 50%.

11.5. From Plithogenic Picture Fuzzy Probability (PPFP) to UniVariate Picture Fuzzy Probability

In conclusion: PSFP(Jenifer) = ((min{0.5, 0.6, 0.8, 0.4), max{0.1, 0.0, 0.1, 0.0}, max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.1, 0.5), or 40% chance to graduate, 10% neutral chance, and 50% chance not to graduate.

11.6. From Plithogenic Spherical Fuzzy Probability (PSFP) to UniVariate Spherical Fuzzy Probability

In conclusion: PSFP(Jenifer) = ((min{0.5, 0.6, 0.8, 0.4), max{0.3, 0.5, 0.3, 0.6}, max{0.2, 0.4, 0.1, 0.5}) = (0.4, 0.6, 0.5), or 40% chance to graduate, 60% hesitant chance, and 50% chance not to graduate.

11.7. From Plithogenic Hybrid Probability (PHP) to UniVariate Hybrid Probability

In conclusion: PHP(Jenifer) = ((min{0.5, 0.7, 0.1, 0.4 or 0.3), max{0.0, 0.1, 0.7, 0.0}, max{0.0, 0.4, 0.2, 0.0}) = (0.1, 0.7, 0.4).

10.8. From Plithogenic Refined Probability (PRP) to UniVariate Refined Probability ???

Taking min of T1's, min T2's, and max of I1's, max of I2's, and max of F1's, F2's, one gets:
PRP(Jenifer) = ((0.3, 0.6), (0.4, 0.7), (0.5, 0.4)).
Whence, Jenifer's chance, with respect to all classes,
to pass the oral test is 30% and the written test is 60%;
indeterminate-chance to pass the oral test is 40% and the written test is 70%;
and chance not to pass the oral test is 50% and the written test 40%.
But, with respect to graduation, we use again the fuzzy conjunction:
PRP(Jenifer) = (min{0.3, 0.6}, max{0.4, 0.7}, max{0.5, 0.4}) = (0.3, 0.7, 0.5), or
Jenifer's chance to graduate is 30%, indeterminate-chance of graduating 70%, and 50% chance not to graduate.

11. Corresponding Applications of Plithogenic Statistics

A prospective is made on the university student population, that was enrolled this semester, in order to determine the chance of the average students to graduate.

Let's take a random sample of the university's student population in order to investigate what's the chance of graduating for an enrolled average student.

By *inference statistics*, we estimate the population's average student to be similar to the sample's average student.

We may have a **classical random sample**, i.e. the sample size is known and all sample individuals belong 100% of the population - i.e. the individuals are full-time students; or a **neutrosophic random sample** {i.e. the sample size may be unknown or only approximately known), and some or all individuals may only partially belonging to the population (for example part-time students), or may have taken some extra classes above the norm.

Even the university's student population is a **neutrosophic population**, since the number of students changes almost continuously (some students drop, others enroll earlier or later), and not all students are 100% enrolled: there are full-time, part-time, and even over-time (i.e. students enrolled in more than the required full time number or credit hour classes).

In a **classical population**, the population size is known, and all population individuals belong 100% to the population.

Let T = truth, with T belongs to [0,1], be the chance to graduate, I = indeterminate, with I belongs to [0,1], be the indeterminate-chance to graduate, and F = falsehood, with F belongs to [0,1], be the chance not to graduate, where $0 \leq T + I + F \leq 3$.

Let's assume, the classical or neutrosophic random sample has the size $n \geq 2$, and a student S_j, $1 \leq j \leq n$, has the plithogenic neutrosophic probabilty of (graduating, indeterminate-graduating, not graduating), respectively (T_j, I_j, F_j), with all T_j, I_j, F_j belong to [0, 1], $0 \leq T_j + I_j + F_j \leq 3$.

Make the average of all sample students, assuming the sample size is $n \geq 2$,

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = (\frac{1}{n}\sum_{j=1}^{n}T_j, \frac{1}{n}\sum_{j=1}^{n}I_j, \frac{1}{n}\sum_{j=1}^{n}F_j) .$$

For the Plithogenic Refined Neutrosophic Probabilities, the average is a straight-forward extension. Let the student S_j, $1 \le j \le n$, have the Plithogenic Refined Neutrosophic Probability:

PRNP(S_j) = $(T_1(j), T_2(j), ..., T_p(j); I_1(j), I_2(j), ..., I_r(j); F_1(j), F_2(j), ..., F_s(j))$

where p, r, s ≥ 0 are integers, and p + r + s ≥ 1.

The refined neutrosophic sub-components with index 0, such as $T_0(j), I_0(j), F_0(j)$, if any, are discarded.

All refined neutrosophic sub-components

$T_k(j), 1 \le k \le p, I_l(j), 1 \le l \le r, F_m(j), 1 \le m \le s$, are single-valued in [0, 1].

Then, the average of PRNPs of the sample students is:

$$\frac{1}{n}\sum_{j=1}^{n} \mathrm{PRNP}\left(\mathrm{S_j}\right) = \frac{1}{n}\sum_{j=1}^{n}(T_1(j), T_2(j), ..., T_p(j); I_1(j), I_2(j), ..., I_r(j); F_1(j), F_2(j), ..., F_s(j))$$

=

$$(\frac{1}{n}\sum_{j=1}^{n}T_1(j), \frac{1}{n}\sum_{j=1}^{n}T_2(j), ..., \frac{1}{n}\sum_{j=1}^{n}T_p(j); \frac{1}{n}\sum_{j=1}^{n}I_1(j), \frac{1}{n}\sum_{j=1}^{n}I_2(j), ..., \frac{1}{n}\sum_{j=1}^{n}I_r(j); \frac{1}{n}\sum_{j=1}^{n}F_1(j), \frac{1}{n}\sum_{j=1}^{n}F_2(j), ..., \frac{1}{n}\sum_{j=1}^{n}F_s(j))$$

And we get the sample average students' plithogenic refined probability to (graduate, indeterminate graduate, not graduate).

For the cases when one or two among T, I, F are missing, we simply discard them.

An average student is not among the best, not among the worst.

Let's consider Jenifer is an average student, whose plithogenic probabilities have been obtained after sampling and computing the average of plithogenic probabilities of all its students - since we have already her data.

Considering the inference statistics, we simply substitute Jenifer by average student.

We consider the simplest case when T,I,F are single-valued neutrosophic (SVN) components in [0,1]. But the cases when T,I,F are hesitant-valued (finite discrete subsets of [0,1]) neutrosophic HVN components, or interval-valued included in [0,1] neutrosophic (IVN) component, or in general subset-valued included in [0,1] neutrosophic (SVN) components.

10.1. Classical MultiVariate Statistics (CMVS)

Since the average student has to pass all four classes, his chance of graduating is min{0.5, 0.6, 0.8, 0.4} = 0.4 or 40% chance. In conclusion: CMVS(average student) = 0.4.

10.2. Plithogenic Neutrosophic Statistics (PNS)

In conclusion: PNP(average student) = ((min{0.5, 0.6, 0.8, 0.4), max{0.9, 0.7, 0.2, 0.3}, max{0.2, 0.4, 0.1, 0.5}) = ((0.4, 0.9, 0.5)).

An average student has 40% chance to graduate, 90% indeterminate chance of graduating, and 50% chance not to graduate.

10.3. Plithogenic Indeterminate Statistics

An average student has (40% or 50%) chance of graduating, 90% indeterminate chance of graduating, and (50% or unknown) chance of not graduating.

10.4. Plithogenic Intuitionistic Fuzzy Statistics

An average student has 40% chance to graduate and 50% chance not to graduate.

10.5. Plithogenic Picture Fuzzy Statistics

An average student has 40% chance to graduate, 10% indeterminate chance to graduate, and 50% chance not to graduate.

10.6. Plithogenic Spherical Fuzzy Statistics

An average student has 40% chance to graduate, 60% indeterminate chance of graduating, and 50% chance not to graduate.

10.7. Plithogenic Hybrid Statistics

An average student has 40% chance to graduate, 60% indeterminate chance of graduating, and 50% chance not to graduate.

10.8. Plithogenic Refined Statistics

An average student's chance to graduate is 30%, indeterminate-chance of graduating 70%, and 50% chance not to graduate.

11. Conclusion

We have presented in this paper many types of Plithogenic Probability and corresponding Plithogenic Statistics, together with some application.

The Plithogenic Probability of an event to occur is composed from the chances that the event occurs with respect to all random variables (parameters) that determine it. Each such a variable is described by a Probability Distribution (Density) Function, which may be a classical, (T,I,F)-neutrosophic, I-neutrosophic, (T,F)-intuitionistic fuzzy, (T,N,F)-picture fuzzy, (T,N,F)-spherical fuzzy, or (other fuzzy extension) distribution function.

The Plithogenic Probability is a generalization of the classical MultiVariate Probability.

The analysis of the events described by the plithogenic probability constitutes the Plithogenic Statistics.

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References

1. F. Smarandache, Plithogeny, Plithogenic Set, Logic, Probability, and Statistics, Pons Editions, Brussels, 141 p., 2017; http://fs.unm.edu/Plithogeny.pdf (all websites were accessed on April 22, 2021)
2. F. Smarandache, Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited, Neutrosophic Sets and Systems, vol. 21, 2018, pp. 153-166. https://doi.org/10.5281/zenodo.1408740, http://fs.unm.edu/NSS/PlithogenicSetAnExtensionOfCrisp.pdf

3. Jeff Kaplan (archivist), Neutrosophic Statistics is a generalization of Classical Statistics, in Internet Archives, San Francisco, California, USA, https://archive.org/details/neutrosophic-statistics?tab=about and https://archive.org/details/neutrosophic-statistics?tab=collection (accessed on April 22nd, 2021)
4. S. Gomathy, D. Nagarajan, S. Broumi, M. Lathamaheswari, Plithogenic sets and their application in decision making, Neutrosophic Sets and Systems, vol. 38, 2020, pp. 453-469. DOI: 10.5281/zenodo.4300565, http://fs.unm.edu/NSS/PlithogenicSetsAndTheirApplication30.pdf
5. Gustavo Álvarez Gómez, Jorge Viteri Moya, Jesús Estupiñán Ricardo, Cristina Belén Viteri Sanchez, Evaluating Strategies of Continuing Education for Academics Supported in the Pedagogical Model and Based on Plithogenic Sets, Neutrosophic Sets and Systems, vol. 37, 2020, pp. 16-23. DOI: 10.5281/zenodo.4121980, http://fs.unm.edu/NSS/3EvaluatingStrategiesContinuing.pdf
6. Rously Eedyah Atencio González, Julio César de Jesús Arrias Añez, Johanna Emperatriz Coronel Piloso, Orlando Iván Ronquillo Riera, Prioritization of the Social Approach of Employment Modeled by Plithogenic Sets , Neutrosophic Sets and Systems, vol. 37, 2020, pp. 184-192. DOI: 10.5281/zenodo.4122116, http://fs.unm.edu/NSS/22PrioritizationSocialApproach.pdf
7. Nivetha Martin and Florentin Smarandache, Introduction to Combined Plithogenic Hypersoft Sets, Neutrosophic Sets and Systems, vol. 35, 2020, pp. 503-510. DOI: 10.5281/zenodo.3951708, http://fs.unm.edu/NSS/IntroductionToCombined28.pdf
8. Nivetha Martin and Florentin Smarandache, Concentric Plithogenic Hypergraph based on Plithogenic Hypersoft sets – A Novel Outlook, Neutrosophic Sets and Systems, vol. 33, 2020, pp. 78-91. DOI: 10.5281/zenodo.3782824, http://fs.unm.edu/NSS/ConcentricPlithogenicHypergraph.pdf
9. Sudipta Gayen, Florentin Smarandache, Sripati Jha, Manoranjan Kumar Singh, Said Broumi and Ranjan Kumar, Introduction to Plithogenic Hypersoft Subgroup, Neutrosophic Sets and Systems, vol. 33, 2020, pp. 208-233. DOI: 10.5281/zenodo.3782897 http://fs.unm.edu/NSS/IntroductionToPlithogenic.pdf (accessed on May 21, 2021)
10. Shawkat Alkhazaleh, Plithogenic Soft Set, Neutrosophic Sets and Systems, vol. 33, 2020, pp. 256-274. DOI: 10.5281/zenodo.3783023, http://fs.unm.edu/NSS/PlithogenicSoftSet.pdf
11. Florentin Smarandache, Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra, Neutrosophic Sets and Systems, vol. 33, 2020, pp. 290-296. DOI: 10.5281/zenodo.3783103, http://fs.unm.edu/NSS/n-SuperHyperGraph-n-HyperAlgebra.pdf
12. Mohamed Grida, Rehab Mohamed and Abdelnaser H. Zaid, A Novel Plithogenic MCDM Framework for Evaluating the Performance of IoT Based Supply Chain, Neutrosophic Sets and Systems, vol. 33, 2020, pp. 321-338. DOI: 10.5281/zenodo.3808980, http://fs.unm.edu/NSS/NovelPlithogenicMCDM.pdf
13. Shazia Rana, Madiha Qayyum, Muhammad Saeed, F. Smarandache, Bakhtawar Ali Khan: Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique, Neutrosophic Sets and Systems, vol. 28, 2019, pp. 34-50. DOI: 10.5281/zenodo.3382507, http://fs.unm.edu/NSS/PlithogenicFuzz%20yWholeHypersoftSet.pdf
14. F. Smarandache: Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set, Neutrosophic Sets and Systems, vol. 22, 2018, pp. 168-170. DOI: 10.5281/zenodo.2159754 http://fs.unm.edu/NSS/ExtensionOfSoftSetToHypersoftSet.pdf

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