

# Tensor Gauge Boson Dark Matter Extension of the Electroweak Sector

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**Abstract:** The existence of dark matter is explained by a new neutral tensor gauge boson,  $Z_{\mu\nu}$ -boson, of mass of 2.3 TeV. The  $Z_{\mu\nu}$ -boson can be predicted by the tensor gauge boson extension of the Electro Weak (EW) theory proposed by G. Savvidy (2005). We compute the self-annihilation cross-section of the  $Z_{\mu\nu}$  dark matter and calculate its relic abundance. We also study the proton-proton scattering by the exchange of massive- $Z_{\mu\nu}$  dark matter at high energy scale is study. The existence of the proposed  $Z_{\mu\nu}$  tensors can be tested by the Large Hadron Collider (LHC). This proposition may have far reaching applications in astrophysics and cosmology.

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## 1. Introduction

Dark matter was proposed in 1933 to explain why galaxies in some clusters move faster than their predicted speed if they contained only baryonic matter [1]. The nature of dark matter, however, is one of the persistent mysteries of modern physics. Several candidate dark matter particles have been suggested, including Light Supersymmetric Particles [2–7], heavy fourth generation neutrinos [8–9], Q-Balls [10–11], mirror particles [12–16], and axion particles – the latter introduced in an attempt to solve the Charge-Parity (CP) violation problem in particle physics [17–18]. Recently, the braneworld idea has been used to furnish new solutions to old problems in particle physics and cosmology [19–33]. Universal Extra Dimensions (UED) models are scenarios that allow all fields to propagate in the bulk [34–35]. UED models provide a viable dark matter candidate, namely the Lightest Kaluza Klein particle (LKP) [36–37]. Gauge–Higgs unification models, based on grand unified gauge theories defined on six-dimensional space-time, have interesting properties. In these models, the extra-dimensional space has the topological structure of a two-sphere orbifold  $S^2/Z_2$  [38–40]. Furthermore, (thin) braneworlds with conical singularities in six-dimensional Einstein-Gauss-Bonnet gravity with a bulk cosmological constant have been investigated [41]. For axially symmetric bulks, however, this model does not provide isotropic braneworld cosmological solutions [41]. Other stable or quasi-stable particles that could emerge in the string theory spectrum have also been suggested in this context: modulinos [42], exotic gauge-charged matter [5], hidden-sector matter composites [2], hidden-sector gauge

composites [43], and wrapped D-branes [44]. One or more of these (or other, not yet imagined) states could contribute to the cosmos' dark matter.

The author recently proposed that cold dark matter (in the form of heavy, neutral, non-regular leptons of an O-order mass (TeV)) can be produced from quarks and leptons, through the process of Electric Charge Swap (ECS) symmetry [45–50]. Furthermore, the ECS symmetry could explain certain properties of lepton families within the framework of superstring theories [51–55].

Models with a vector DM, especially in the non-abelian case, are the least explored, despite the fact that the gauge principle can guide and constrain the possible theoretical constructions (for a discussion of non-abelian DM in different set-ups, in those of non-renormalisable kinetic mixing terms or Higgs portal scenarios, see [56–71]).

More recently, N.Masi 2021 [72] proposed a new criterion to extend the Standard Model (SM) of particle physics from a straightforward algebraic conjecture: the symmetries of physical microscopic forces originate from the automorphism groups of main Cayley–Dickson algebras, from complex numbers to octonions and sedenions. From the automorphism of octonion (and sedenion) algebra, the exceptional symmetry group  $G(2)$  that could solve the DM problem can be pinpointed.

A.Belyaev et.al 2022 [73,74], introduced the Fermion Portal Vector Dark Matter: a new class of renormalisable models, consisting of a dark  $SU(2)_D$  (Dark-Isospin) gauge sector connected to the SM through a Vector-Like fermion mediator without the need for a Higgs portal. In these models, a massive vector boson is the Dark Matter (DM) candidate.

A satisfactory theory of higher-spin gauge fields was constructed by G. Savvidy (2005) [75–78]. The goal of the present paper is to investigate the possibility that the occurrence of dark matter can be explained by a new neutral tensor gauge boson (the  $Z_{\mu\nu}$ -boson, of a mass of 2.3 TeV) that can be predicted by the tensor gauge boson extension of the Electro Weak (EW) [75-78]. We compute the self-annihilation cross section of the tensor gauge boson  $Z_{\mu\nu}$ -dark matter and calculate its relic abundance. We also study the proton-proton scattering by the exchange of a massive, tensor  $Z_{\mu\nu}$ -dark matter at a high energy scale. The existence of these proposed  $Z_{\mu\nu}$ -tensors can be tested at the Large Hadron Collider (LHC). The current proposition may have far reaching applications in astrophysics and cosmology.

## 2. Non-Abelian tensor gauge bosons dark matter

Following G. Savvidy 2005 [75–78], we first consider a model whereby the  $SU(2)_L$  group is extended to higher spins but the  $U(1)_Y$  group is not extended. The  $W^\pm$ ,  $Z$  gauge bosons receive their higher-spin descendance

$$\left(W^\pm, Z\right)_\mu, \left(\tilde{W}^\pm, \tilde{Z}\right)_{\mu\lambda}, \dots, \quad (1)$$

and the doublet of complex Higgs scalars appear together with their higher-spin partners:

$$\left(\begin{matrix} \phi^+ \\ \phi^0 \end{matrix}\right), \left(\begin{matrix} \phi^+ \\ \phi^0 \end{matrix}\right)_\lambda, \left(\begin{matrix} \phi^+ \\ \phi^0 \end{matrix}\right)_{\lambda\rho}, \dots \quad Y = +1. \quad (2)$$

The Lagrangian that describes the interaction of the tensor gauge bosons with the scalar fields and tensor bosons is:

$$\begin{aligned}
\mathfrak{L} = & -\frac{1}{4}G_{\mu\nu}^i G_{\mu\nu}^i - \frac{1}{4}G_{\mu\nu} G_{\mu\nu} - \left( \partial_\mu + \frac{ig'}{2}B_\mu + \frac{ig}{2}\tau^i A_\mu^i \right) \phi^\dagger \left( \partial_\mu - \frac{ig'}{2}B_\mu - \frac{ig}{2}\tau^i A_\mu^i \right) \phi \\
& + g_2 \left\{ -\frac{1}{4}G_{\mu\nu,\lambda}^i G_{\mu\nu,\lambda}^i - \frac{1}{4}G_{\mu\nu}^i G_{\mu\nu,\lambda\lambda}^i \right\} - b_2 \left\{ \frac{g^2}{4} \phi^\dagger \tau^i A_{\mu\lambda}^i \tau^j A_{\mu\lambda}^j \phi \right. \\
& + \nabla_\mu \phi_\lambda^\dagger \nabla_\mu \phi_\lambda + \frac{1}{2} \nabla_\mu \phi_{\lambda\lambda}^\dagger \nabla_\mu \phi + \frac{1}{2} \nabla_\mu \phi^\dagger \nabla_\mu \phi_{\lambda\lambda} \\
& - ig \nabla_\mu \phi^\dagger A_{\mu\lambda} \phi_\lambda + ig \phi_\lambda^\dagger A_{\mu\lambda} \nabla_\mu \phi - ig \nabla_\mu \phi_\lambda^\dagger A_{\mu\lambda} \phi + ig \phi_\lambda^\dagger A_{\mu\lambda} \nabla_\mu \phi_\lambda \\
& \left. - \frac{1}{2} ig \nabla_\mu \phi^\dagger A_{\mu\lambda\lambda} \phi + \frac{1}{2} ig \phi_\lambda^\dagger A_{\mu\lambda\lambda} \nabla_\mu \phi \right\} - U(\phi)
\end{aligned} \tag{3}$$

where

$$\nabla_\mu = \partial_\mu - \frac{ig'Y}{2}B_\mu - igT^i A_\mu^i. \tag{4}$$

In equations (3) and (4),  $Y$  is the hypercharge, so the electric charge is ( $Q = T_3 + Y/2$ ), and, for isospinor fields,  $T_i = \tau_i/2$ . In the Lagrangian (3),  $g_2$  is the tensor gauge boson coupling constant, and is a real positive parameter. The three terms in the first line of (3) represent the standard electroweak model, and the rest of the terms represent the higher-spin generalization of this model. Therefore, all parameters of the standard model are incorporated in the tensor extension.

When the scalar fields acquire the vacuum expectation value  $\eta$ ,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \eta + \chi(x) \end{pmatrix}. \tag{5}$$

and

$$\tilde{Z}_{\mu\lambda} = A_{\mu\lambda}^3. \tag{6}$$

$$\tilde{W}_{\mu\lambda}^\pm = \frac{1}{\sqrt{2}} (A_{\mu\lambda}^1 \pm iA_{\mu\lambda}^2), \tag{7}$$

the third term in the second line of Equation (3) generates the masses of the tensor ( $\tilde{W}^\pm, \tilde{Z}^0$ ) gauge bosons:

$$\frac{1}{8} \left( \frac{b_2}{g_2} \right) g^2 \eta^2 \left[ (A_{\mu\lambda}^3)^2 + 2A_{\mu\lambda}^+ A_{\mu\lambda}^- \right]. \tag{8}$$

Thus all intermediate spin-2 bosons acquire the same mass:

$$M_{\tilde{W},\tilde{Z}}^2 = \left( \frac{b_2}{g_2} \right) M_{W,Z}^2. \quad (9)$$

The rest of the terms in equation (3) describe the interaction between old and new particles [75-78]. The non-Abelian tensor gauge boson  $Z_{\mu\nu}$ , given by Equation (7), is a real field. The tensor boson  $Z_{\mu\nu}$ , therefore, is its own antiparticle. For this reason, field  $Z_{\mu\nu}$  has no electrical charge. In this article, we propose this neutral tensor gauge boson  $Z_{\mu\nu}$  as a new dark matter candidate.

### 3. The Annihilation cross section of the tensor gauge boson $Z_{\mu\nu}$ -dark mater

The annihilation modes of the proposed  $Z_{\mu\nu}$ -dark matter particles differ from these of other dark matter candidates, such as neutralinos in supersymmetric models [2–9]. The annihilation of neutralinos to fermions is chirality-suppressed by a factor of  $m_f^2/m_\chi^2$ , and thus does not produce electron-positron ( $e^+e^-$ ) pairs directly. By contrast,  $Z_{\mu\nu}$ -dark matter, being a boson, is not similarly suppressed and can annihilate directly to lepton  $e^+e^-e^+e^-$ ,  $\mu^+\mu^-\mu^+\mu^-$  and  $\tau^+\tau^-\tau^+\tau^-$  pairs. Each of these particles yields a large number of high-energy electrons and positrons. Other dominant modes include the annihilation to up-type quarks ( $u^+u^+u^-u^-$ ), ( $c^+c^+c^-c^-$ ) and ( $t^+t^+t^-t^-$ ), and down-type quarks ( $d^+d^+d^-d^-$ ), ( $s^+s^+s^-s^-$ ), ( $b^+b^+b^-b^-$ ). Based on [60], the total averaged annihilation cross section times the relative velocity of  $Z_{\mu\nu}$ -dark matter is given by:

$$\langle \sigma u_{rel} \rangle \simeq \frac{g_2^4}{M_{\tilde{Z}}^2} = \frac{g_2^4}{\left( \frac{b_2}{g_2} \right) M_W^2} = \frac{g_2^5}{b_2 M_W^2}, \quad (10)$$

where  $M_{\tilde{Z}}$  is derived from Equation (9). Following [60], for the proposed tensor gauge boson  $Z_{\mu\nu}$ -dark matter, the relic density should be:

$$\Omega_{\tilde{Z}} h^2 = \frac{0.1 pb}{\langle \sigma u_{rel} \rangle} = 0.11, \quad (11)$$

where

$$\langle \sigma u_{rel} \rangle \simeq \frac{g_2^4}{M_{\tilde{Z}}^2} = \frac{g_D^4}{\left( \frac{b_2}{g_D} \right) M_W^2} = \frac{g_D^5}{b_2 M_W^2} = 0.81. \quad (12)$$

The tensor dark matter coupling constant ( $g_2 = g_D$ ) and the positive real parameter  $b_2$  should be fixed in order to derive the correct value of ( $\langle \sigma u_{rel} \rangle = 0.81$ ). Here,  $M_W = 80 GeV$  is the mass of W-gauge boson as predicted by the Standard Model. The tensor gauge boson-dark matter possible masses can thus be calculated for the different values of parameters ( $g_D, b_2$ ) that satisfy the conditions of Equation (12).

Analysis of the three-year Wilkinson Microwave Anisotropy Probe (WMAP) data suggests that the density of dark matter is  $\Omega_{DM} h^2 = 0.102 \pm 0.009$  (where  $\Omega_{DM} = \rho_{DM}/\rho_{crit}$ , with  $\rho_{crit}$

being the density corresponding to a flat universe [79], and  $h$  being the Hubble constant, in units of  $100 \text{ km s}^{-1} \cdot \text{Mpc}^{-1}$  [80].

A cold dark matter candidate produced at the LHC should, therefore, have this annihilation cross section. This quantity leads us to the second method of measuring the coupling of dark matter from SM particles: through the search for the products of dark matter annihilation or decay, originating from high-density regions of the Universe, such as the center of galaxies [81]. Since WMAP results provide good information about  $\langle \sigma u_{rel} \rangle$ , the uncertainties in this approach stem from our sketchy knowledge of the exact density of dark matter in the center of galaxies, and the difficulty of separating the dark-matter annihilation signal from possible background signals.

In the SM, there already exists a particle that is accidentally stable: the proton [82]. There is, therefore, no reason why the tensor particle could not also be stable [82–83]. It follows that the SM (and hidden sector) gauge symmetries allow no dimension-five operator between the dark matter tensor field candidate and SM fields [82–84]. If the SM (and hidden sector) gauge symmetries allow dimension-6 operators, the lifetime of this tensor dark matter candidate is of the order of  $10^{26}$  sec if  $10^{14}$  GeV [82], which is close to the GUT scale. In other words, an accidentally stable tensor dark matter candidate that can be destabilised by a dimension-six GUT scale-induced interaction results in a flux of cosmic rays of the observed order – potentially, therefore, to a rich phenomenology [81]. If the tensor particle is accidentally stable, and since it also interacts weakly with baryonic matter, it can be a good Weakly Interacting Massive Particle (WIMP) candidate. We do not discuss the stability of the tensor particle further in this article; we leave this intriguing question open to future research. In any case, however, the tensor particle of mass around 2.3 TeV predicted by the proposed tensor dark matter model, also provides the correct abundance of dark matter in the universe. This encouraging theoretical suggestion is testable through LHC evidence.

#### 4. Proton-proton scattering by the exchange of massive, tensor $Z_{\mu\nu}$ -dark matter at a high energy scale

Base on G.Savvidy [75–78], we symmetrise the extended gauge transformation given in [75] as follows:

$$\tilde{\delta} A_{\mu\nu}^a = \left( \delta^{ab} \partial_\mu + g f^{acb} A_\mu^c \right) \xi_\nu^b + g f^{acb} A_{\mu\nu}^c \xi^b. \quad (13)$$

We explicitly symmetrize the right-hand side of these equations over all space-time indices:

$$\left( \delta^{ab} \partial_\mu + g f^{acb} A_\mu^c \right) \xi_\nu^b = \left( \delta^{ab} \partial_\mu + g f^{acb} A_\mu^c \right) \xi_\nu^b + \left( \delta^{ab} \partial_\nu + g f^{acb} A_\nu^c \right) \xi_\mu^b \quad (14)$$

Here,  $\xi_\lambda^a$  are total symmetric gauge parameters. The field strength-curvature transformation flow from Equation (14) is given by:

$$\tilde{\delta} G_{\mu\nu,\lambda}^a = g f^{abc} \left( G_{\mu\nu,\lambda}^b \xi^c + G_{\mu\nu}^b \xi_\lambda^c \right) + \nabla_\mu^{ab} \nabla_\lambda^{bc} \xi_\nu^c - \nabla_\nu^{ab} \nabla_\lambda^{bc} \xi_\mu^c. \quad (15)$$

Gauge transformation (14) respects the symmetry properties of the tensor field  $\tilde{A}_{\mu\nu} = \tilde{A}_{\nu\mu}$ , which implies the symmetric tensor dark matter gauge field  $\tilde{Z}_{\mu\nu} = \tilde{Z}_{\nu\mu}$ . Furthermore, for the

Z-dark matter field, we impose the traceless condition  $\tilde{Z}_\mu^\mu = 0$ . We note that the Z-dark matter does not coincide with the graviton because it has different gauge symmetries and interactions from those of the graviton.

Here, the proposed Z-dark matter field is expressed as a real second-rank symmetric traceless tensor  $Z_{\mu\nu}$  that is assumed to be coupled predominantly to the Quantum Chromo-Dynamic (QCD) energy-momentum tensor  $T_{\mu\nu}$  [85]:

$$S = g_D \int d^4x \tilde{Z}_{\mu\nu} T^{\mu\nu}. \quad (16)$$

The proton-tensor dark matter-proton vertex can then be extracted from the matrix element of the energy-momentum tensor  $T_{\mu\nu}$  between the proton states,

$$\langle p', s' | T^{\mu\nu}(0) | p, s \rangle. \quad (17)$$

Considering the symmetry and conservation of  $T_{\mu\nu}$ , Equation (17) can be generally expressed in terms of three form factors [86, 85]:

$$\begin{aligned} \langle p', s' | T^{\mu\nu}(0) | p, s \rangle = & \bar{u}(p', s') \left[ A(t) \frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) k^\rho}{4M_{\tilde{Z}}} \right. \\ & \left. + C(t) \frac{(k_\mu k_\nu - \eta_{\mu\nu} k^2)}{4M_{\tilde{Z}}} \right] u(p, s), \end{aligned} \quad (18)$$

where  $k = p' - p$ ,  $t = k^2$  and  $P = (p + p')/2$ . At zero momentum transfer, due to the fact that the proton has a spin of 1/2 and mass  $m_p$ , there are two constraints on the form factors:  $A(0) = 1$  and  $B(0) = 0$ . We consider the scattering of pp (or p $\bar{p}$ ) by the exchange of massive, spin-2 dark matter. Only the t-channel needs to be taken into account, since this is the dominant channel in the high energy regime.

The massive spin-2 dark matter propagator can be written as follows [87, 85]:

$$\Delta(k)_{\alpha\beta\gamma\delta}^{ab} = \delta^{ab} \frac{d_{\alpha\beta\gamma\delta}(k)}{k^2 - M_{\tilde{Z}}^2}, \quad (19)$$

where  $\alpha$  and  $\beta$  are the Lorentz indices contracted at one side,  $\gamma$  and  $\delta$  are the Lorentz indices contracted at the other side,  $M_{\tilde{Z}}$  is the mass of the tensor dark matter, and  $d_{\alpha\beta\gamma\delta}$  can be explicitly expressed as follows:

$$\begin{aligned} d_{\alpha\beta\gamma\delta} = & \frac{1}{2} (\eta_{\alpha\gamma} \eta_{\beta\delta} + \eta_{\alpha\delta} \eta_{\beta\gamma}) - \frac{1}{2M_{\tilde{Z}}^2} (k_\alpha k_\delta \eta_{\beta\gamma} + k_\alpha k_\gamma \eta_{\beta\delta} + k_\beta k_\delta \eta_{\alpha\gamma} + k_\beta k_\gamma \eta_{\alpha\delta}) \\ & + \frac{1}{24} \left[ \left( \frac{k^2}{M_{\tilde{Z}}^2} \right)^2 - \left( \frac{k^2}{M_{\tilde{Z}}^2} \right) - 6 \right] \eta_{\alpha\beta} \eta_{\gamma\delta} - \frac{k^2 - 3M_{\tilde{Z}}^2}{6M_{\tilde{Z}}^4} (k_\alpha k_\beta \eta_{\gamma\delta} + k_\gamma k_\delta \eta_{\alpha\beta}) \\ & + \frac{2k_\alpha k_\beta k_\gamma k_\delta}{3M_{\tilde{Z}}^4}. \end{aligned} \quad (20)$$

By combing the form factors in Equation (18) and the propagator in Equation (19), the amplitude can be written as:

$$\begin{aligned}
M_{\tilde{Z}} &= \frac{g_D^2 d_{\alpha\beta\gamma\delta}}{4(t - M_{\tilde{Z}}^2)} \\
&\times \left[ A(t)(\bar{u}_1 \gamma^\alpha u_3)(p_1 + p_3)^\beta + \frac{iB(t)}{2m_p}(p_1 + p_3)^\beta k_\rho (\bar{u}_1 \sigma^{\alpha\rho} u_3) + \frac{C(t)}{m_p}(\bar{u}_1 u_3)(k^\alpha k^\beta - \eta^{\alpha\beta} t) \right] \\
&\times \left[ A(t)(\bar{u}_2 \gamma^\gamma u_4)(p_2 + p_4)^\delta + \frac{iB(t)}{2m_p}(p_2 + p_4)^\delta k_\lambda (\bar{u}_2 \sigma^{\gamma\lambda} u_4) + \frac{C(t)}{m_p}(\bar{u}_2 u_4)(k^\gamma k^\delta - \eta^{\gamma\delta} t) \right]. \quad (21)
\end{aligned}$$

Using the condition  $s \gg |t|$ , it can be seen that the contributions from the C(t)-related terms are suppressed by the factor  $|t|/s$ , while the contributions from the B(t)-related terms are negligible compared to those from the A(t)-related terms. Therefore, in the high-energy regime, Equation (21) is greatly reduced, and only the terms containing the form factor A(t) need to be considered. By applying these approximations, Equation (21) can be expanded and rearranged into the following form:

$$M_{\tilde{Z}} = \frac{g_D^2}{8(t - M_{\tilde{Z}}^2)} \left[ 2sA(t)^2 (\bar{u}_1 \gamma^\alpha u_3)(\bar{u}_2 \gamma_\alpha u_4) + 4A(t)^2 p_2^\alpha p_1^\beta (\bar{u}_1 \gamma_\alpha u_3)(\bar{u}_2 \gamma_\beta u_4) \right]. \quad (22)$$

Since the differential cross-section is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |M_{\tilde{Z}}(s, t)|^2, \quad (23)$$

the expression for the process under consideration can be derived from the modulus and spin-averaged sum of Equation (22). We thus obtain:

$$\frac{d\sigma}{dt} = \frac{g_D^4 s^2 A(t)^4}{16\pi(t - M_{\tilde{Z}}^2)^2}. \quad (24)$$

This differential cross-section represents only the exchange of the tensor dark matter particle. Here, the gravitational-like form factor is approximated by the dipole form factor

$A(t) = \left(1 - t/M_d^2\right)^{-2}$  [85], where the dipole mass  $M_d$  is the one of the four model parameters.

The dipole form factor was originally proposed to fit the elastic electron-proton scattering data at large angles [88, 85].

## 5. Conclusions

We suggest that a new, neutral tensor gauge boson ( $Z_{\mu\nu}$ -boson) can explain the existence of dark matter in our Universe. The  $Z_{\mu\nu}$ -boson can be predicted by the tensor gauge boson extension of the Electro Weak (EW) theory proposed by G. Savvidy (2005). We compute the self-annihilation cross-section of the  $Z_{\mu\nu}$ -boson-dark matter, and calculate its relic abundance. We also study the proton-proton scattering by the exchange of a massive,  $Z_{\mu\nu}$ -boson-dark matter at a high energy scale. The existence of these proposed  $Z_{\mu\nu}$ -boson is testable at the

Large Hadron Collider (LHC). This proposition may have far-reaching applications in astrophysics and cosmology.

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