WAMS - Winternitz Abstract Merkle Signature Scheme

Author: Herman Schoenfeld
Version: 1.1
Date: 2020-07-20

Abstract

A quantum-resistant, many-time signature scheme combining Winternitz and Merkle-Signature schemes is proposed. This construction is compatible with the Abstract Merkle Signature (AMS) Scheme [1] and thus is an AMS-algorithm called "WAMS".

1. Introduction

WAMS is a specialization of the AMS [1] scheme parameterized with the standard Winternitz one-time signature scheme (W-OTS). WAMS is a quantum-resistant cryptographic scheme suitable for blockchain-based applications.

This document focuses only on the OTS-layer of WAMS. The merkle signatures themselves are performed as part of the AMS-layer of WAMS which is defined in the AMS document [1]. The reader should familiarize themselves with the AMS document as it provides the background context for AMS-algorithms of which WAMS is one.

2. WAMS Scheme

The Winternitz Abstracted Merkle Signature (WAMS) Scheme is a general purpose, quantum-resistant digital signature scheme. WAMS is an AMS algorithm that selects the standard Winternitz OTS (W-OTS) as the OTS parameter. As part of the parameter set inherited from AMS, WAMS includes the additional parameters `H`, a cryptographic hash function and `w`, the Winternitz parameter.

The selected cryptographic hash function `H` is fundamental to the security of WAMS, an analysis of which is not provided in this paper. So long as the user selects a standard CHF such as `SHA2-256` or `Blake2b`, the security of WAMS is equivalent to standard W-OTS constructions. For performance, the use of W-OTS# [2] may be used in conjunction with `Blake2b-128` in order to reduce signature sizes without introducing vulnerability to birthday-class attacks.

In the presented construction herein, the Winternitz parameter `w` refers to the number of bits being simultaneously signed as famously proposed by Merkle [3] (who was inspired by Winternitz). Varying the parameter `w` changes the size/speed trade-off without affecting security. For example, the higher the value the more expensive (and slower) the computations but the shorter the signature and private key size. The lower the value the faster the computation but larger the signature and key size. The range of values for `w` supported in WAMS is `1 <= w <= 16`.

Since the WAMS scheme inherits the AMS scheme, it is required to define the following:

- The OTS private key which is a standard W-OTS private key.
- The OTS public key is a standard W-OTS public key (hash).
- Definitions for `GenOTSSig` and `VerOTSSig` which generate and verify W-OTS signatures in accordance to the WAMS [1] specification.

Definitions for all of the above are provided below.

2.1 Notation & Definitions

1. Notations and definitions from AMS [1] are inherited by this document.
2. `ReadBits(arr, N, M)` is a function that skips `N` bits and then reads `M` bits from the byte array `arr` and re-interprets the bits as a big-endian unsigned 32-bit integer.
3. `WriteBits(x, arr, N, M)` is a function that converts unsigned 32-bit integer `x` to big-endian byte array of 4 bytes and writes the first `M` bits of the array into array `arr` start at bit offset `N`.
4. Bit-ordering in (2) and (3) is such that bit `i` of `arr` maps to byte `arr[i SHR 3]` and to in-byte bit-index `(i SHR 3) - (i SHL 3)`. Explained below:

```
Bit-ordering within `ReadBits` and `WriteBits` are such that the
   least-significant bit (LSB) is the left-most bit of that byte.

   For example, consider an array of two bytes C = [A,B]:

   Memory layout of C=[a,b] with their in-byte indexes marked.
   A:[7][6][5][4][3][2][1][0]  B:[7][6][5][4][3][2][1][0]
   C:[0][1][2][3][4][5][6][7]    [8][9]...

   The bit indexes of the 16-bit array C are such that:
     Bit 0 maps to A[7]
     Bit 1 maps to A[6]
     Bit 7 maps to A[0]
     Bit 8 maps to B[7]
     Bit 16 maps to B[0]
```

2.2 WAMS Parameters

Parameters	Description	Bits
`h`	Tree height (used in AMS layer)	8
`w`	Winternitz parameter, how many bits are simultaneously signed via the Winternitz gadget	8
`H`	Cryptographic hash function, and security parameter for the scheme (digest length)	8

Note that the Winternitz `w` and `H` are stored in the RESERVED part of the AMS private key. The cryptographic hash function is stored as a code, defined as follows:

2.2.1 Cryptographic Hash Function Code

Value	Cryptographic Hash Function
0	*user specified*
1	`SHA2-256`
2	`Blake2b-256`
3	`Blake2b-160`
4	`Blake2b-128`

The author reserves the right to update this list as new use-cases emerge.

2.3 WAMS Variables

During key generation, signing and verification the following variables are calculated based on the parameter set.

Variable	Formula	Description
`U`	`sizeof(H(x)) * 8` for any x	Security parameter for the scheme (and number of bits in a hash `H`)
`DigitBase`	`2^w`	The number of values a signed "digit" can take
`SigDigits`	`ceil(256 / w)`	Number of digits in the message-digest being signed
`CheckDigits`	`Log((2^w - 1) * (256/w))_{2^w}`	Number of digits in checksum being signed
`OTS_KeyDigits`	`SigDigits + CheckDigits`	Number of "digit keys" in a W-OTS private key (used by AMS-layer)
`OTS_SigLen`	`OTS_KeyDigits`	Number of "digit signatures" in a W-OTS sig (used by AMS-layer)

2.4 W-OTS Theory Basics

The W-OTS scheme follows the Lamport [4] signature approach but allows a signer to sign `w` bits of a message-digest simultaneously rather than 1. This collection of bits is a treated as a "digit" of base `2^w`.

For example, in the case of `w=8` the digits simply become bytes since each digit can take any value within `0..255`. The fundamental cryptographic mechanism in W-OTS is the ability to sign individual digits using a unique "digit private key".

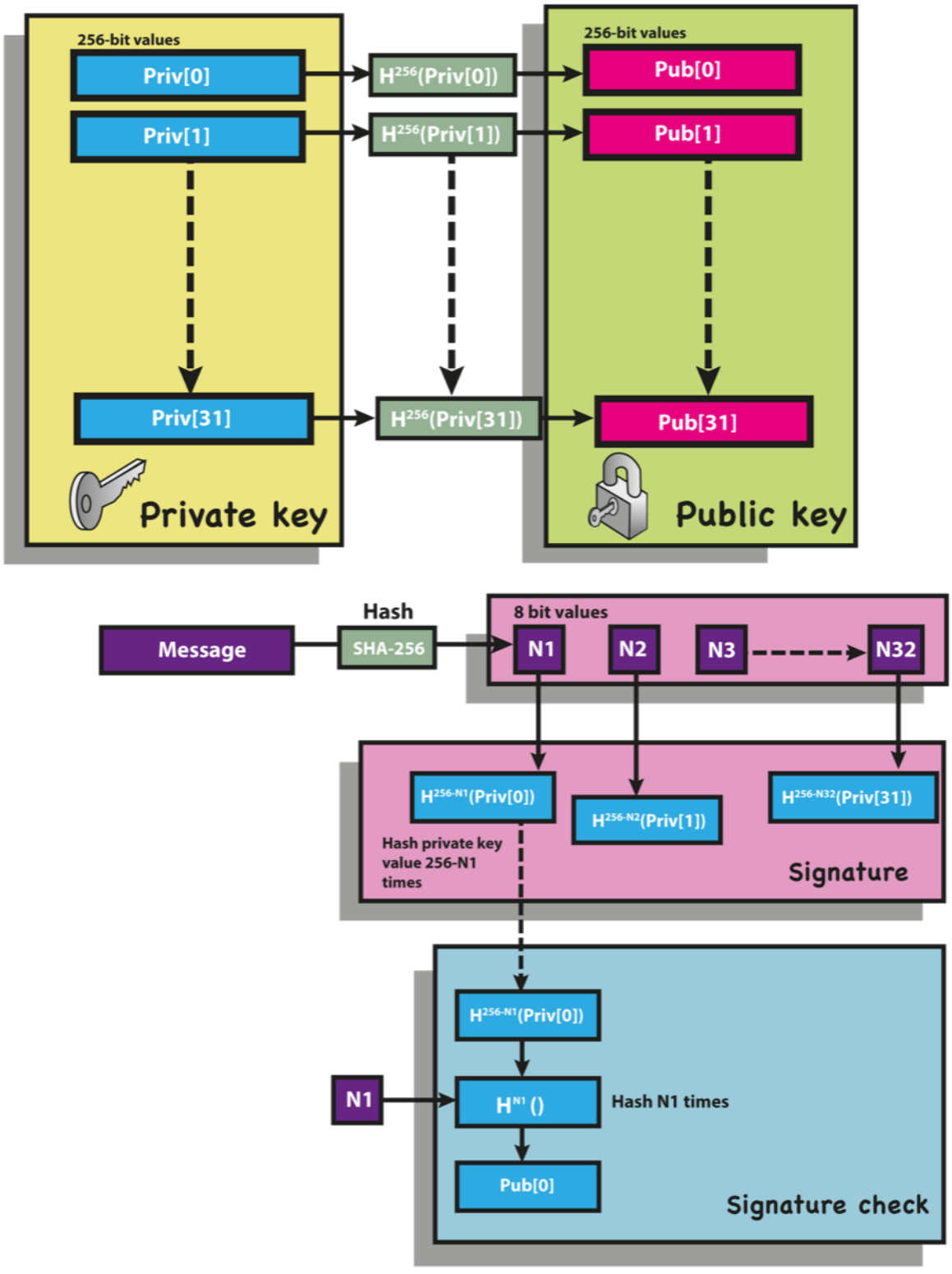
For example, to sign the byte `b` (for `w=8`), a signer first derives a "private digit key" as `K = H(secret)` and a "public digit key" `P = H^255(K)`. Notice that all the values of `b` map to a unique hash in that chain of hashes. The signer advertises the "public digit key" prior to signing any digit. When signing a digit `b`, the signer provides the verifier the value `S=H^(255-b)(K)` referred to as the "signature of `b`". The verifier need only perform `b` more iterations on the signature `s` to arrive at the public key `P`, since `H^b(S) = H^b(H^(255-b)(K)) = H^255(K) = P`.

At this point, the verifier has cryptographically determined the signer had knowledge of `K` since the signature `S` was the `b'th` pre-image of `P`. This process of signing digits is repeated for each digit in the message and each digit signature is concatenated to form the signature. The message being signed is always a digest of an actual logical message, and thus referred to as the "message-digest".

In W-OTS, the individual "digit keys" and "digit signatures" are concatenated to comprise the "key" and "signatures" respectively. This results in order of magnitude larger key and signature objects when compared to traditional elliptic-curve / discrete logarithm schemes. This is a significant down-side of OTS schemes when used in post-quantum cryptography (PQC) use cases. The burden of large keys can be optimized by using the hash of a public key as WAMSD prescribes. The burden of large signatures can be halved by choosing shorter hash functions without impacting security, as prescribed by the W-OTS#[2] variant. .

NOTE In order to prevent signature forgeries arising from digit signature re-use for prior messages, a checksum is calculated and appended to the message-digest and co-signed. The checksum is calculated in such a way that any increment to a message digit necessarily decreases a checksum digit. Thus it is impossible to forge a signature since it requires the pre-image of at least one checksum digit signature.

The reader can further their understanding of the theory and basics of W-OTS by reviewing the literature and through the below diagram[5].


256-bit values
Priv[0]
Priv[1]
Priv[31]
Private key
H256(Priv[0])
H256(Priv[1])
H256(Priv[31])
256-bit values
Pub[0]
Pub[1]
Pub[31]
Public key
Message
Hash
SHA-256
8 bit values
N1
N2
N3
N32
H256-N1(Priv[0])
H256-N2(Priv[1])
H256-N32(Priv[31])
Hash private key value 256-N1 times
Signature
H256-N1(Priv[0])
N1
HN1 ()
Hash N1 times
Pub[0]
Signature check

2.4.1 W-OTS Private Key

A W-OTS private key `P'` is a one-time key used to generate W-OTS signatures and defined as follows:

```
1: P' = byte-array[OTS_KeyDigits, U/8]
2: for n in {0, OTS_KeyDigits - 1}
3:    P'[n] = cryptographically random U bits
```

The W-OTS private key is an array of `OTS_KeyDigits` "digit keys" each of `U/8` bytes in length. The total size of the W-OTS private key is thus `(OTS_KeyDigits) * (U/8)` bytes.

Whilst the W-OTS scheme requires that private keys be cryptographically random, they can be deterministically derived from a secret seed. In WAMS the AMS Private Key is used (see below).

2.4.2 W-OTS Public Key Hash

A W-OTS public key hash `K'` is a one-time key used to verify W-OTS signatures signed by a W-OTS private key `P'` and defined as follows:

```
1: k = byte-array[OTS_KeyDigits, U/8]
2: for n in {0, OTS_KeyDigits - 1}
3:    k[n] = H^(DigitBase - 1)( P'[n] )
4: K' = H( k[0] || k[1] || ... || k[OTS_KeyDigits - 1] )
```

The length of a W-OTS public key hash is `U/8` bytes.

NOTE In WAMS, the W-OTS public key hash is used rather than the W-OTS public key since signature verification always rebuilds the public key from the signature. Since the verifier derives the public key it can derive the public key hash with one additional step. By using the hash rather than the key in the AMS signature, a ~50% space saving is made to the AMS signature length.

NOTE 2 Since the OTS layer passes the public key hash to the AMS layer, the AMS layer does not need hash the public keys when building the hash-tree of OTS keys, it simply re-uses the OTS public key value which is itself a hash digest (saving `2^h` hash computations when computing a batch).

2.5 WAMS Key Generation

Given a AMS Private Key `P` and batch number `B`, the `i'th` W-OTS key-pair (`P'`, `K'`) are derived as follows:

```
 1: algorithm GenOTSKeys
 2:   Input:
 3:         P: AMS Private Key
 4:         B: batch number (UInt64)
 5:         i: index (UInt16)
 6:   Output:
 7:         P': the W-OTS private key that derives K'
 8:         K': the i'th W-OTS public key hash in the batch
 9:   Pseudo-Code:
10:     P' = byte-array[OTS_KeyDigits, U/8]
```

```
11:     k = byte-array[OTS_KeyDigits, U/8]
12:     let seed = ToBytes(i) || ToBytes(B) || P
13:     for n in {0, OTS_KeyDigits - 1}
14:        P'[n] = H^2( n || seed )
15:        k[n] = H^(DigitBase - 1) ( P'[n] )
16:     K' = H( k[0] || k[1] || ... || k[OTS_KeyDigits - 1] )
17: end algorithm
```

2.6 W-OTS Signature Generation

A W-OTS signature is an 2D array of bytes of dimensions `[OTS_KeyDigits, U/8]` and generated as follows:

```
 1: algorithm GenOTSSig
 2:   Input:
 3:         m: a message-digest (U/8 bytes)
 4:         P': a W-OTS private key
 5:   Output:
 6:         S': a W-OTS signature
 7:   Pseudo-Code:
 8:     S' = byte-array[OTS_KeyDigits, U/8]
 9:     // sign message part
10:     let c = 0                      ; checksum value
11:     for n in {0, SigDigits - 1}
12:        let v = 2^w - ReadBits(m, w*n, w) - 1
13:        c = c + v;
14:        S'[n] = H^v( P'[n] )
15:
16:     // sign checksum part
17:     let c_bytes = byte-array[4]
19:     WriteBits(c, c_bytes, 0, 32)
20:     for n in {0, CheckDigits - 1}
21:        let v = 2^w - ReadBits(c_bytes, w*n, w) - 1
22:        S'[SigDigits + n] = H^v( P'[SigDigits + n] )
24: end algorithm
```

2.7 W-OTS Signature Verification

Here a W-OTS signature is verified to a W-OTS public key hash by rebuilding the W-OTS public key from the signature, hashing it and comparing with public key hash provided by the AMS layer.

```
 1: algorithm VerOTSSig
 2:   Input:
 3:        S': a W-OTS signature (byte[ OTS_KeyDigits, U/8 ])
 4:        m: a message-digest (byte[U/8])
 5:        K': W-OTS public key/hash (byte[U/8])
 6:   Output: Boolean
 7:   Pseudo-Code:
 8:        k = byte[ OTS_KeyDigits, U/8 ]    ; the W-OTS public key
 9:        ; verify message part
```

```
10:        let c = 0                          ; checksum value
11:        for n in {0, SigLen - 1}
12:           let d = ReadBits(m, w * n, w)   ; note: d + v = 2^w - 1
13:           c = 2^w + d - 1
14:           k[n] = H^d( S'[n] )             ; note: k[n] = H^d(H^c(P'[n]))
15:
16:        ; verify checksum part
17:        let c_bytes = byte-array[4]
18:        WriteBits(c, c_bytes, 0, 32)
19:        for n in {0, CheckDigits - 1}
20:           let d = ReadBits(c_bytes, w * n, w)
21:           k[SigDigits + n] = H^d( S'[SigDigits + n] )
22:
23:        ; compare pub key hash
24:        let PKH = H( k[0] || k[1] || ... || k[OTS_KeyDigits - 1] )
25:        return (K' = PKH)                  ; check sig rebuilt the public key hash
26: end algorithm
```

3. WAMS#

WAMS# is a variant of WAMS which selects W-OTS# [2] rather than W-OTS as the OTS. W-OTS# is virtually identical to W-OTS except the message-digest is salted to harden the signature security to a sufficient level that thwarts birthday-class attacks. This allows the selection of shorter hash functions which produce shorter and faster signatures for same security as W-OTS.

The WAMS# implementation is virtually identical to WAMS except for the following changes:

1. A cryptographically random salt `R` of `U`-bits is generated during signing.
2. For any message `m`, the signer signs the "sig-mac" `SMAC(m, R)` rather than the message-digest `H(m)` which is defined as `SMAC(m, R) = H(R || H (R || H(m)))`.
3. `R` is appended to the signature.
4. During verification, the verifier similarly verifies `SMAC(m, R)` rather than the ordinary message-digest.

The reader is referred to the reference implementation of WAMS# which succinctly overloads WAMS with these minor changes.

4. Object Lengths & Throughput

A C# implementation in .NET 7 was developed and object lengths and performance metrics are measured below. All tests were performed on a single thread on an Intel Core i9-10900K CPU 3.70 GHz with 32GB RAM. The implementation was not performance tuned so the throughput metrics are useful when compared relative to each other.

OTS	CHF bits	Winternitz `w`	Height `h`	Public Key Length (b)	Signature Length (b)	Sign Throughput	Verify Throughput
W-OTS	128	2	0	32	2163	3620	18098

OTS	CHF bits	Winternitz `w`	Height `h`	Public Key Length (b)	Signature Length (b)	Sign Throughput	Verify Throughput
W-OTS#	128	2	0	32	2211	3425	13139
W-OTS	128	2	8	32	2163	3832	17919
W-OTS#	128	2	8	32	2211	3523	12056
W-OTS	128	2	16	32	2163	3759	18137
W-OTS#	128	2	16	32	2211	3528	12111
W-OTS	128	4	0	32	1107	2821	11479
W-OTS#	128	4	0	32	1155	2619	10403
W-OTS	128	4	8	32	1107	2803	13454
W-OTS#	128	4	8	32	1155	2610	9861
W-OTS	128	4	16	32	1107	2810	13470
W-OTS#	128	4	16	32	1155	2602	9515
W-OTS	128	8	0	32	579	432	2406
W-OTS#	128	8	0	32	627	414	2079
W-OTS	128	8	8	32	579	434	2403
W-OTS#	128	8	8	32	627	419	2749
W-OTS	128	8	16	32	579	432	2411
W-OTS#	128	8	16	32	627	404	2749
W-OTS	160	2	0	36	2703	3850	17026
W-OTS#	160	2	0	36	2763	3607	11620

OTS	CHF bits	Winternitz `w`	Height `h`	Public Key Length (b)	Signature Length (b)	Sign Throughput	Verify Throughput
W-OTS	160	2	8	36	2703	3828	16875
W-OTS#	160	2	8	36	2763	3567	11800
W-OTS	160	2	16	36	2703	3871	16969
W-OTS#	160	2	16	36	2763	3551	11572
W-OTS	160	4	0	36	1383	2864	12419
W-OTS#	160	4	0	36	1443	2702	9318
W-OTS	160	4	8	36	1383	2841	12564
W-OTS#	160	4	8	36	1443	2685	9841
W-OTS	160	4	16	36	1383	2854	12586
W-OTS#	160	4	16	36	1443	2680	9120
W-OTS	160	8	0	36	723	434	2154
W-OTS#	160	8	0	36	783	417	2184
W-OTS	160	8	8	36	723	428	2145
W-OTS#	160	8	8	36	783	422	2425
W-OTS	160	8	16	36	723	427	2156
W-OTS#	160	8	16	36	783	421	2032
W-OTS	256	2	0	48	4323	3937	12474
W-OTS#	256	2	0	48	4419	3662	9235
W-OTS	256	2	8	48	4323	3951	12275

OTS	CHF bits	Winternitz `w`	Height `h`	Public Key Length (b)	Signature Length (b)	Sign Throughput	Verify Throughput
W-OTS#	256	2	8	48	4419	3620	8829
W-OTS	256	2	16	48	4323	3905	12373
W-OTS#	256	2	16	48	4419	3666	9168
W-OTS	256	4	0	48	2211	3059	8653
W-OTS#	256	4	0	48	2307	2885	7711
W-OTS	256	4	8	48	2211	3081	8549
W-OTS#	256	4	8	48	2307	2873	7151
W-OTS	256	4	16	48	2211	3025	8527
W-OTS#	256	4	16	48	2307	2865	7035
W-OTS	256	8	0	48	1155	485	1299
W-OTS#	256	8	0	48	1251	464	1849
W-OTS	256	8	8	48	1155	489	1345
W-OTS#	256	8	8	48	1251	471	1372
W-OTS	256	8	16	48	1155	487	1331
W-OTS#	256	8	16	48	1251	458	1502

Throughput is measured in "Signatures Per Second"

5. Reference Implementation

This section contains snippets for the full reference implementation[6] . The reference implementation is part of the PQC library within the Hydrogen Framework[7] .

5.1 AMS-Compatible W-OTS

This implementation of W-OTS can be used as an OTS within the AMS implementation [1].

```
public class WOTS : IOTSAlgorithm {

    public WOTS()
        : this(Configuration.Default) {
    }

    public WOTS(int w, bool usePublicKeyHashOptimization = false)
        : this(w, Configuration.Default.HashFunction, usePublicKeyHashOptimization)
{
    }

    public WOTS(int w, CHF hashFunction, bool usePublicKeyHashOptimization = false)
        : this(new Configuration(w, hashFunction, usePublicKeyHashOptimization)) {
    }

    public WOTS(Configuration config) {
        Config = (Configuration)config.Clone();
    }

    public Configuration Config { get; }

    OTSConfig IOTSAlgorithm.Config => Config;

    public void SerializeParameters(Span<byte> buffer) {
        buffer[0] = (byte)Config.W;
    }

    public byte[,] GeneratePrivateKey()
        => GenerateKeys().PrivateKey;

    public byte[,] DerivePublicKey(byte[,] privateKey) {
        var publicKey = new byte[Config.KeySize.Length, Config.KeySize.Width];
        for (var i = 0; i < Config.KeySize.Length; i++) {
            publicKey.SetRow(i,  Hashers.Iterate(Config.HashFunction,
privateKey.GetRow(i), Config.ChainLength));
        }
        return Config.UsePublicKeyHashOptimization ? ToOptimizedPublicKey(publicKey)
: publicKey;
    }

    public OTSKeyPair GenerateKeys()
        =>
GenerateKeys(Tools.Crypto.GenerateCryptographicallyRandomBytes(Config.DigestSize -
1));

    public OTSKeyPair GenerateKeys(ReadOnlySpan<byte> seed) {
        var enumeratedSeed = new byte[seed.Length + 1];
```

```
        seed.CopyTo(enumeratedSeed.AsSpan(1));
        return GenerateKeys(i => {
            enumeratedSeed[0] = (byte)i;
            return Hashers.Iterate(Config.HashFunction, enumeratedSeed, 2);
        });
    }

    public OTSKeyPair GenerateKeys(Func<int, byte[]> gen) {
        var priv = new byte[Config.KeySize.Length, Config.KeySize.Width];
        var pub = new byte[Config.KeySize.Length, Config.KeySize.Width];  // actual
W-OTS pubkey is same size as priv key, we may optimize below
        for (var i = 0; i < Config.KeySize.Length; i++) {
            var randomBytes = gen(i);
            priv.SetRow(i, randomBytes);
            pub.SetRow(i, Hashers.Iterate(Config.HashFunction, randomBytes,
Config.ChainLength));
        }

        IFuture<byte[]> pubKeyHash;
        if (Config.UsePublicKeyHashOptimization) {
            pub = ToOptimizedPublicKey(pub);
            pubKeyHash = ExplicitFuture<byte[]>.For(pub.ToFlatArray());
        } else {
            pubKeyHash = LazyLoad<byte[]>.From(() =>
ToOptimizedPublicKey(pub).ToFlatArray());
        }

        return new OTSKeyPair(priv, pub, pubKeyHash);
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    public virtual byte[,] Sign(byte[,] privateKey, ReadOnlySpan<byte> message)
        => SignDigest(privateKey, ComputeMessageDigest(message));

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    public virtual byte[,] SignDigest(byte[,] privateKey, ReadOnlySpan<byte> digest)
{
        var signature = new byte[Config.SignatureSize.Length,
Config.SignatureSize.Width];

        // Sign the digest (and build the checksum in process)
        uint checksum = 0U;
        for (var i = 0; i < Config.SignatureDigits; i++) {
            var signValue = (int)Bits.ReadBinaryNumber(digest, Config.W * i,
Config.W, IterateDirection.LeftToRight);
            var c = Config.ChainLength - signValue;
            checksum += (uint)c;
            signature.SetRow(i, Hashers.Iterate(Config.HashFunction,
privateKey.GetRow(i), c));
        }

        // Sign the checksum
```

```
        var checksumBytes = new byte[4];
        Bits.WriteBinaryNumber(checksum, checksumBytes, 0, 32,
IterateDirection.LeftToRight);
        for (var i = 0; i < Config.ChecksumDigits; i++) {
            var signValue = (int)Bits.ReadBinaryNumber(checksumBytes, Config.W * i,
Config.W, IterateDirection.LeftToRight);
            var c = Config.ChainLength - signValue;
            var row = Config.SignatureDigits + i;
            signature.SetRow(row, Hashers.Iterate(Config.HashFunction,
privateKey.GetRow(row), c));
        }

        return signature;
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    public bool Verify(byte[,] signature, byte[,] publicKey, ReadOnlySpan<byte>
message)
        =>  VerifyDigest(signature, publicKey, ComputeMessageDigest(message));

    public virtual bool VerifyDigest(byte[,] signature, byte[,] publicKey,
ReadOnlySpan<byte> digest) {
        var verify = new byte[Config.KeySize.Length, Config.KeySize.Width];

        // Verify Digest
        uint checksum = 0U;
        for (var i = 0; i < Config.SignatureDigits; i++) {
            var signValue = (int)Bits.ReadBinaryNumber(digest, Config.W * i,
Config.W, IterateDirection.LeftToRight);
            var c = Config.ChainLength - signValue;
            checksum += (uint)c;
            verify.SetRow(i, Hashers.Iterate(Config.HashFunction,
signature.GetRow(i), signValue));
        }

        // Verify checksum
        var checksumBytes = new byte[4];
        Bits.WriteBinaryNumber(checksum, checksumBytes, 0, 32,
IterateDirection.LeftToRight);
        for (var i = 0; i < Config.ChecksumDigits; i++) {
            var signValue = (int)Bits.ReadBinaryNumber(checksumBytes, Config.W * i,
Config.W, IterateDirection.LeftToRight);
            var row = Config.SignatureDigits + i;
            verify.SetRow(row, Hashers.Iterate(Config.HashFunction,
signature.GetRow(row), signValue));
        }

        return (Config.UsePublicKeyHashOptimization ? this.ComputeKeyHash(verify) :
verify.AsFlatSpan()).SequenceEqual(publicKey.AsFlatSpan());
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
```

```
    public void ComputeKeyHash(byte[,] key, Span<byte> result) {
        ComputeKeyHash(key.AsFlatSpan(), result);
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    public void ComputeKeyHash(ReadOnlySpan<byte> key, Span<byte> result) {
        Hashers.Hash(Config.HashFunction, key, result);
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    protected byte[,] ToOptimizedPublicKey(byte[,] publicKey) {
        var publicKeyHash = new byte[1, Config.DigestSize];
        ComputeKeyHash(publicKey, publicKeyHash.AsFlatSpan());
        return publicKeyHash;
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    public byte[] ComputeMessageDigest(ReadOnlySpan<byte> message)
        => Hashers.Hash(Config.HashFunction, message);


    public class Configuration : OTSConfig {
        public static readonly Configuration Default;

        static Configuration() {
            Default = new Configuration(8, CHF.SHA2_256, false);
        }

        public Configuration() : this(Default.W, Default.HashFunction,
Default.UsePublicKeyHashOptimization) {
        }

        public Configuration(int w, CHF hasher, bool usePubKeyHashOptimization)
            : this(
                  w,
                  hasher,
                  usePubKeyHashOptimization,
                  AMSOTS.WOTS,
                  Hashers.GetDigestSizeBytes(hasher),
                  new OTSKeySize(
                      Hashers.GetDigestSizeBytes(hasher),
                      (int)Math.Ceiling(256.0 / w) + (int)Math.Floor(Math.Log(((1 <<
w) - 1) * (256 / w), 1 << w)) + 1
                  ),
                new OTSKeySize(
                    Hashers.GetDigestSizeBytes(hasher),
                    usePubKeyHashOptimization ? 1 : (int)Math.Ceiling(256.0 / w) +
(int)Math.Floor(Math.Log(((1 << w) - 1) * (256 / w), 1 << w)) + 1
                ),
                new OTSKeySize(
                    Hashers.GetDigestSizeBytes(hasher),
```

```
                    (int)Math.Ceiling(256.0 / w) + (int)Math.Floor(Math.Log(((1 <<
w) - 1) * (256 / w), 1 << w)) + 1
                )
            ) {
        }

        protected Configuration(int w, CHF hasher, bool usePubKeyHashOptimization,
AMSOTS id, int digestSize, OTSKeySize keySize, OTSKeySize publicKeySize, OTSKeySize
signatureSize)
            : base(id, hasher, digestSize, usePubKeyHashOptimization, keySize,
publicKeySize, signatureSize) {
            Guard.ArgumentInRange(w, 1, 16, nameof(w));
            W = (byte)w;
            ChainLength = (1 << w) - 1; // 2^w - 1 (length of Winternitz chain)
            SignatureDigits = (int)Math.Ceiling(256.0 / w); // how many chains
required;
            ChecksumDigits = (int)Math.Floor(Math.Log(((1 << w) - 1) * (256 / w), 1
<< w)) + 1; // floor ( log_b (2^w - 1) * (256/w) ) where b = 2^w
        }

        public int W { get; }

        public int ChainLength { get; }

        public int SignatureDigits { get; }

        public int ChecksumDigits { get; }

        public override object Clone() => new Configuration(W, HashFunction,
UsePublicKeyHashOptimization, AMSID, DigestSize, KeySize, PublicKeySize,
SignatureSize);

    }

}
```

5.2 AMS-Compatible W-OTS#

This implementation of W-OTS# can be used as an OTS within the AMS implementation [1].

```
public class WOTSSharp : WOTS {

    public WOTSSharp()
        : this(WOTSSharp.Configuration.Default) {
    }

    public WOTSSharp(int w, bool usePublicKeyHashOptimization = false)
        : this(w, Configuration.Default.HashFunction, usePublicKeyHashOptimization)
{
    }
```

```
    public WOTSSharp(int w, CHF hashFunction, bool usePublicKeyHashOptimization =
false)
        : this(new Configuration(w, hashFunction, usePublicKeyHashOptimization)) {
    }

    public WOTSSharp(Configuration config)
        : base(config) {
    }

    public override byte[,] SignDigest(byte[,] privateKey, ReadOnlySpan<byte>
digest)
        => SignDigest(privateKey, digest,
Tools.Crypto.GenerateCryptographicallyRandomBytes(digest.Length));

    public byte[,] SignDigest(byte[,] privateKey, ReadOnlySpan<byte> digest,
ReadOnlySpan<byte> seed) {
        Guard.Argument(seed.Length == digest.Length, nameof(seed), "Must be same
size as digest");
        var wotsSig = base.SignDigest(privateKey, HMAC(digest, seed));
        Debug.Assert(wotsSig.Length == Config.SignatureSize.Length *
Config.SignatureSize.Width);
        seed.CopyTo(wotsSig.GetRow(Config.SignatureSize.Length - 1)); // concat seed
to sig
        return wotsSig;
    }

    public override bool VerifyDigest(byte[,] signature, byte[,] publicKey,
ReadOnlySpan<byte> digest) {
        Debug.Assert(signature.Length == Config.SignatureSize.Length *
Config.SignatureSize.Width);
        var seed = signature.GetRow(Config.SignatureSize.Length - 1);
        return base.VerifyDigest(signature, publicKey, HMAC(digest, seed));
    }

    [MethodImpl(MethodImplOptions.AggressiveInlining)]
    private byte[] SMAC(ReadOnlySpan<byte> message, ReadOnlySpan<byte> seed)
        => HMAC(ComputeMessageDigest(message), seed);

    private byte[] HMAC(ReadOnlySpan<byte> digest, ReadOnlySpan<byte> seed) {
        using (Hashers.BorrowHasher(Config.HashFunction, out var hasher)) {
            hasher.Transform(seed);
            hasher.Transform(digest);
            var innerHash = hasher.GetResult();
            hasher.Transform(seed);
            hasher.Transform(innerHash);
            return hasher.GetResult();
        }
    }

    public new class Configuration : WOTS.Configuration {
        public new static readonly Configuration Default;
```

```
        static Configuration() {
            Default = new Configuration(4, CHF.Blake2b_128, true);
        }

        public Configuration()
            : this(Default.W, Default.HashFunction,
Default.UsePublicKeyHashOptimization) {
        }

        public Configuration(int w, CHF hasher, bool usePubKeyHashOptimization)
            : base(
                w,
                hasher,
                usePubKeyHashOptimization,
                AMSOTS.WOTS_Sharp,
                Hashers.GetDigestSizeBytes(hasher),
                new OTSKeySize(
                    Hashers.GetDigestSizeBytes(hasher),
                    (int)Math.Ceiling(256.0 / w) + (int)Math.Floor(Math.Log(((1 <<
w) - 1) * (256 / w), 1 << w)) + 1
                ),
                new OTSKeySize(
                    Hashers.GetDigestSizeBytes(hasher),
                    usePubKeyHashOptimization ? 1 : (int)Math.Ceiling(256.0 / w) +
(int)Math.Floor(Math.Log(((1 << w) - 1) * (256 / w), 1 << w)) + 1
                ),
                new OTSKeySize(
                    Hashers.GetDigestSizeBytes(hasher),
                    (int)Math.Ceiling(256.0 / w) + (int)Math.Floor(Math.Log(((1 <<
w) - 1) * (256 / w), 1 << w)) + 1  + 1 // Adds extra row for seed here
                )
            ) {
        }
    }
}
```

6. References

1. Herman Schoenfeld. "AMS - Abstract Merkle Signature Scheme", 2020. URL: https://vixra.org/abs/2212.0019 ↩ ↩ ↩ ↩ ↩ ↩ ↩

2. Herman Schoenfeld. "W-OTS# - Shorter and Faster Winternitz Signatures". URL: https://vixra.org/abs/2007.0194. Accessed on: 2020-07-20. ↩ ↩ ↩

3. Ralph Merkle. "Secrecy, authentication and public key systems / A certified digital signature". Ph.D. dissertation, Dept. of Electrical Engineering, Stanford University, 1979. URL: http://www.merkle.com/papers/Certified1979.pdf. ↩

4. L. Lamport. "Constructing digital signatures from a one-way function". Technical Report SRI-CSL-98, SRI International Computer Science Laboratory, Oct. 1979. ↩

5. Sphere 10 Software. "Winternitz One-Time Signature Scheme". URL: https://sphere10.com/articles/cryptography/pqc/wots. Accessed on: 2023-05-09. ↩

6. Sphere 10 Software. PQC Library. Url: https://github.com/Sphere10/Hydrogen/tree/master/src/Hydrogen/Crypto/PQC. Accessed 2023-05-09. ↩

7. Sphere 10 Software. Hydrogen Framework. Url: https://github.com/Sphere10/Hydrogen. Accessed 2023-05-09 ↩