A somewhat intuitive visual representation of the formulae for π^3 and Ramanujan's π^4=97.5-1/11+1.2491..x10^{-7}

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Mojstrana, November 2022

Abstract

In the article, I show a visual representation of the formula π^3=31.00627.., respectively visualization how π^3 is close to 31. I show this using the area of a circle with radius π that is compared with the area that is quite simply composed of squares and triangles. In the same way the Ramanujan formula π^4=97.5-1/11+1.2491..x10^{-7} is visualized. At the end, I mention once again the challenge to explain the Ramanujan formula for za π^4.

1. Visualization of the relation π^3=31.00627..

We draw a circle with radius π, and the area of this circle is equal to π^3, that is 31.00627..[1]. Now we ask ourselves how to imagine this area with the help of the unit squares with sides 1 and with the help of simple triangles, so that the total area is 31. Let us call it as *Structure 31* (S31). Let us try to make S31 as easily as possible, so with as simple geometric shapes as possible, but also with as good coverage as possible for this circle. The first condition to achieve this is that S31 should be symmetrical enough .

Let us make a square 6 x 6, and take away 4 squares 1 x 1 at corners. So we have already obtained an area of 32, which is close to 31, Fig. 1. Then we symmetrically take away another 8 right triangles with a total area of 1, as in Fig. 1. It is important that this S31 is very symmetrical, it consists of 8 symmetrical parts.

[1] Two dots mean the following digits. That means also that the last written digit is not rounded up.

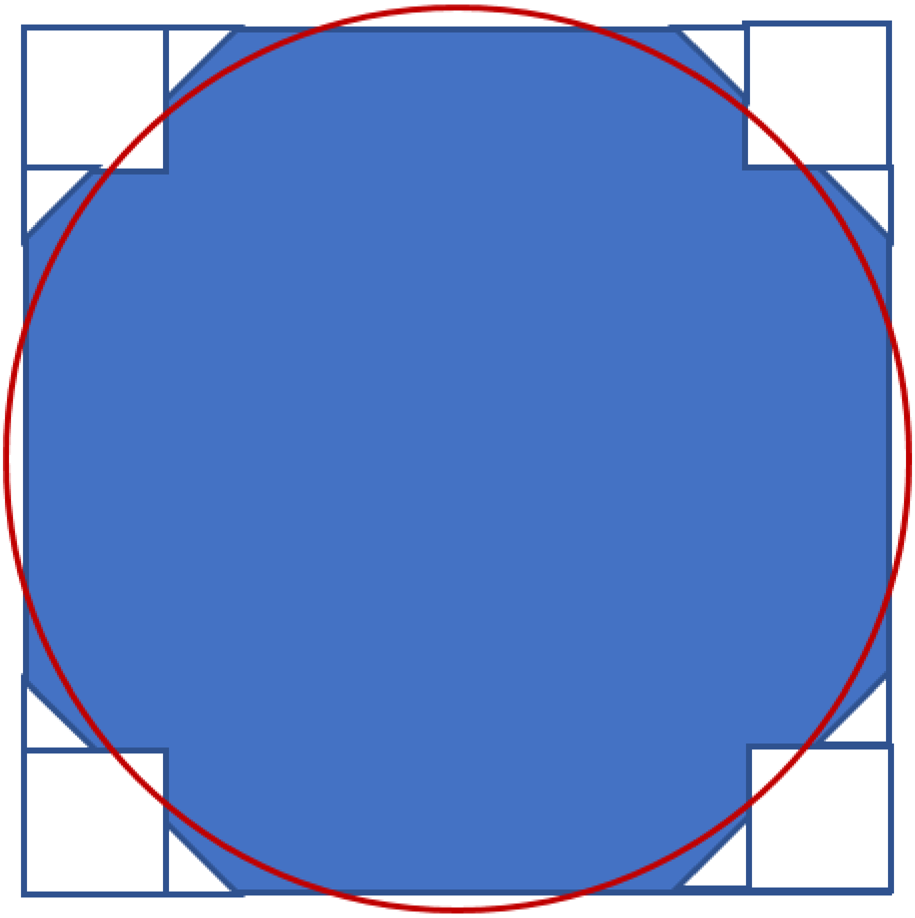

Fig. 1 The red circle has radius π, the area of the largest square is 6 x 6, and the blue area is equal to 31. Each white square has an area of 1 x 1, and each white right triangle, next to it, has an area of 1/8, their number is 8.

Now we make another correction to the current S31 to add the same right triangles as before to the removed edge squares so that they cover the circle in these parts. Then we compensate for this next to the previously removed right triangles, which we increase according to Fig. 1, i.e. we add a white triangle with an area of 1/16 to each one. Let see Fig. 2.

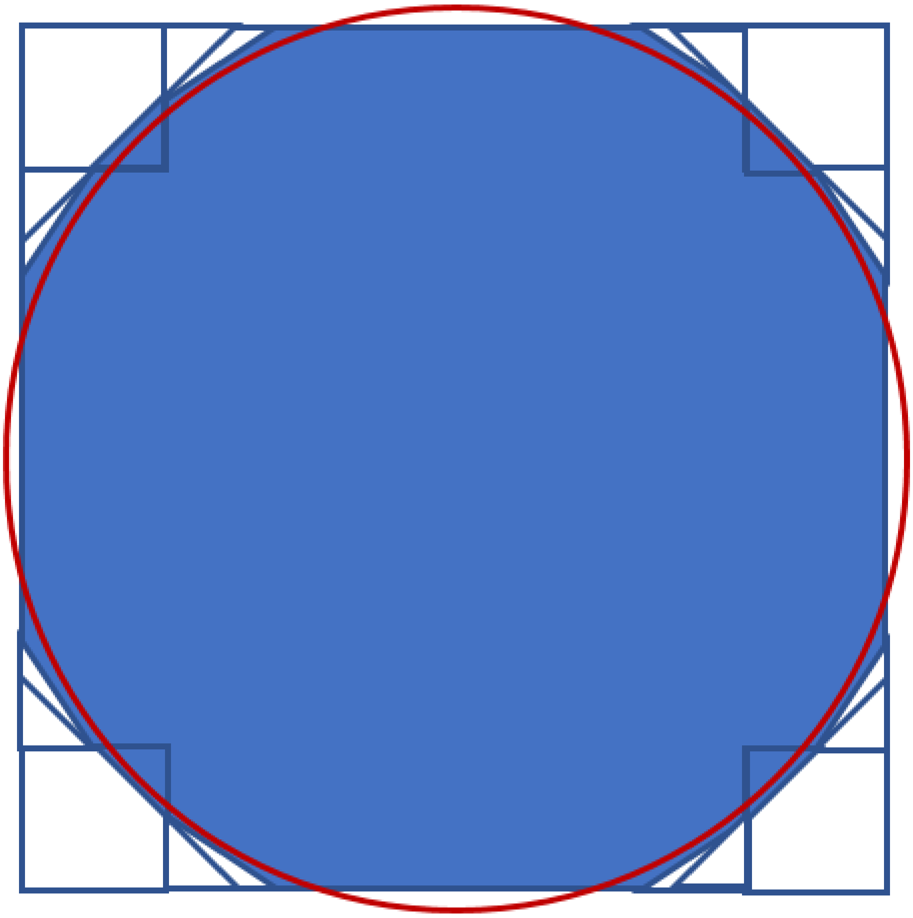

Fig 2. The red circle has radius π, the area of the largest square is 6 x 6, and the marked blue area is equal to 31. According to Fig. 1, the new blue rectangular triangle on each corner square is added, each equals to 1/8, their number is four, and the whitened triangles are inserted, where each has an area of 1/16, their number is eight.

We obtained S31, which is simple and overlaps quite well to this circle area.

Now let's compare these mismatch surfaces, but only on the circular sector of 45°, because the rest is symmetrical and repeated. This means that central line of this circular sector is tilted 22,5° left from the top, as evident in Fig. 3.

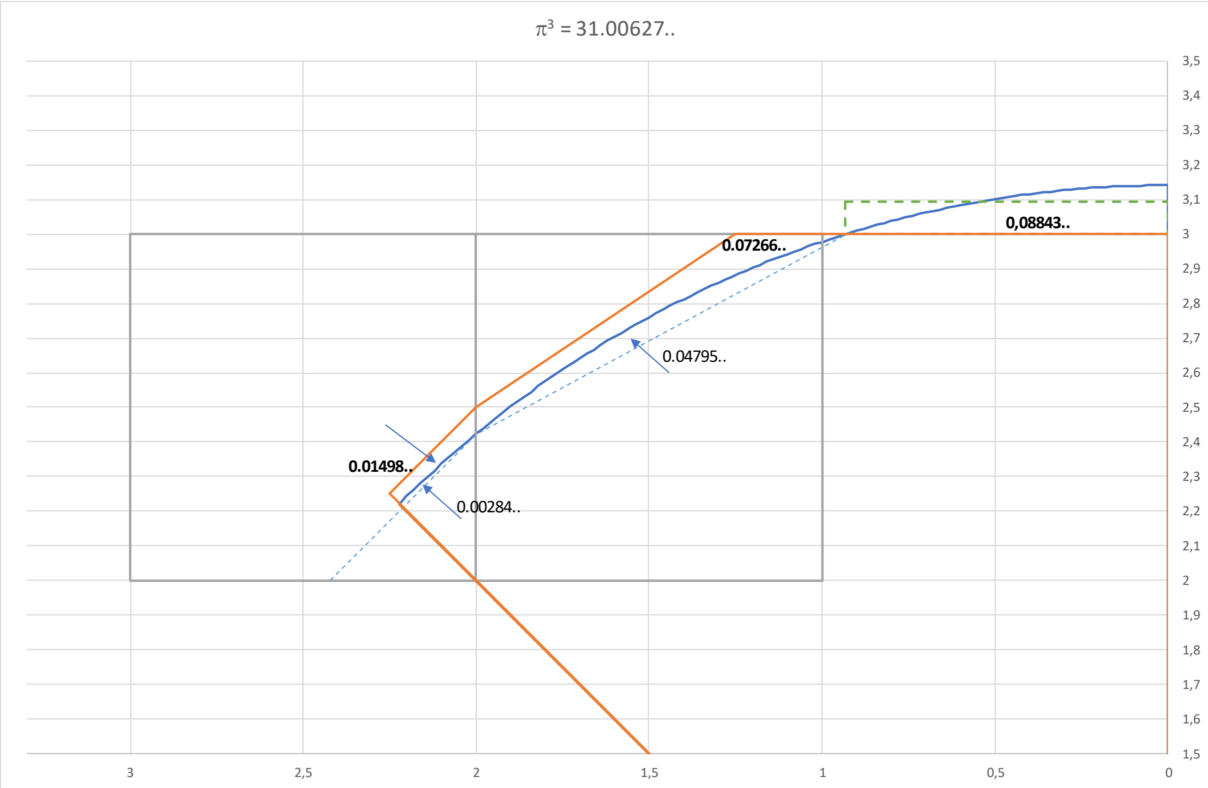

Fig. 3. The circular sector with angle 45° is zoomed at areas where S31/8 is not completely covered by this circular sector or it does not completely cover it. The blue solid line means the arc. The orange polygon means the edge of S31/8. Bold numbers mean three areas where S31/8 and the circular sector disagree. The blue dotted lines indicate chords of two circular segments [1]. The left non-bold number means half of area of the left circular segment, the right non-bold number means area of the middle circular segment. The green dotted line means the rectangle, which has the same width and the same area as the half of the circular segment that is partially overlapped by it.

As further, let us recognize three non-overlapped parts in S31/8 in Fig. 3. The first part is the half circular segment where the circle exceeds S31/8 (the right-hand side), the third part is inside of the corner square (the left-hand side), i.e. between the edge of the circle and the edge of S31. The rest is the second part where we also look the area between edge of the circle and the edge of S31.

In the first part, the circular segment protrudes from S31/8 where half of its area equals **0.08843..**. In the second part, there is blue area of **0.07266..** out of the circle, and in the third part of S31/8 there is a blue area **0.01498..** out of the circle. This adds up to **0.000875..**, and this is precisely equal to 0.00627../8. We already seen the number 0.00627.., thus a circle has 0.00627.. more area than 31.

In the second part, a circular segment with an area of 0.04795.. is located under the blue excess surface, and in the third part, half of a circular segment with an area of 0.00284.. is located under the blue excess surface (We must know these areas that we are able to calculate the above excesses.) This the last area of 0.00284.. is the smallest of all mentioned areas here, it is barely noticeable, this segment has a height of 0.01425.., but it is 3.62.. times the area 0.000785.., which is the excess of area of S31/8 according to 1/8 of the circle. On this way we can imagine this excess.

It is also important here that half the width of the first circular segment (in the right-hand side) is equal to 0.93252.., i.e. a little less than the width of the unit square. If we change this circular segment to a rectangle with the same width and area, the height of it is equal to 0.14159..x 0.66973.. =0.09482..[2].
Namely, this rectangle is easier comparable with the excesses in the second and the third part.

The circumference intersects the inserted unit square at a distance of 0.42272.. from the inner vertex, i.e. a little less than 0.5, where it touches the vertex of the inserted right triangle.

Thus, for intuitive understanding of the above formulae we need these numbers.

2. A similar visualization of the Ramanujan formula for π^4

The Ramanujan formula
π^4=97.5-1/11+1.2491..x10^{-7}
can also be treated similarly as for π^3.

Let us create a circle with radius $\pi^{1.5}$=5.56832… Then, let us create a square with a side 11 and it is evident, that the circle protrudes from the square only a little, with the areas 4 x 0.13664.., it is lesser than before. Area of the square is 121, if we trim it to 97.5-1/11 (let us name this S97), we should trim the sum 3-1/16+1/88 in every 1/8 of the square. (In Section 1, the reduction was 5/8 of 1/8 of the square, which was 4.5).

Fig. 4 shows zoom of S97/8, using the orange polygon, but it is still without reduction 1/88. Dark red line corrects this S97/8 so that triangles are used for better smoothness. Where the line is doubled, reduction for 1/88 is done, so that this thin band is used for this.

[2] 0.14159.. is the height of this circular segment (i.e. π-3), and 0.66973.. is a factor with which we reduce this height that we obtain the rectangle with the same area and the same width.

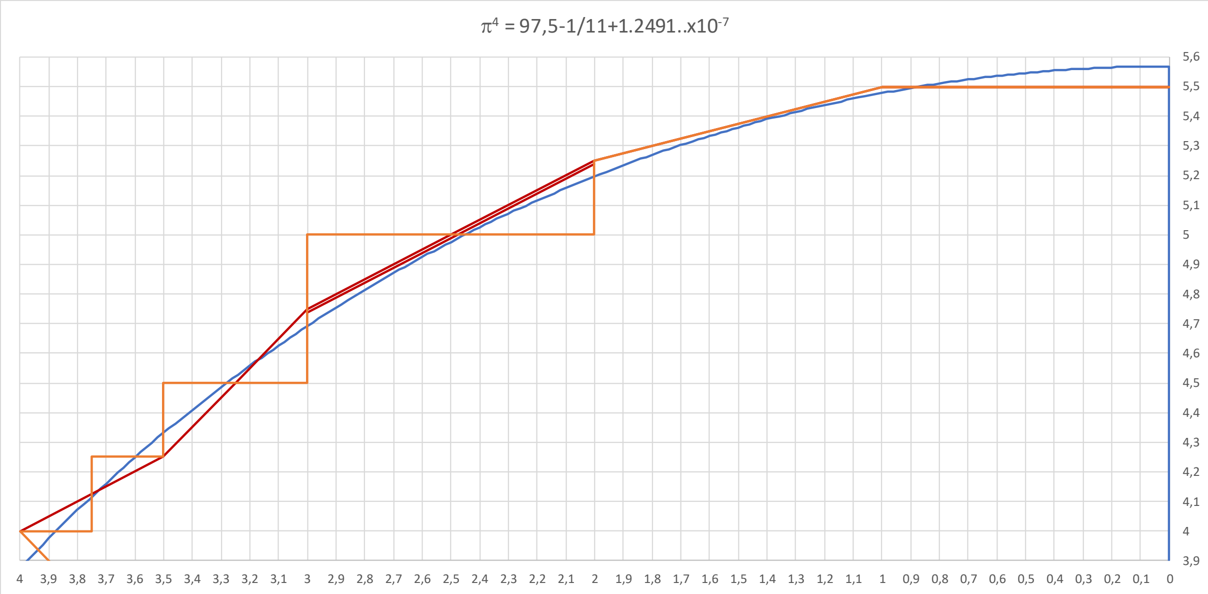

Fig. 4. Here, the essential part of S97/8 and of the circular segment are zoomed. The blue curve is a part of arc of 45°. The orange polygon is the essential part of the edge of S97/8. (1/88 is not yet respected.) The dark red polygon is obtained so that the orange polygon is smoothed with the triangles. Where the line is doubled, the area 1/88 is taken away.

In comparison with Fig. 3 it is evident that non-overlapped areas are now additionally smaller (if we compare parts according to the whole areas.)

The Ramanujan formula can also be visualized with a sphere with a radius π, and it can be compared with a cube with a side 6, where unit cubes or triangles would be removed, or similar simple bodies. Another option would be to use a cube with a side of 2π and before mentioned bodies could be removed. There are a lot of such or different options.

It is important that S31 and S97 are very similar to a circle, but it is also important that this can be achieved by a short and simple path, thus we can crumble further, but these first approximations are also important.

Solutions for such geometrical problems can also be found by artificial intelligence.

This graphical solution above is somewhat similar to solving a problem called *squaring the circle*.

3. The challenge for next approximations of the Ramanujan formula

However, I will still repeat an important question from the Ref. [2], namely, there are two options:

1. Either the Ramanujan formula is only a statistical accident [2]?[3]
2. Or a next approximation for the Ramanujan formula exists that π^4 can be calculated arbitrarily precisely? This means that this approximation can be written with a finite formula, and that it has a similar pattern as the Ramanujan formula now.

The question is which option is correct? The above visualizations can help at this.

This is an important question. Namely, this formula gives π more precisely than the formula for π^3. But it is also interesting that all the results of the formulae for π^2, π^3, and π^4 are close to some integer or to a simple correction (for π^4), and that the accuracies increase drastically - from power 2 to 4. Maybe the first two formulae are coincidences, but it is very possible that the Ramanujan formula is not a coincidence.

I have not yet seen this question or the answer to this question.

4. As curiosity: one interesting coincidence

One interesting coincidence: This Fig. 5 has a circle which is also put on a square 11x11.

[3] Wrong grammar in this and the next sentence is intentional because of clarity.

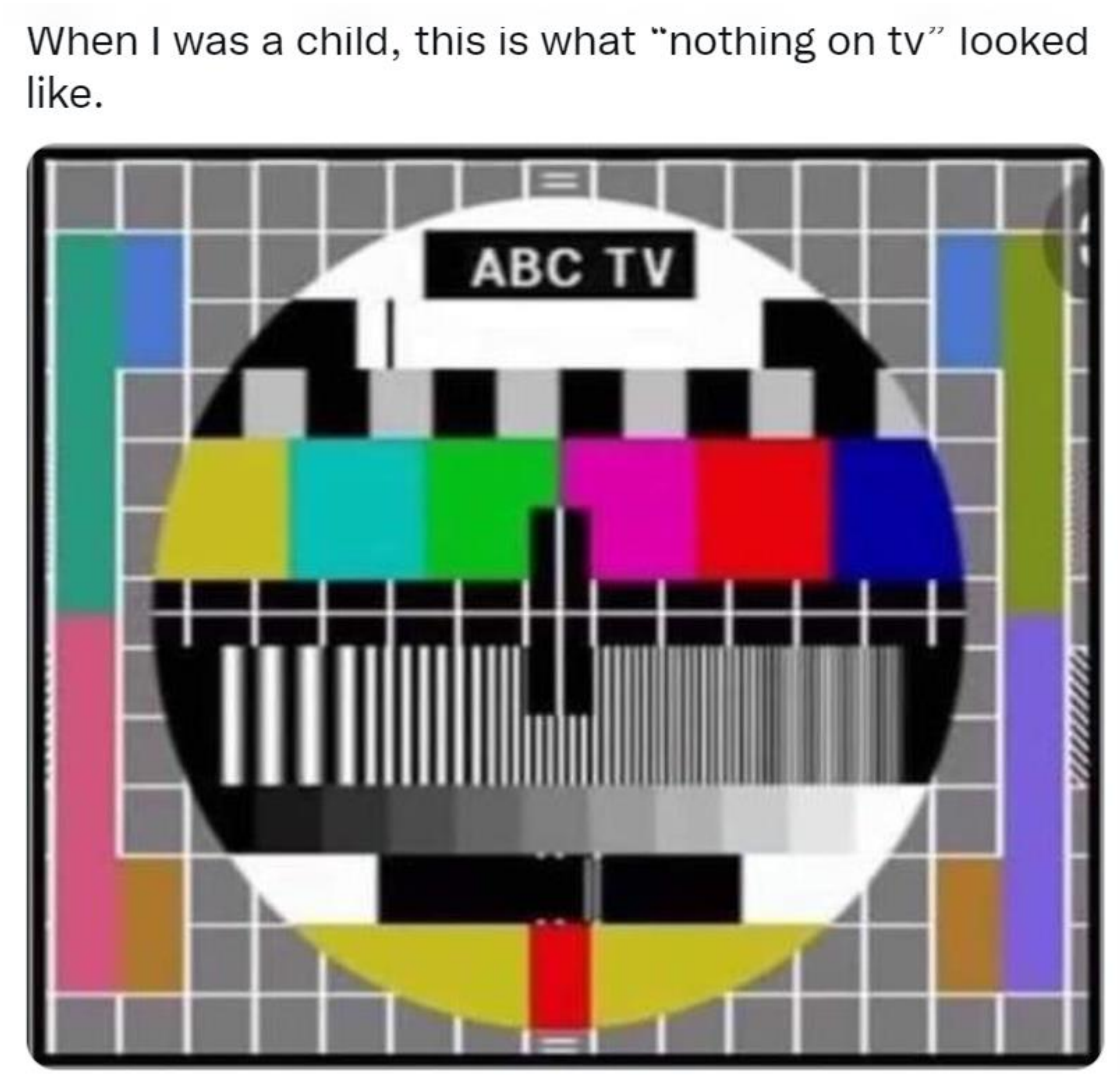

Fig. 5. Ref. [3].

The disagreement is only that the circle is a little bigger than our circle with radius $\pi^{1.5}$, Fig. 4. In other words, four circular segments which protrudes out of square 11x11, are now bigger, each width is ≅5 and <5, but in Fig. 4, each width is ≅2 and <2.

References

[1] Wikipedia, "Circular segment" *Link* https://en.wikipedia.org/wiki/Circular_segment.

[2] Janko Kokosar, “Similarity of a Ramanujan Formula for pi with Plouffe's Formulae, and Use of This for Searching of Physical Background for Some Guessed Formula for the Elementary Physical Constants,” 1-12 (2022); *Preprint* https://vixra.org/abs/2206.0147.

[3] Twitter, *Link*: https://twitter.com/nerds_feed/status/1461041942226214912