# Complete Bulgarian-English Dictionary and the Graphical law

Anindya Kumar Biswas*

*Department of Physics;*

*North-Eastern Hill University,*

*Mawkynroh-Umshing, Shillong-793022.*

(Dated: December 1, 2022)

## Abstract

We study the Complete Bulgarian-English Dictionary by Constantine Stephanove. We draw the natural logarithm of the number of head words, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that the Dictionary can be characterised by BP(4,$\beta H$=0.02) i.e. a magnetisation curve in the Bethe-Peierls approximation of the Ising model with four nearest neighbours with $\beta H$=0.02. H is external magnetic field, $\beta$ is $\frac{1}{k_B T}$ where, T is temperature and $k_B$ is the tiny Boltzmann constant. Moreover, the Bulgarian language appears to be dual to the Greek language to large extent.

* anindya@nehu.ac.in

## I. INTRODUCTION

From the Black Sea to the East, Serbia, Macedonia to the West, Romania to the North with the Danube as the natural boundary, Greece, Turkey to the South with the Constantinople almost outside the South East boundary, the rectangular Bulgaria between $40^0$ and $44^0$ N offers huge diversity in climate, flora and fauna with the Bulgarian language being the integrating factor. Situated at the intersection of the ancient Silk route and the migration route of the birds, Sofia is the Capital of this country of approximately seven million people. Being the origin of the Cyrillic script from the Saint Cyril, the Bulgarian language is written in the Bulgarian Cyrillic script. To have a glimpse we open the Complete Bulgarian-English Dictionary by Constantine Stephanove, [1], written way back in the year 1914. The Cyrillic script has thirty one letters as in the dictionary, [1].

We count all the head words one by one to probe for the magnetic field pattern. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical law.

Then, we moved on to investigate into, [3], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[4] and the basque language[5]. This was pursued by finding of the graphical law behind the Romanian language, [6], five more disciplines of knowledge, [7], Onsager core of Abor-Miri, Mising languages,[8], Onsager Core of Romanised Bengali language,[9], the graphical law behind the Little Oxford English Dictionary, [10], the Oxford Dictionary of Social Work and Social Care, [11], the Visayan-English Dictionary, [12], Garo to English School Dictionary, [13], Mursi-English-Amharic Dictionary, [14] and Names of Minor Planets, [15], A Dictionary of Tibetan and English, [16], Khasi English Dictionary, [17], Turkmen-English Dictionary, [18], Websters Universal Spanish-English Dictionary, [19], A Dictionary of Modern Italian, [20], Langenscheidt's German-English Dictionary, [21], Essential Dutch dictionary by G. Quist and D. Strik, [22], Swahili-English dictionary by C. W. Rechenbach, [23], Larousse Dictionnaire De Poche for the French, [24], the Onsager's solution behind the Arabic, [25], the graphical law behind Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung, [26], the graphical law behind the NTC's Hebrew and English

Dictionary by Arie Comey and Naomi Tsur, [27], the graphical law behind the Oxford Dictionary Of Media and Communication, [28], the graphical law behind the Oxford Dictionary Of Mathematics, Penguin Dictionary Of Mathematics, [29], the Onsager's solution behind the Arabic Second part, [30], the graphical law behind the Penguin Dictionary Of Sociology, [31], behind the Concise Oxford Dictionary Of Politics, [32], a Dictionary Of Critical Theory by Ian Buchanan, [33], the Penguin Dictionary Of Economics, [34], the Concise Gojri-English Dictionary by Dr. Rafeeq Anjum, [35], A Dictionary of the Kachin Language by Rev.O.Hanson, [36], A Dictionary Of World History by Edmund Wright, [37], Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool, [38], A Dictionary of Plant Sciences by Michael Allaby, [39], respectively. The graphical law was pursued more in Along the side of the Onsager's solution, the Ekagi language ,[40], Along the side of the Onsager's solution, the Ekagi language-Part Three, [41], Oxford Dictionary of Biology by Robert S. Hine and the Graphical law, [42], A Dictionary of the Mikir Language by G. D. Walker and the Graphical law, [43], A Dictionary of Zoology by Michael Allaby and the Graphical Law, [44], Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical Law, [45], Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law, [46], A Greek and English Lexicon by H.G.Liddle et al simplified by Didier Fontaine and the Graphical law, [47], Learner's Mongol-English Dictionary and the Graphical law, [48], respectively.

We describe how the graphical law is hidden within the Complete Bulgarian-English Dictionary by Constantine Stephanove, [1], in this article. The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe the analysis of the Complete Bulgarian-English Dictionary by Constantine Stephanove, [1]. The section IV is the comparison of the two neighbouring languages, the Bulgarian and the Greek. The section V is Acknowledgment. The last section is Bibliography.

## II. MAGNETISATION

### A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferro magnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where $\sigma_i$ is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where $N_+$ is the number of up spins, $N_-$ is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , $M$ is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, $\mu N L$, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[49], for the lattice of spins, setting $\mu$ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [50], $2\epsilon\gamma\bar{\sigma} + 2H$, where $\gamma$ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [51]. In the Bragg-Williams approximation,[52], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L + c}{\frac{T}{T_c}} \qquad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$ , $T_c = \gamma\epsilon/k_B$, [53]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of $L$ vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [50]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

### B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [49],[50],[51],[52],[53], due to Bethe-Peierls, [54], reduced magnetisation varies with reduced temperature, for $\gamma$ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \qquad (2)$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

| BW | BW(c=0.01) | BP(4,$\beta H$ = 0) | reduced magnetisation |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0.435 | 0.439 | 0.563 | 0.978 |
| 0.439 | 0.443 | 0.568 | 0.977 |
| 0.491 | 0.495 | 0.624 | 0.961 |
| 0.501 | 0.507 | 0.630 | 0.957 |
| 0.514 | 0.519 | 0.648 | 0.952 |
| 0.559 | 0.566 | 0.654 | 0.931 |
| 0.566 | 0.573 | 0.7 | 0.927 |
| 0.584 | 0.590 | 0.7 | 0.917 |
| 0.601 | 0.607 | 0.722 | 0.907 |
| 0.607 | 0.613 | 0.729 | 0.903 |
| 0.653 | 0.661 | 0.770 | 0.869 |
| 0.659 | 0.668 | 0.773 | 0.865 |
| 0.669 | 0.676 | 0.784 | 0.856 |
| 0.679 | 0.688 | 0.792 | 0.847 |
| 0.701 | 0.710 | 0.807 | 0.828 |
| 0.723 | 0.731 | 0.828 | 0.805 |
| 0.732 | 0.743 | 0.832 | 0.796 |
| 0.756 | 0.766 | 0.845 | 0.772 |
| 0.779 | 0.788 | 0.864 | 0.740 |
| 0.838 | 0.853 | 0.911 | 0.651 |
| 0.850 | 0.861 | 0.911 | 0.628 |
| 0.870 | 0.885 | 0.923 | 0.592 |
| 0.883 | 0.895 | 0.928 | 0.564 |
| 0.899 | 0.918 | | 0.527 |
| 0.904 | 0.926 | 0.941 | 0.513 |
| 0.946 | 0.968 | 0.965 | 0.400 |
| 0.967 | 0.998 | 0.965 | 0.300 |
| 0.987 | | 1 | 0.200 |
| 0.997 | | 1 | 0.100 |
| 1 | 1 | 1 | 0 |

TABLE I. Reduced magnetisation vs reduced temperature datas for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours .

datas generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those datas. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

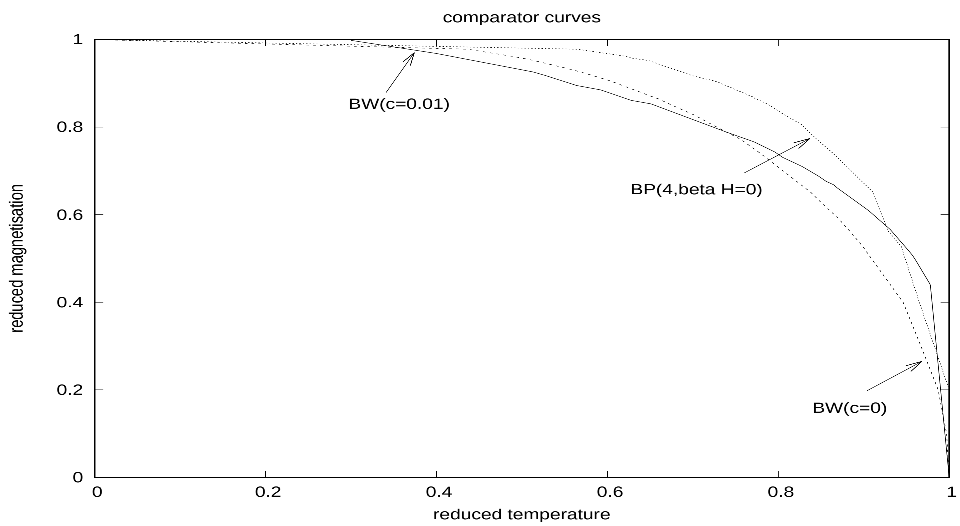


FIG. 1. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

### C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [54], reduced magnetisation varies with reduced temperature, for $\gamma$ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

Derivation of this formula ala [54] is given in the appendix of [7].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

In the following, we describe datas in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that

$\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.2. Similarly, we plot fig.3. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

| BP(m=0.03) | BP(m=0.025) | BP(m=0.02) | BP(m=0.01) | BP(m=0.005) | reduced magnetisation |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0.583 | 0.580 | 0.577 | 0.572 | 0.569 | 0.978 |
| 0.587 | 0.584 | 0.581 | 0.575 | 0.572 | 0.977 |
| 0.647 | 0.643 | 0.639 | 0.632 | 0.628 | 0.961 |
| 0.657 | 0.653 | 0.649 | 0.641 | 0.637 | 0.957 |
| 0.671 | 0.667 | | 0.654 | 0.650 | 0.952 |
| | 0.716 | | | 0.696 | 0.931 |
| 0.723 | 0.718 | 0.713 | 0.702 | 0.697 | 0.927 |
| 0.743 | 0.737 | 0.731 | 0.720 | 0.714 | 0.917 |
| 0.762 | 0.756 | 0.749 | 0.737 | 0.731 | 0.907 |
| 0.770 | 0.764 | 0.757 | 0.745 | 0.738 | 0.903 |
| 0.816 | 0.808 | 0.800 | 0.785 | 0.778 | 0.869 |
| 0.821 | 0.813 | 0.805 | 0.789 | 0.782 | 0.865 |
| 0.832 | 0.823 | 0.815 | 0.799 | 0.791 | 0.856 |
| 0.841 | 0.833 | 0.824 | 0.807 | 0.799 | 0.847 |
| 0.863 | 0.853 | 0.844 | 0.826 | 0.817 | 0.828 |
| 0.887 | 0.876 | 0.866 | 0.846 | 0.836 | 0.805 |
| 0.895 | 0.884 | 0.873 | 0.852 | 0.842 | 0.796 |
| 0.916 | 0.904 | 0.892 | 0.869 | 0.858 | 0.772 |
| 0.940 | 0.926 | 0.914 | 0.888 | 0.876 | 0.740 |
| | 0.929 | | | 0.877 | 0.735 |
| | 0.936 | | | 0.883 | 0.730 |
| | 0.944 | | | 0.889 | 0.720 |
| | 0.945 | | | | 0.710 |
| | 0.955 | | | 0.897 | 0.700 |
| | 0.963 | | | 0.903 | 0.690 |
| | 0.973 | | | 0.910 | 0.680 |
| | | | | 0.909 | 0.670 |
| | 0.993 | | | 0.925 | 0.650 |
| | | 0.976 | 0.942 | | 0.651 |
| | 1.00 | | | | 0.640 |
| | | 0.983 | 0.946 | 0.928 | 0.628 |
| | | 1.00 | 0.963 | 0.943 | 0.592 |
| | | | 0.972 | 0.951 | 0.564 |
| | | | 0.990 | 0.967 | 0.527 |
| | | | | 0.964 | 0.513 |
| | | | 1.00 | | 0.500 |
| | | | | 1.00 | 0.400 |
| | | | | | 0.300 |
| | | | | | 0.200 |
| | | | | | 0.100 |
| | | | | | 0 |

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

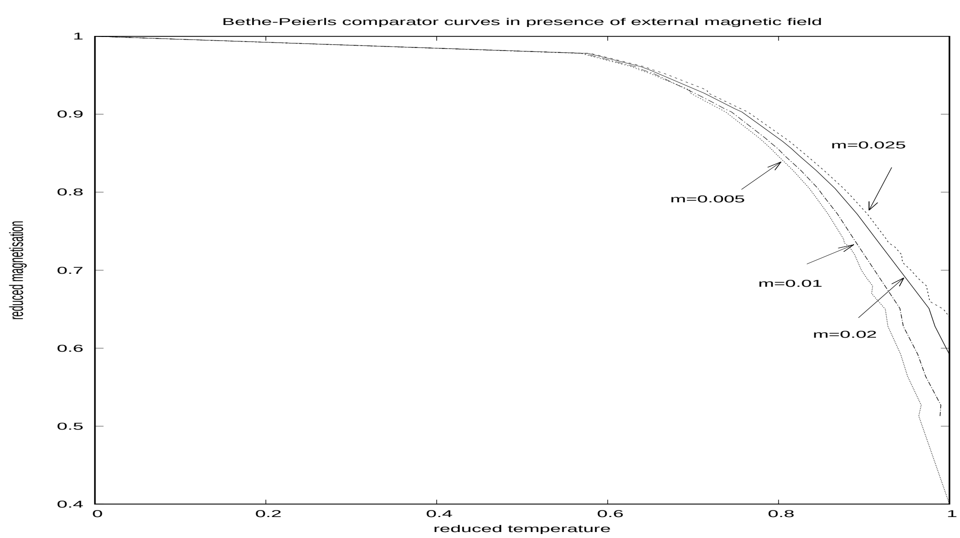


FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| 301 | 870 | 897 | 619 | 993 | 265 | 172 | 1073 | 965 | 1325 | 445 | 873 | 1821 | 1578 | 3915 | 1046 | 2054 | 698 | 564 | 215 | 345 | 201 | 399 | 213 | 76 | 0 | 0 | 1 | 50 | 112 | 7 |
| a | b | v | g | d | e | s(zh) | z | i | k | l | m | n | o | p | r | s | t | oo | f | ch | ts | ch | sh | sht | u | y | | yu | ya | |

TABLE III. Complete Bulgarian-English Dictionary head words: the first row represents letters of the Bulgarian Cyrillic script in the serial order, the second row represents the number of respective head words, the third row represents the equvalent English letter(s), [56].

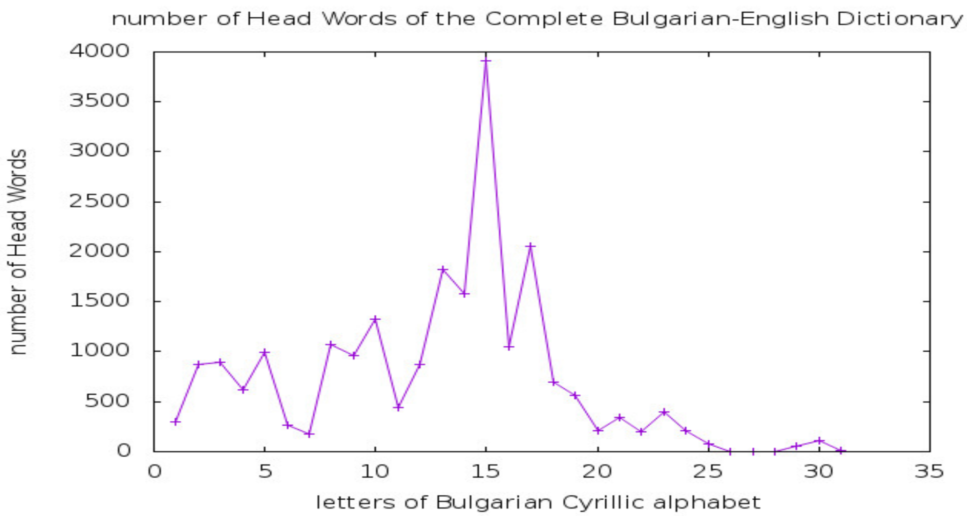


FIG. 3. The vertical axis is number of head words of the Complete Bulgarian-English Dictionary,[1]. The horizontal axis is the letters of the Bulgarian Cyrillic script. Letters are represented by the sequence number in the alphabet.

## III. ANALYSIS OF THE HEAD WORDS OF THE COMPLETE BULGARIAN-ENGLISH DICTIONARY

In the Complete Bulgarian-English Dictionary, [1], we have counted the head words, one by one from the beginning to the end, starting with different letters. The result is the table, III. Highest number of words, three thousand nine hundred fifteen starts with the Cyrillic letter equivalent of English P followed by words numbering two thousand fifty four beginning with the Cyrillic letter equivalent of English S as in "sound", [56], one thousand eight hundred twenty one initiating with the Cyrillic letter equivalent of English N. To visualise we plot the number of words against respective letters in the dictionary sequence,[1] in the figure fig.3. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by $f$ and the respective rank, denoted by $k$. $k$ is a positive integer starting from one. Moreover, the limiting rank, $k_{lim}$ is twenty nine and the limiting number of words is one. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.4. We then ignore the letter with the highest of words, tabulate in the adjoining table, IV and redo the plot, normalising the $lnf$s with next-to-maximum $lnf_{n-max}$, and starting from $k = 2$ in the figure fig.5. Normalising the $lnf$s with next-to-next-to-maximum $lnf_{2n-max}$, we tabulate in the adjoining table, IV, and starting from $k = 3$ we draw in the figure fig.6. Normalising the $lnf$s with next-to-next-to-next-to-maximum $lnf_{3n-max}$ we record in the adjoining table, IV, and plot starting from $k = 4$ in the figure fig.7. Normalising the $lnf$s with next-to-next-to-next-to-next-to-maximum $lnf_{4n-max}$ we record in the adjoining table, IV, and plot starting from $k = 5$ in the figure fig.8. Normalising the $lnf$s with nextnextnextnextnext-maximum $lnf_{5n-max}$ we record in the adjoining table, IV, and plot starting from $k = 6$ in the figure fig.9.

| k | lnk | lnk/$lnk_{lim}$ | f | lnf | lnf/$lnf_{max}$ | lnf/$lnf_{n-max}$ | lnf/$lnf_{2n-max}$ | lnf/$lnf_{3n-max}$ | lnf/$lnf_{4n-max}$ | lnf/$lnf_{5n-max}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 3915 | 8.273 | 1 | Blank | Blank | Blank | Blank | Blank |
| 2 | 0.69 | 0.205 | 2054 | 7.628 | 0.922 | 1 | Blank | Blank | Blank | Blank |
| 3 | 1.10 | 0.326 | 1821 | 7.507 | 0.907 | 0.984 | 1 | Blank | Blank | Blank |
| 4 | 1.39 | 0.412 | 1578 | 7.364 | 0.890 | 0.965 | 0.981 | 1 | Blank | Blank |
| 5 | 1.61 | 0.478 | 1325 | 7.189 | 0.869 | 0.942 | 0.958 | 0.976 | 1 | Blank |
| 6 | 1.79 | 0.531 | 1073 | 6.978 | 0.843 | 0.915 | 0.930 | 0.948 | 0.971 | 1 |
| 7 | 1.95 | 0.579 | 1046 | 6.953 | 0.840 | 0.912 | 0.926 | 0.944 | 0.967 | 0.996 |
| 8 | 2.08 | 0.617 | 993 | 6.901 | 0.834 | 0.905 | 0.919 | 0.937 | 0.960 | 0.989 |
| 9 | 2.20 | 0.653 | 965 | 6.872 | 0.831 | 0.901 | 0.915 | 0.933 | 0.956 | 0.985 |
| 10 | 2.30 | 0.682 | 897 | 6.799 | 0.822 | 0.891 | 0.906 | 0.923 | 0.946 | 0.974 |
| 11 | 2.40 | 0.712 | 873 | 6.772 | 0.819 | 0.888 | 0.902 | 0.920 | 0.942 | 0.970 |
| 12 | 2.48 | 0.736 | 870 | 6.768 | 0.818 | 0.887 | 0.902 | 0.919 | 0.941 | 0.970 |
| 13 | 2.56 | 0.760 | 698 | 6.548 | 0.791 | 0.858 | 0.872 | 0.889 | 0.911 | 0.938 |
| 14 | 2.64 | 0.783 | 619 | 6.428 | 0.777 | 0.843 | 0.856 | 0.873 | 0.894 | 0.921 |
| 15 | 2.71 | 0.804 | 564 | 6.335 | 0.766 | 0.830 | 0.844 | 0.860 | 0.881 | 0.908 |
| 16 | 2.77 | 0.822 | 445 | 6.098 | 0.737 | 0.799 | 0.812 | 0.828 | 0.848 | 0.874 |
| 17 | 2.83 | 0.840 | 399 | 5.989 | 0.724 | 0.785 | 0.798 | 0.813 | 0.833 | 0.858 |
| 18 | 2.89 | 0.858 | 345 | 5.844 | 0.706 | 0.766 | 0.778 | 0.794 | 0.813 | 0.837 |
| 19 | 2.94 | 0.872 | 301 | 5.707 | 0.690 | 0.748 | 0.760 | 0.775 | 0.794 | 0.818 |
| 20 | 3.00 | 0.890 | 265 | 5.580 | 0.674 | 0.732 | 0.743 | 0.758 | 0.776 | 0.800 |
| 21 | 3.04 | 0.902 | 215 | 5.371 | 0.649 | 0.704 | 0.715 | 0.729 | 0.747 | 0.770 |
| 22 | 3.09 | 0.917 | 213 | 5.361 | 0.648 | 0.703 | 0.714 | 0.728 | 0.746 | 0.768 |
| 23 | 3.14 | 0.932 | 201 | 5.303 | 0.641 | 0.695 | 0.706 | 0.720 | 0.738 | 0.760 |
| 24 | 3.18 | 0.944 | 172 | 5.147 | 0.622 | 0.675 | 0.686 | 0.699 | 0.716 | 0.738 |
| 25 | 3.22 | 0.955 | 112 | 4.718 | 0.570 | 0.619 | 0.628 | 0.641 | 0.656 | 0.676 |
| 26 | 3.26 | 0.967 | 76 | 4.331 | 0.524 | 0.568 | 0.577 | 0.588 | 0.602 | 0.621 |
| 27 | 3.30 | 0.979 | 50 | 3.912 | 0.473 | 0.513 | 0.521 | 0.531 | 0.544 | 0.561 |
| 28 | 3.33 | 0.988 | 7 | 1.946 | 0.235 | 0.255 | 0.259 | 0.264 | 0.271 | 0.279 |
| 29 | 3.37 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE IV. Bulgarian words: ranking, natural logarithm, normalisations

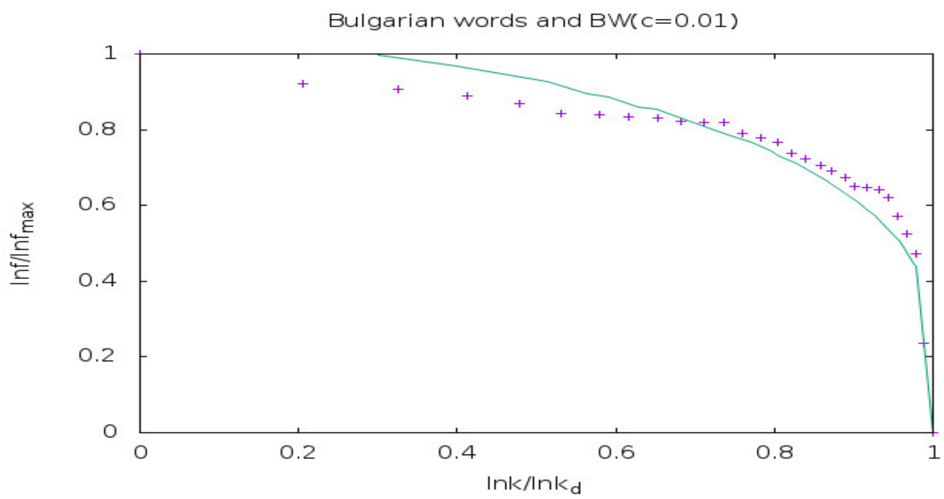


FIG. 4. Vertical axis is $\frac{lnf}{lnf_{max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Bulgarian language with the fit curve being the Bragg-Williams approximation curve in the presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$.

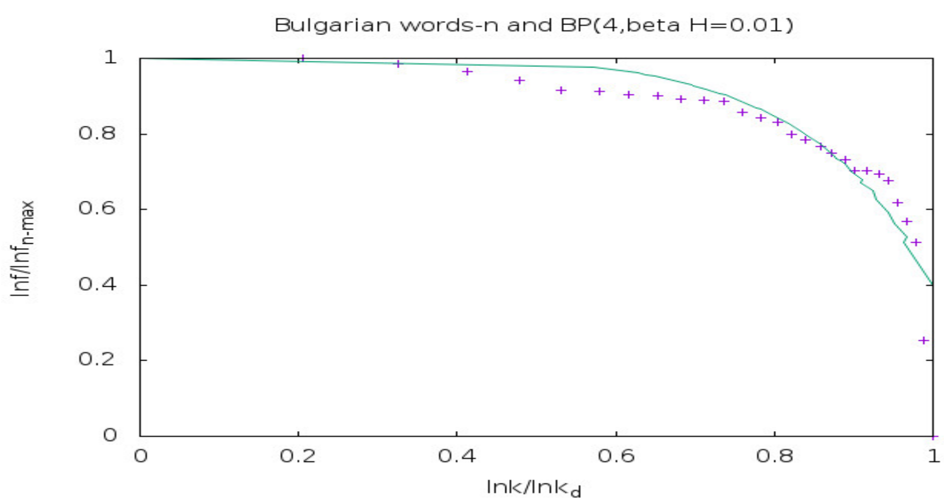


FIG. 5. Vertical axis is $\frac{lnf}{lnf_{n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Bulgarian language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.005 or, $\beta H = 0.01$.

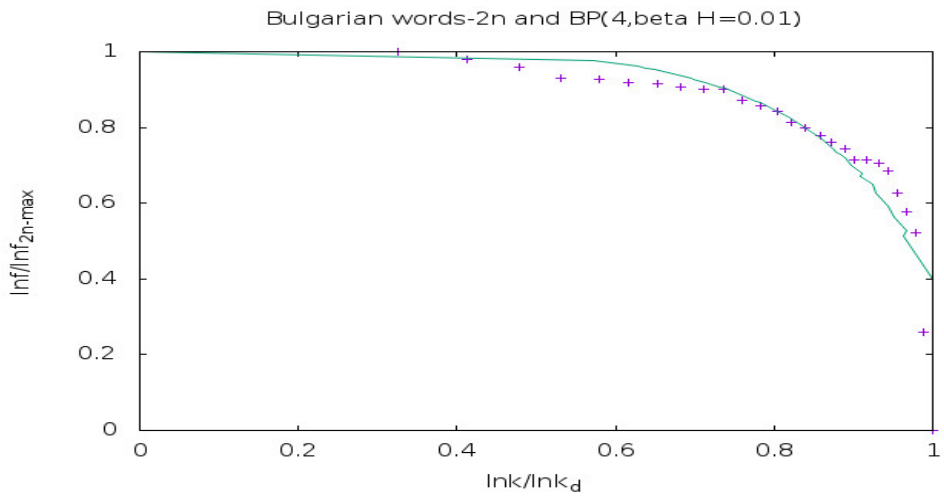


FIG. 6. Vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Bulgarian language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.005 or, $\beta H = 0.01$.

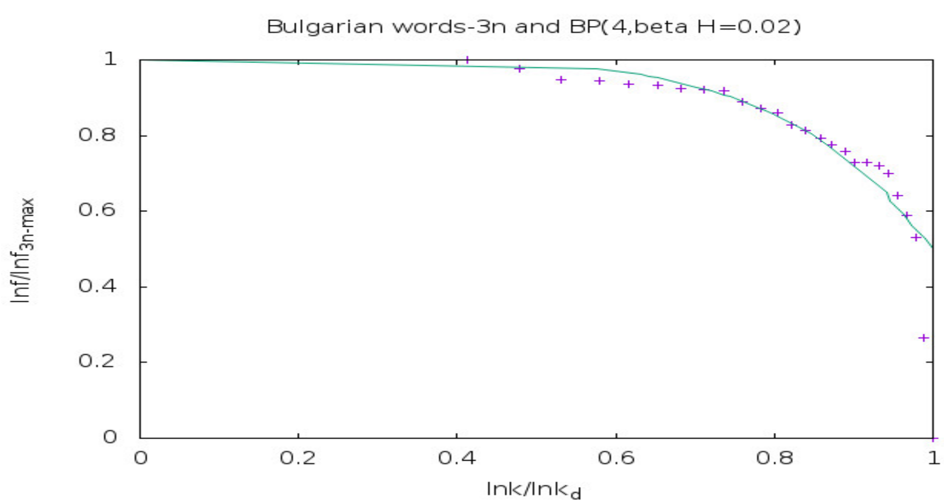


FIG. 7. Vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Bulgarian language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.01 or, $\beta H = 0.02$.

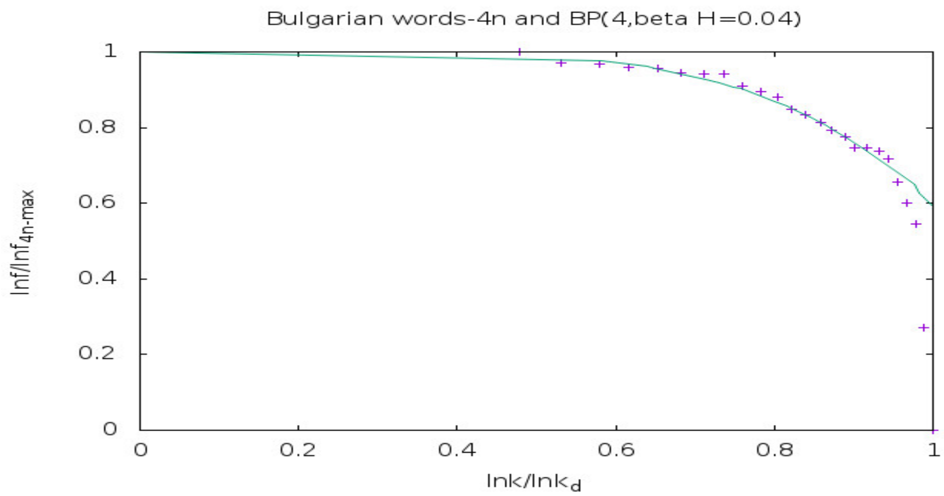


FIG. 8. Vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Bulgarian language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.02 or, $\beta H = 0.04$.

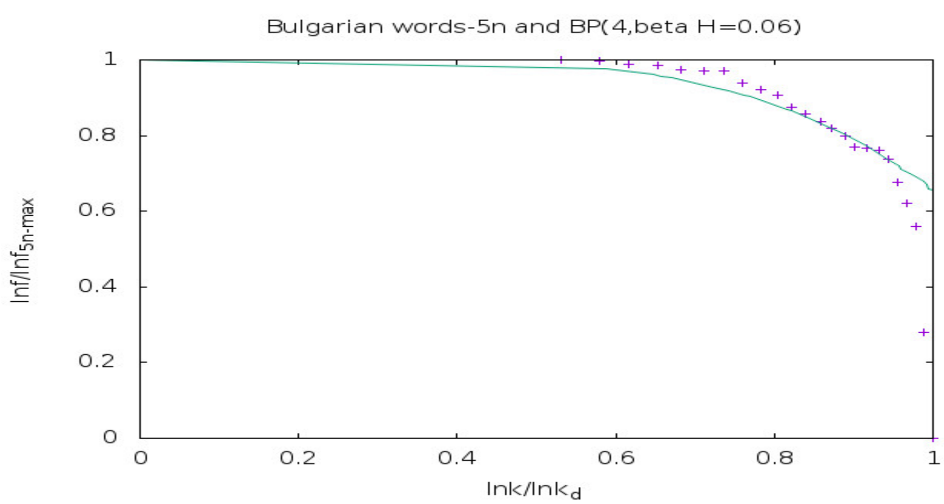


FIG. 9. Vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Bulgarian language with fit curve being the Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field, m=0.03 or, $\beta H = 0.06$.

### A. conclusion

From the figures (fig.4-fig.9), we observe that there is a curve of magnetisation, behind the entries of Bulgarian language,[1]. This is the magnetisation curve, BP(4,$\beta H$=0.02), in the Bethe-Peierls approximation in presence of little external magnetic field.

Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{3n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [57].

Moreover, on successive higher normalisations, the words of Bulgarian language, do not go over to, the Onsager solution.

Interestingly, $\frac{lnf}{lnf_{max}}$ vs $\frac{lnk}{lnk_{lim}}$ is matched by BW(c=0.01) as in the Greek, Turkmen, Tibetan, Basque, Romanian, Khasi languages respectively. Also the Greek,[47], Mikir,[43], Mursi,[14], Garo ,[13],Visayan languages,[12], are charcterised by BP(4,$\beta H$=0.02).

| | a | b | c | ch | d | e | f | g | h | i | j | k | l | m | n | o | oo | p |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Greek | 5800 | 0 | 0 | 667 | 0 | 4805 | 876 | 58 | 0 | 1926 | 0 | 2726 | 863 | 1578 | 584 | 1452 | 0 | 83 |
| Bulgarian | 301 | 870 | 0 | 546 | 993 | 265 | 215 | 619 | 0 | 965 | 0 | 1325 | 445 | 873 | 1821 | 1578 | 564 | 3915 |
| | ps | q | r | s | sh | sht | t | th | ts | u | v | w | x | y | yu | ya | yut | z |
| Greek | 181 | 0 | 262 | 2654 | 0 | 0 | 1194 | 2053 | 0 | 0 | 660 | 0 | 116 | 0 | 0 | 0 | 0 | 129 |
| Bulgarian | 0 | 0 | 1046 | 2226 | 213 | 76 | 698 | 0 | 201 | 0 | 897 | 0 | 0 | 0 | 50 | 112 | 7 | 1073 |

TABLE V. Number of entries of the Greek and the headwords of the Bulgarian dictionaries against the equivalent embedding English alphabet.

## IV. COMPARISON WITH THE GREEK

To compare the Bulgarian language written in the Cyrillic script and the Greek language written in the Greek alphabet, we take the English equivalents of the letters using [56] and [58]. As a result we get an augmented English alphabet and the entries for each letter for both the dictionaries, [1], [59]. This we present in the table, V.

To visualise we plot both the set of entries against the equivalent embedding English alphabet in the figure, fig.10. We notice from the figure, fig.10, that the two lines are almost complementary. Moreover, both the languages are characterised by BP(4,$\beta H$=0.02). For both the languages, $\frac{lnf}{lnf_{max}}$ vs $\frac{lnk}{lnk_{lim}}$ is matched by BW(c=0.01). Hence, we surmise that the Bulgarian and the Greek languages as represented by the dictionaries, [1] and [59], are dual to each other to large extent.

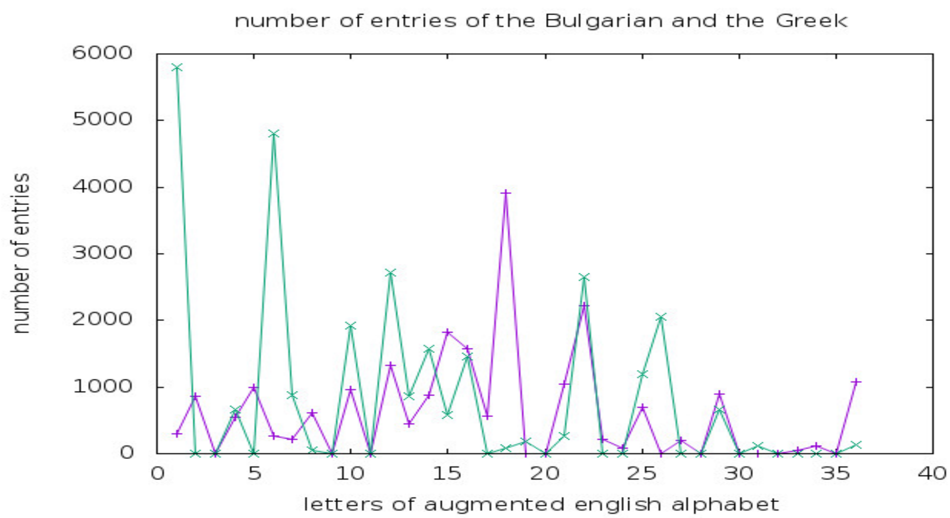


FIG. 10. The vertical axis is number of entries. The Green line represents the Bulgarian language head words and the red line corresponds to the Greek language entries. The horizontal axis is the letters of the augmented embedding English alphabet. Letters are represented by the sequence number in the augmented alphabet. Maximum of one language is falling on the almost minimum of the other language.

## V. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper. We have benefited from reading related web pages of wiki travel, Wikipedia, BBC and Encyclopedia Britannica.

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А(а) # 301

Б(б) # 870

В(в) # 897

Г(г) # 619

Д(д) # 993

Е(е) # 265

Ж(ж) # 172

З(з) # 1073

И(и) # 965

К(к) # 1325

Л(л) # 445

М(м) # 873

Н(н) # 1821

О(о) # 1578

П(п) # 3915

Р(р) # 1046

С(с) # 2054

Т(т) # 698

У(у) # 564

Ф(ф) # 215

Х(х) # 345

Ц(ц) # 201

Ч(ч) # 399

Ш(ш) # 213

Щ(щ) # 76

Ъ # 0

Ь # 0

Ѣ # ~~0~~1

Ю # 50

Я(я) # 112

Ѫ(ѫ) # 7