Geometrical optics as an Abelian U(1) local gauge theory

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We point out that geometrical optics can be treated as an Abelian $U(1)$ local gauge theory and we observe what it implies.

Keywords: *geometrical optics, Abelian $U(1)$ local gauge theory, eikonal equation, gauge potential.*

The treatment of the geometrical optics as an Abelian $U(1)$ local gauge theory implies that *the gauge potential of the geometrical optics and Maxwell's theory are the same, i.e. both are the Abelian $U(1)$ gauge potential* written as

$$\vec{A}_\mu^{\,U(1)} = \vec{a}_\mu \; e^{i\psi} \tag{1}$$

where $\vec{A}_\mu^{\,U(1)}$ is *a complex*[1,2] gauge potential, $\vec{a}_\mu$ is *a complex amplitude*[2], a slowly varying function of space coordinates and time[3], ψ is *the eikonal* (*a real phase*[2]), a function of space coordinates and time, and $e^{i\psi}$ is *a complex scalar function.*

The gauge potential, $\vec{A}_\mu^{\,U(1)}$ consists of the electric scalar potential, ϕ, and the magnetic vector potential, $\vec{A}$, defined[4] as

$$\vec{A}_\mu^{\,U(1)} \equiv (\phi, \vec{A}) \tag{2}$$

$\vec{A}_\mu^{\,U(1)}$ is also called *the four-vector potential* or *gauge field*[4]. We consider gauge field as *gauge potential of the field strength.*

If we substitute (2) into (1), we obtain

$$(\phi, \vec{A}) = \vec{a}_\mu \; e^{i\psi} \tag{3}$$

Eq.(3) has a consequence that we need to write a complex amplitude as

$$\vec{a}_\mu = (a, \vec{a}) \tag{4}$$

where a and $\vec{a}$ are *complex scalar and complex vector amplitudes*, respectively. By substituting eq.(4) into (3), we obtain

$$(\phi, \vec{A}) = (a, \vec{a}) \; e^{i\psi} \tag{5}$$

Eq.(5) can be written also as

$$\phi = a \; e^{i\psi} \tag{6}$$

$$\vec{A} = \vec{a} \; e^{i\psi} \tag{7}$$

The Abelian $U(1)$ gauge potential of the geometrical optics, instead of eq.(1), can be written as[5,6]

$$\vec{A}_\mu^{\,U(1)} = \vec{a}_\mu \; e^{i\frac{f_\theta}{c}\left(\int_{x_1}^{x_2} n \; d^3x - ct\right)} \tag{8}$$

where f_θ is the angular frequency, c is the speed of light in a vacuum, n is the refractive index. Eq.(8) shows explicitly the relation between the refractive index and the gauge potential.

Eq.(8) implies that eqs.(6), (7) can be written as

$$\phi = a \; e^{i\frac{f_\theta}{c}\left(\int_{x_1}^{x_2} n \; d^3x - ct\right)} \tag{9}$$

$$\vec{A} = \vec{a} \; e^{i\frac{f_\theta}{c}\left(\int_{x_1}^{x_2} n \; d^3x - ct\right)} \tag{10}$$

Eqs.(9), (10) show explicitly the relation between the refractive index, the scalar and the vector potentials, respectively. We see from eqs.(9), (10) *the scalar and the vector values of the potential depend on the value of complex amplitude.*

Is there analogy of $\vec{A}_\mu^{\,U(1)}$ in quantum electrodynamics? What is $\vec{A}_\mu^{\,U(1)}$ in quantum electrodynamics? If we assume that gauge field, $\vec{A}_\mu^{\,U(1)}$, is gauge boson (gauge potential of boson), i.e. (gauge potential of) photon, does photon have a structure? This consideration appears due to the fact that the gauge potential consists of the scalar and the vector potentials, respectively, as shown in eq.(2).

The work is still in progress.

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