A VISUAL PROOF THAT $e^s > s^e, \forall s \in [0, \infty)\setminus\{e\}$

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Abstract

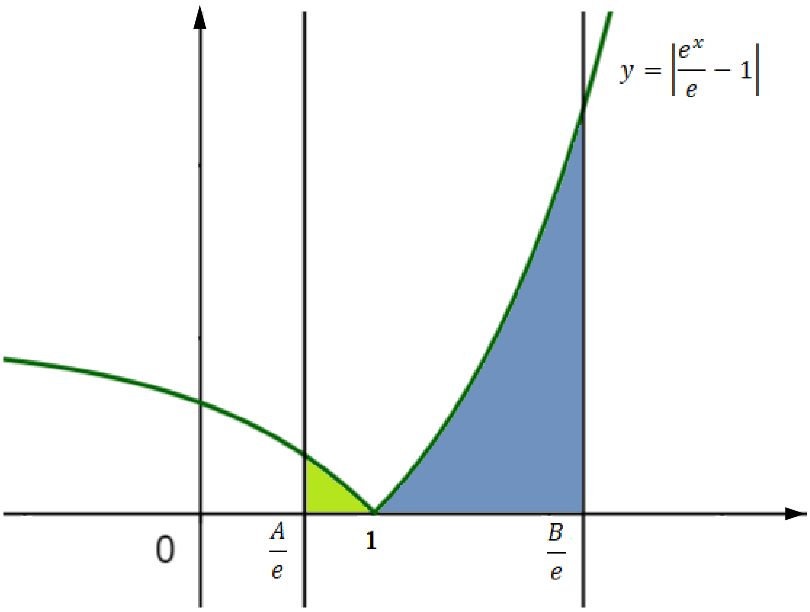
The beauty of mathematics fascinates humans and when we are dealing with some special constants that surely encourage us to understand some classy relations through numerous visual proofs of inequalities and when the constants are e and π then nothing more to say There are visual proofs of the beautiful inequality $\pi^e < e^\pi$. We have provided an alternative visual proof for this inequality using an area argument with the help of the theorem, for all real numbers $s \geq 0$ *and* $s \neq e, e^s > s^e$.

Keywords: Visual Proof, $e, \pi, \pi^e < e^\pi$

Theorem

For all real numbers $s \geq 0$ *and* $s \neq e, e^s > s^e$.

Proof:



$$Let\ us\ take\ a\ function, \psi: \mathbb{R} \to [0,\infty)\ \ defined\ by\ \psi(x) = \left|\frac{e^x}{e} - 1\right| , \forall x \in \mathbb{R}$$

$$if\ 0 \le A < e < B\ , then\ \frac{A}{e} < 1\ and\ \frac{B}{e} > 1$$

$$Clearly\ \int_{\frac{A}{e}}^{\frac{B}{e}} \psi(x)\, dx \quad > 0\ , \forall x \ge 0$$

$$\Rightarrow \int_{\frac{A}{e}}^{1} \psi(x)\, dx\ > 0\ and\ \int_{1}^{\frac{B}{e}} \psi(x)\, dx \quad > 0$$

$$\Rightarrow \int_{\frac{A}{e}}^{1} \left(1 - \frac{e^x}{e}\right) dx\ > 0\ and\ \int_{1}^{\frac{B}{e}} \left(\frac{e^x}{e} - 1\right) dx \quad > 0$$

$$\Rightarrow \left[x - \frac{e^x}{e}\right]_{\frac{A}{e}}^{1} > 0\ and\ \left[\frac{e^x}{e} - x\right]_{1}^{\frac{B}{e}} > 0$$

$$\Rightarrow e^{\frac{A}{e}} > A\ ,\ \ \forall A \in [0,e)\ \ and\ \ e^{\frac{B}{e}} > B\ ,\ \ \forall B \in (e,\infty)$$

$$\Rightarrow e^A > A^e\ ,\ \ \forall A \in [0,e)\ \ and\ \ e^B > B^e\ ,\ \ \forall B \in (e,\infty)$$

$$Therefore, e^s > s^e\ , \forall s \ge 0\ and\ s \ne e$$

Corollary: $\pi^e < e^\pi$

$Since, \pi > e$ we have from above theorem $e^\pi > \pi^e$

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