A method to check the reliability oF one-to-one correspondence

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Abstract: The method of checking whether the established one-to-one correspondence is correct was given. It is proved by the verification that any infinite set cannot be in one-to-one correspondence with any of its proper subsets.

Key words: mathematic fundation; infinite set; proper subset; one-to-one correspondence; check method

We all know the importance of check calculation in mathematics. For example, when we do addition, we often need to use subtraction to check calculation; when we do multiplication, we need to use division to check calculation.

When establishing a one-to-one correspondence[1] between two sets, we actually need to check to see if the bijective function we have established is really a bijective function between these two sets.

The method of checking is actually very simple. Take the establishment of a bijective function between sets A and B as an example, if we think that we have found a bijective function b=f(a) between sets A and B, the bijective function is correct when both

$$B = \{b \mid b = f(a),\ a \in A\} \tag{1}$$

And

$$A = \{a \mid a = f^{-1}(b),\ b \in B\} \tag{2}$$

hold true.

In other words, if eq.(1) or eq.(2) dose not hold true, the bijective function is incorrect

For example, between the set $N_1=\{0\}\cup N$ and its proper subset N={1, 2, 3...}, if we think that we have obtained the bijective function n'=n-1, then we can examine whether the set

$$N' = \{n' \mid n-1,\ n \in N\}$$

is the set N_1.

It is not difficult to find that the both lists of sets of N_1 and N' are {0, 1, 2, 3...}, however, for any $n \in N$, there is one and only one $n'=n-1 \in N'$, so N' has the same number of elements as N. But clearly, the number of elements of N_1 is more than that of N according to definition $N_1=\{0\}\cup N$, so the number of elements of N_1 is more than that of N', N_1 and N' cannot be the same set, and the verification fails.

Although the above verification is carried out on a special case, it is easy to generalize to

general situations, so that it can be strictly proved that any infinite set cannot correspond to any of its proper subsets one-to-one.

[1] Cantor. The theoretical basis of transfinite numbers, second edition, Commercial Press(in Chinese), 2016