An Example of the Division by Zero Calculus Appeared in Conformal Mappings

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Abstract: We introduce an interesting example of conformal mappings (Joukowski transform) from the view point of the division by zero calculus. We give an interpretation of the identity, for $a > b > 0$

$$\frac{\rho + 1/\rho}{\rho - 1/\rho} = \frac{a}{b}, \quad \rho = \sqrt{\frac{a+b}{a-b}},$$

for the case $a = b$.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, conformal mapping, Joukowski transform.

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1 A new type example

We introduce an interesting example of conformal mappings from the view point of the division by zero calculus.

For $a > b > 0$, we consider the elementary mapping

$$W = \frac{c}{2}\left(z + \frac{1}{z}\right) \tag{1.1}$$

with

$$c = \sqrt{a^2 - b^2}$$

on the complex $z = x + iy$ plane. Then, with

$$\rho = \sqrt{\frac{a+b}{a-b}},$$

the annulus

$$1 < |z| < \rho$$

is mapped conformally to the elliptic domain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$$

deleted the segment

$$[-c, c].$$

Then, the points $z = \rho, i\rho$ are mapped to the points $W = a, b$, respectively, furthermore we have the identity

$$\frac{\rho + 1/\rho}{\rho - 1/\rho} = \frac{a}{b}. \tag{1.2}$$

Then, if $a = b$, by the division by zero calculus

$$\rho^2 = \frac{a+b}{a-b} = 1.$$

Then, from

$$\frac{\rho + 1/\rho}{\rho - 1/\rho} = \frac{\rho^2 + 1}{\rho^2 - 1},$$

by the division by zero calculus we have the good result

$$\left(\frac{\rho^2 + 1}{\rho^2 - 1}\right)_{\rho^2 = 1} = 1.$$

2 Conclusion

For the identity with $a > b > 0$

$$\frac{\rho + 1/\rho}{\rho - 1/\rho} = \frac{a}{b}, \quad \rho = \sqrt{\frac{a+b}{a-b}},$$

we gave an interpretation for $a = b$, by means of the division by zero calculus.

3 Essence of division by zero calculus

We state the essence of division by zero calculus.

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n (z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n (z-a)^n, \tag{3.1}$$

we will **define**

$$f(a) = C_0. \tag{3.2}$$

For the correspondence (3.2) for the function $f(z)$, we will call it **the division by zero calculus**. By considering derivatives in (3.1), we **can define** any order derivatives of the function f at the singular point a; that is,

$$f^{(n)}(a) = n! C_n.$$

However, we can consider the more general definition of the division by zero calculus.

For a function $y = f(x)$ which is n order differentiable at $x = a$, we will **define** the value of the function, for $n > 0$

$$\frac{f(x)}{(x-a)^n}$$

at the point $x = a$ by the value

$$\frac{f^{(n)}(a)}{n!}.$$

For the important case of $n = 1$,

$$\frac{f(x)}{x-a}|_{x=a} = f'(a). \tag{3.3}$$

In particular, the values of the functions $y = 1/x$ and $y = 0/x$ at the origin $x = 0$ are zero. **We write them as $1/0 = 0$ and $0/0 = 0$, respectively.** Of course, the definitions of $1/0 = 0$ and $0/0 = 0$ are not usual ones in the sense: $0 \cdot x = b$ and $x = b/0$. Our division by zero is given in this sense and is not given by the usual sense as in stated in [1, 2, 3, 4].

In particular, note that for $a > 0$

$$\left[\frac{a^n}{n}\right]_{n=0} = \log a.$$

This will mean that the concept of division by zero calculus is important.

Note that

$$(x^n)' = nx^{n-1}$$

and so

$$\left(\frac{x^n}{n}\right)' = x^{n-1}.$$

Here, we obtain the right result for $n = 0$

$$(\log x)' = \frac{1}{x}$$

by the division by zero calculus.

References

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