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Chaos Theory and Intentional Teaching in Evolution

Nicholas Hoggard [1]

[1] No affiliation; nick.hoggard+evolution@gmail.com

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1. Abstract

Gärdenfors and Hedberg have identified 6 levels of Intentional Teaching which have arisen in sequence, in synchrony with tool innovations during human evolution. Remarkably, these events appear at time intervals that decrease at a constant ratio equal to the Feigenbaum constant, 4.66920… . This constant is the signature of period-doubling cascades, which can be found in many phenomena in nature and arise in iterative nonlinear processes in an environment of limited resources. Darwinian evolution is just such a process.

Cherry picking is not an issue because the 6 levels of teaching were discovered by studying the evolution of human behaviour, without regard for when the behaviour arose.

The pattern apparently extends both forwards in time from cultural evolution to information technology (movable-type printing, and computers), as well as backwards to physical and biological evolution (Big bang, life, sexual reproduction). All of these milestones mark new ways to transmit information to coming generations, first as DNA, then via teaching, and finally by information technology.

If confirmed, this apparent pattern in evolution may have major consequences for evolution theory. Other studies showing that rates of genetic change and speciation are largely unaffected by climate support the idea that a regular pattern is possible.

2. *Introduction*

One of the biggest mysteries in science is how life and intelligence evolved. Are they completely random accidents, or are there laws that make evolution predictable in some ways. The random hypothesis is the one favoured by science today, but it is difficult to observe directly what has occurred in the distant past. The mystery applies not only to biological evolution, but also the pre-biotic evolution of life, and to cultural evolution.

The conclusion of this study is that there are indications of a pattern in evolution,, Furthermore the pattern is familiar to population ecologists and is simply explained by using mathematical models based on Chaos Theory.

Co-evolution of tools, language, and biology

It is thought that intelligence in early humans evolved because they used tools. Tools gave them a competitive advantage, but with tools came a need for communication and language to evolve in order to pass on the knowledge of tools to the next generation, and also biological evolution to improve physical dexterity for handling tools, and larger brains to handle, among other things, language.

Six Levels of Intentional Teaching

Gärdenfors[1] argues that intentional teaching was necessary to maintain the cumulative culture that evolved largely through the development of tools. He identifies six levels of intentional teaching:

- 1. intentional evaluative feedback,
- 2. drawing attention (aka referential gestures),
- 3. demonstrating,
- 4. communicating concepts,
- 5. explaining relations between concepts, and
- 6. narrating.

Teaching ←→ Technology

Gärdenfors and Högberg suggest that these levels evolved separately, one after the other. They associate level 3 (demonstrating) with Oldowan technology (removing flakes of flint with another stone), and level 4 (communicating concepts) with late Acheulean technology. Frustratingly, they do not associate the other teaching levels with tool development levels, but the only possibilities are:

Teaching method	**Corresponding technology**
1. intentional evaluative feedback	Tool use
2. drawing attention (aka referential gestures)	Tool-making by hand
3. demonstrating	Oldowan technology (Making tools with tools)
4. communicating concepts	Late Acheulean technology (Concept-based tools)
5. explaining relations between concepts	New inventions (Tools not found in nature)
6. narrating	Transition from hunter-gatherer to other livelihoods.

New Inventions

So now we have mappings to technology for all levels of teaching. But was are New Inventions? Tool use began with objects found in the environment being used as tools for various purposes. There followed 3 phases of tool manufacture that led to improved tools, but no new tools. This whole process took well over 60 million years. Then came the first of many new inventions: the harpoon, and also a tool used for making clothes (though the clothes themselves have not survived). These are the first tools that are not improved versions of tools found in nature, but new tools created by the imagination. They would have required a new form of teaching - Teaching level 5: "Explaining relationships between concepts".

The distinction between on the one hand tools found in nature and improved manufactured versions of them, and on the other hand "new inventions", referring to manufactured tools with novel functions, seems to be a distinction rarely made in archaeology.

Tools for making clothes, dated to somewhere between 120,000 and 90,000 years ago[17], and harpoons dated to somewhere between 110,000 and 80,000 years ago[18][19] have been found.

A seventh level?

There are three potential issues with the list as it stands:

- Intentional Evaluative Feedback as a teaching method does not require the use of tools, and can be associated with tool-less Cumulative Culture. This leaves Tool Use without a unique form of teaching.
- Also, I shall be looking at dates for the various levels of teaching. The date of tool-less Cumulative Culture is not known as there is no conclusive fossil record, so I will not look at this level of teaching.
- However, there is another form of intentional teaching, associated with Tool Use, which arguably belongs on Gärdenfors's list. It is called Tool Transfer, and consists of the teacher simply giving the tool to the pupil. This behaviour has been observed in chimpanzees and it fulfils the definition of teaching[2].

After adding Tool-Transfer, the levels can be summarized in table GT1.

Level of Intentional Teaching	**Technology level**	**Date of technology level (years before 2000)**
Intentional evaluative feedback	Cumulative Culture	*Unknown*
Tool transfer	Tool-use	65 to 55 million years ago [3][4]
Referential gestures	Tool-making	13 to 9 million years[5][6]
Demonstrating	Oldowan technology (making tools with tools)	2.60 to 2.55 million years [7]
Communicating concepts	Late Acheulean technology (concept-based tools)	550,000 to 450,000 years [8][9][10]
Explaining relations between concepts	New inventions	Tools for making clothes (somewhere between 120,000 and 90,000 years ago[17]). Harpoons (somewhere between 110,000 and 80,000 years ago[18][19].
Narrating	New livelihoods (domestication)	Domestication (of the dog) 26,000–23,000 years[11]

Table GT1. Levels of Intentional Teaching and corresponding technology levels.

There are 6 technology levels in a row for which we know the dates – from "2. Tool Transfer" to "6. Domestication". If we plot the 5 ***intervals*** between those 6 technology levels on a logarithmic scale, we get figure gh2.

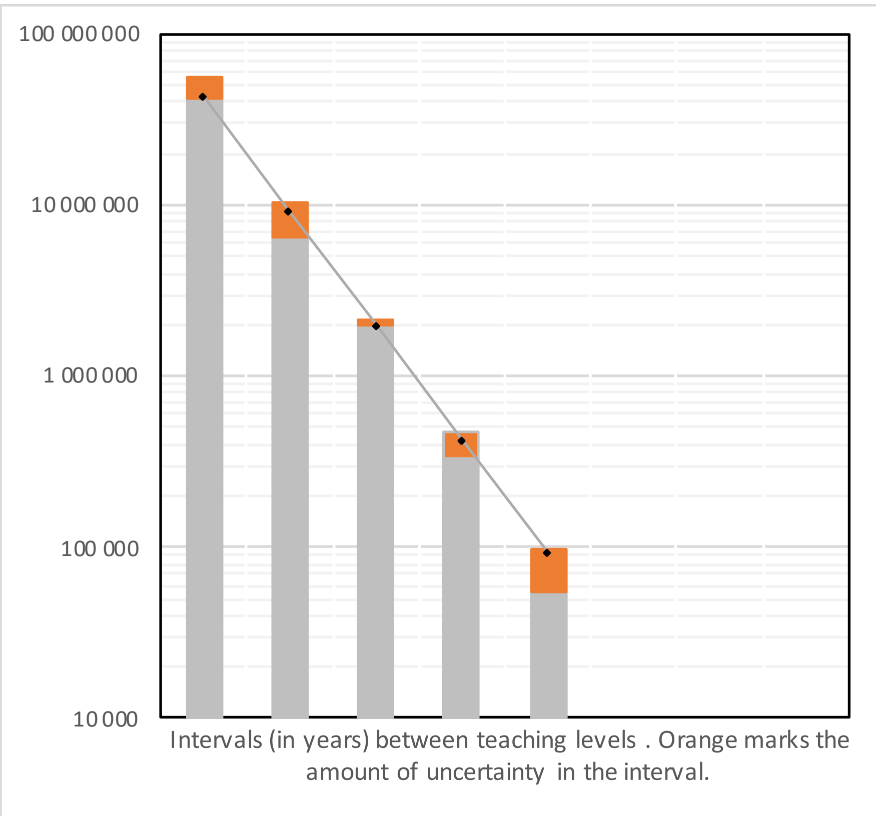


Figure gh2. intervals between teaching levels are getting shrinking at a uniform rate.

Figure gh2 shows that the intervals between the levels are getting smaller. Not only that, but they appear to be getting smaller at the same rate. The average rate of shrinking from one interval to the next is 4.79.

It should be emphasized that the teaching levels in Gärdenfors's list have been arrived at by careful observation of humans in learning situations and of tool-using animals. The events have not been selected with the time dimension in mind.

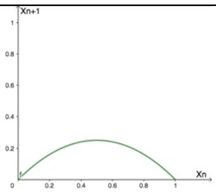
2.1. Where do the decreasing intervals come from?: Period-doubling cascades

There is a phenomenon well-known in Chaos Theory called the Period-doubling Cascade. The signature of such a cascade is a series of so-called bifurcations occurring at decreasing intervals. Decreasing intervals is exactly what we observe in the levels of Intentional Teaching. Furthermore, the ratio by which the intervals decrease in period-doubling cascades always converges to 4.66920… . This number is known as the Feigenbaum constant. The rate of shrinkage we found in the teaching intervals is 4.79, which is within 2.5% of the Feigenbaum constant.

The Feigenbaum constant is found in all period-doubling cascades, and period-doubling cascades are found in many natural phenomena, from dripping taps[12] to the way the eye transmits images[13]. They are found in iterated nonlinear dynamic processes. One example of an iterated nonlinear dynamic process is evolution.

Causation

The relationship between the limited resources curve and the period-doubling cascade is often found in nature, and it crucial for understanding the connection with evolution. I show it, for emphasis, in figure ca1.

The limited resources curve 

, **which affects all populations**, amplified by the growth-rate "*a*",

when iterated thousands of times,

creates the period-doubling cascade as "*a*" increases:

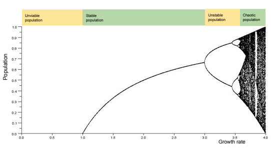


Figure ca1. The limitation of resources (logistic map or similar), when iterated many times, produces the attractor with period-doubling cascades.

Limited resources – the source of the cascade

The period-doubling cascade comes ultimately from a situation of limited resources. In the case of a species of animal in an ecosystem, there is a limit to the size of population of that species that the ecosystem will support (called the carrying capacity). The larger the population, the more food is consumed and each individual has to spend more energy to find the remaining food. There is less food for the energy spent.

The limitations of a typical ecosystem can be represented by the kind of curve shown in figure jj2.

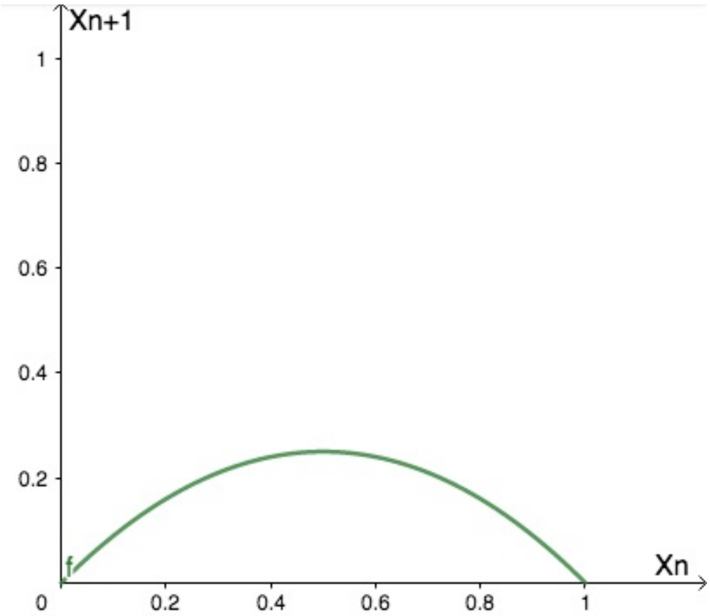


Figure jj2. The logistic map $x_{n+1} = a \, . \, x_n \, (1 - x_n/K)$ for growth rate a = 1.0 and carrying capacity K = 1.0. This curve, and similar ones, when multiplied by growth ratee and iterated many times, creates the cascade of bifurcations.

This curve predicts next year's population from this year's population. Next year's population depends on three factors: the current population, the carrying capacity, and the growth rate of the species.

(For the curve in figure jj2, the growth rate, a, is 1.0, which is not enough to sustain the species, and the population will die out in a few years.)

Iterating the logistic map

If we iterate the logistic map a few thousand times for increasing values of growth rate a, we get the result shown in figure jj3.

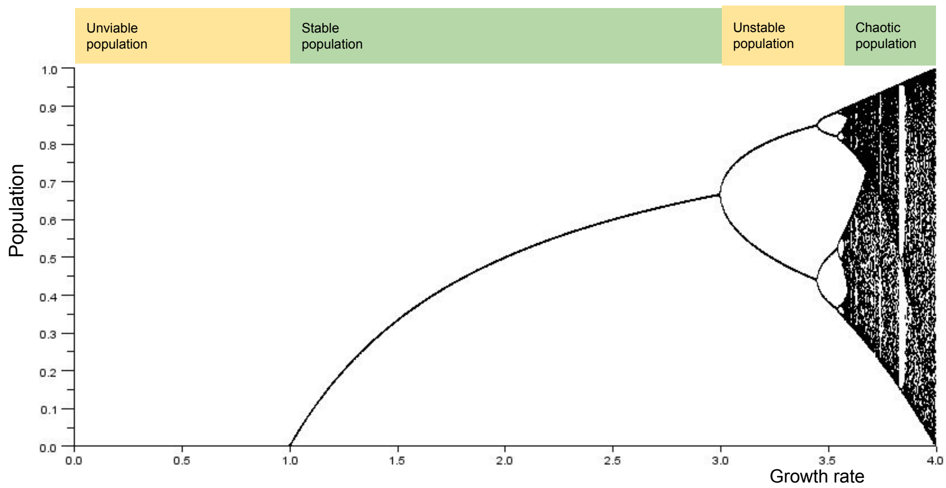


Figure jj3. The attractor for iterations of the logistic map x → a.x(1 – x) for various values of growth rate *a* from 1.0 to 4.0. (4.0 is the maximum for growth rate for the logistic function. More sophisticated functions can handle higher growth rates, but all show the same bifurcations.)

Interpreation

Figure jj3 shows the complex behavior that can result from a simple system if it is iterated a few thousand times. The points shown in the figure are the levels of population that the species settles down to after many iterations, given a random starting population.

Growth rate "*a*"	**Population (see figure jj3)**
< 1.0	Population dies out
1.0 to < 3.0	Stable population, increases with growth rate, but slows down.
3.0 to about 3.5	Population becomes unstable and a period-2 bifurcation occurs, which means that the population alternates between a high and a low value on alternate years.
A bit higher	Period-4 bifurcation – population cycles between 4 different levels on successive years.
A bit higher	Period-8 bifurcation
A bit higher	Period doubles, at higher and higher values of growth rate *a*, to 16, 32, 64 etc.

A bit higher	Period becomes non-repeating and we have reached the zone of chaotic population levels.

So we can see that by iterating the relatively simple logistic map, which is commonly used for modelling populations in resource-limited ecosystems, we get complex behavior and bifurcations. The exact curve does not matter as a large number of different "single hump curves" give the same rate of decrease in intervals in the so-called bifurcation parameter. This is true as long as the map used has a single peak with a quadratic maximum – the so-called bifurcation velocity is always 4.66920…, which is why the Feigenbaum constant is called a universal constant.

What does this mean for Teaching?

From the pattern we see in Intentional Teaching, we can see that for each new innovation in teaching, there is a population bifurcation. This is a phenomenon similar to what can be seen in Population Dynamics, a subject closely related to Evolution studies. (It is relevant at this point to remember that Darwin got the idea for his theory of Natural Selection after reading a treatise on population by Thomas Malthus.) Population Dynamics is mostly used by the likes of commercial fish farms to maximize their production. They find that by increasing the birth rate, a population can consume too much of their food supply which the result that the population dips in the following year as the animals run out of food. At the first bifurcation (period-2) the population settles down to alternating between 2 numbers. At the next bifurcation (period-4) the population alternates between 4 levels every 4 years. And so on through 8, 16, 32, etc.

How does this apply to Teaching?

Intentional Teaching is necessary in order to pass on cultural innovations. Also, there was probably co-evolution between cultural innovation, intentional teaching, language, and biological traits such as manual dexterity and brain size/structure. Cultural innovations were the driving force of this co-evolution and can be seen as a form of artificial adaptation that generates variation much faster than random genetic adaptation and is subject to cultural selection (although natural selection is still in play). If that is the case, then each new stage in Intentional Teaching passes on to the next generation a new kind of information that wasn't passed on before the new form of Intentional Teaching arose. This additional information is the extra information needed for each of the stages in cultural evolution (*Cumulative Culture, Using Tools, Making Tools, Making Tools With Tools, Concept-Based Tools, New Inventions,* and *New Livelihoods*).

Each of these stages is not actually a cultural innovation in itself but a *category* of cultural innovations. For example, in the category "Making Tools" there will be a number of different tools which will appear at different historical dates. The first of these tools will be the one that ushers in the new category of found tools and which will be the first cultural innovation that requires the new form of Intentional Teaching associated with the cultural innovation category.

Hypothetical Population Stability

Let us assume a scenario where there are a number of species competing for the same resources in an ecosystem, and none of the species has yet invented Cumulative Culture, the first level in our teaching hierarchy. Let us also assume they are all multicellular animals using the same method of biological adaptation and evolution, namely sexual reproduction, and therefore adapt to change at a similar rate to one another. It may well be that similar rates of adaptation lead to an equilibrium over evolutionary time whereby, although relative population numbers change, no one species gains the upper hand, and populations remain stable, meaning that they do not bifurcate.

In support of such a hypothetical population equilibrium is a phenomenon called the "cost of complexity", which means that animals that are more complex are not only better adapted, they also evolve more slowly because random genetic change is more likely to make then more poorly adapted than better adapted. As some species draw ahead of others, they also slow down, and the lesser-adapted slow runners have time to catch up.

Cultural Adaptation could cause Population Instabilities/Bifurcations

What happens if a species now invents Intentional Teaching and Cumulative Culture? Cumulative Culture leads to increased Darwinian fitness in the same way as pure genetic evolution does. As a means of adaptation, Cumulative Culture is much faster than genetic evolution. It also provides a way for species that have stagnated, (with the Cost of Complexity), to break free and evolve in a new direction using the new cultural adaptation mechanism. Cumulative culture is just as much a Darwinian process (inherited traits with variation) as sexual or asexual reproduction are, but with different mechanisms.

Being much faster that genetic evolution, Cumulative Culture is likely to give a species a permanent advantage compared with those species that don't have it. This may have the consequence that the species' fitness becomes much higher than the other species it is competing with, to the point where it overconsumes the food it relies on, leaving too little food for the following year. This would cause the population to drop the following year, crossing the threshold where period-2 bifurcations occur.

Learning from Population Dynamics

The mechanism of overconsumption causing a population bifurcation also happens in simulations of Population Dynamics as the Birth Rate increases. In the case of Population Dynamics, the Birth Rate is the so-called Bifurcation Parameter. As the Bifurcation Parameter increases, further population bifurcations occur at decreasing intervals of birth rate. At the first bifurcation the birth rate increases the population to an unstainable threshold where it where it eats all of the food available for the current year and beginning to deplete the food needed for the following year. The following year there is insufficient food and the population drops. Assuming all parameters, including the Birth Rate, remain the same, the population settles into a period-2 oscillation on a long term basis.

Increasing Birth Rate to the next threshold, the population eats into the food supply of not only the following year, but also the year after that. This results in a second bifurcation and a long-term period oscillation of the population.

Increasing the Birth Rate further causes further bifurcations at birth-rate intervals that decrease by the Feigenbaum ratio 4.66920… .

Translated to Intentional Teaching

The case of Intentional Teaching follows the same pattern of population bifurcation as in Population Dynamics, which the following differences:

- Instead of Birth Rate, the bifurcation parameter for Intentional Teaching is *growth rate*, which increases monotonically with *complexity*, which increases monotonically with *time*.
- Instead of a Birth Rate threshold, each threshold is a cultural innovation. The thresholds are these corresponding to the 7 levels of Intentional Teaching, namely *Cumulative Culture, Using Tools, Making Tools, Making Tools With Tools, Making Concept-Based Tools, Making New Inventions,* and *Inventing New Livelihoods).* Each of these cultural innovations is also a new Darwinian process, using different mechanisms for inheritance with variation, and all use cultural selection. Each Darwinian Process is additional, and does not replace earlier Darwinian processes.
- In Population Dynamics, increased Birth Rate beyond a critical threshold leads to overconsumption and subsequent population bifurcation.
- In Intentional Teaching, elapsed time on an evolutionary timescale leads to increased complexity, in turn leading to new categories of cultural innovation, resulting in step increases in adaptation rate, causing overconsumption and population bifurcation

Linearity

As mentioned, elapsed evolutionary time causes an increase in complexity which in turn leads to 7 new categories of cultural innovations and associated population bifurcations at intervals decreasing by 4.66920… . "Complexity" is used in a loose sense because there is no universal definition of complexity or how to measure it. Normally the relationship between time, complexity, and the decreasing intervals would be important, but any monotonic, smooth relationship that contains a decreasing series of intervals naturally converges to linearity as the intervals get smaller.

A Loose End

The date of the first teaching event, "Intentional Evaluative Feedback" is not known as there is no conclusive fossil record. Extrapolating the intervals according to the observed interval ratio gives a date of 261 million years ago. Probably the most social of animals at this time were Cynodonts, animals that were the ancestors of mammals. They lived in communal burrows with mixed generations, which indicates that they may have been highly social, and thereby could have practiced Intentional Evaluative Feedback and had Cumulative Culture.

2.2. *Extending forwards*

A period-doubling cascade does not just stop. There are an infinite number of them and the interval will shrink to zero, at which point in time the population will become aperiodic, otherwise called chaotic. (The sum of an infinite series can be finite, as in the case 1 = ½ + ¼ + 1/8 + and so on.)

The established pattern, if extended, predicts the following events:

Innovation	Predicted date	Actual date
Written Language (used for Hand-written Documents)	3500 to 3200 BCE [26][27]	3210 BCE
Movable-type Printing (used for Mass-produced Literature)	1039 CE [28]	1039 CE
Computers (used for Automating Information)	1948 CE [29]	1948 CE
Unknown future events		

How do these fit in with teaching?

The invention of Writing.

Writing allows accumulation of knowledge. Reading and Writing must be taught, and so belong to the series of innovations in Intentional Teaching.

The invention of Movable-type Printing

The invention of Movable-type Printing is related to teaching insofar as it is concerned with the transmission of knowledge, as is teaching. However, printed literature is predominantly used for self-teaching, and belongs to a new mode of information transmission. Movable-type Printing was invented in China, 450 years before it appeared in Europe.

The invention of the Computer

Like the printing machine, the computer and the computer network is a new way of distributing knowledge that requires little or no teaching. (The delay of the Movable-type Printing Machine in Europe does not seem to have slowed development, as the first fully Turing-complete (including stored-program) computer was made at Manchester University in 1948.

2.3. *A Wider Perspective – prior to teaching*

Following the bifurcation pattern backwards, finds the following events

Information Transmission	Best known date from the historical record (upper and lower limit)	Date calculated from Feigenbaum constant 4.66920…	Deviation of known date from Feigenbaum constant
No information transmission (Big Bang)	13.82 to 13.78 billion years BCE [21]	26.6 billion years BCE	-52%
Information transmission by DNA in Asexual Reproduction of Single-celled Organisms.	4.28 to 3.77 billion years BCE [22]	5.70 billion years BCE	-25%
Information transmission by DNA in Sexual Reproduction of Complex Multicellular Organisms	1.0 to 1.2 billion years BCE [23] [24]	1.22 billion years BCE	-1.6%

Again, these are different from teaching, although Information Transmission is still central to these processes. The Big Bang is the beginning of the whole process, the first part of which is the physical evolution of the universe, culminating in stars, planets, and complex molecules which are gradually evolved into life on Earth (and perhaps in space).

The dates of these events do not match the Feigenbaum constant at first, but quickly converge to it around the time of sexual reproduction. This initial different followed by rapid convergence to the Feigenbaum constant is the rule rather than the exception for period-doubling cascades.

The whole pattern

Evolutionary phases	Description	Number of stages	Information Transmission
Big Bang	Physical evolution	1 stage	No information Transmission
Life	Biological evolution	2 stages	DNA
Intentional Teaching	Cultural evolution, co-evolving with biology	8 stages	Intentional Teaching
Information Technology	Technological evolution	2 stages so far	Information technology

3. *Evolution may not be so random*

3.1. How is this pattern possible when evolution is random?

Eminent evolutionary biologist Stephen Jay Gould posited that in the interplay between mutation and natural selection, random mutation dominates and that if the tape of evolution were to be rerun, the results would be very different[14]. He also argued, together with Niles Eldredge, in their theory of Punctuated Equilibria, that evolution is not gradual, as Darwin claimed, but proceeds at different speeds governed by environment and climate[15].

However, both of these theories have been challenged in subsequent studies. A recent experiment with yeast cells indicates that although mutation is random, natural selection always ends up picking the same mutations, and that evolution reaches the same endpoint every time the experiment is run[16]. This in turn implies that mutation provides all variants necessary for natural selection to always choose the same solution. If the mutations were only to generate say 1 % of the possible variations, then we may expect this to limit evolution. This is the Gould scenarario, which may not always be valid.

Other there is research that indicates that the rate of evolution is hardly affected by environmental changes. and that, while environment and climate affect abundance, they have little effect on rates of speciation or extinction[17]. In other words, not all species necessarily experience equilibrium in their evolution.

These results together suggest that predictable evolution may still be a possibility.

It is significant that Gould and Eldredge were so aware that they were going against the mainstream in evolutionary biology – which mostly subscribed to the model of gradual change – that in their Punctuated Equilibria paper they urged evolutionary biologists to reconsider old facts in the light of new ideas. It is only fair that the same consideration is applied to this paper.

4. *The Whole Picture*

Figure 1 shows all the events together on a logarithmic time scale.

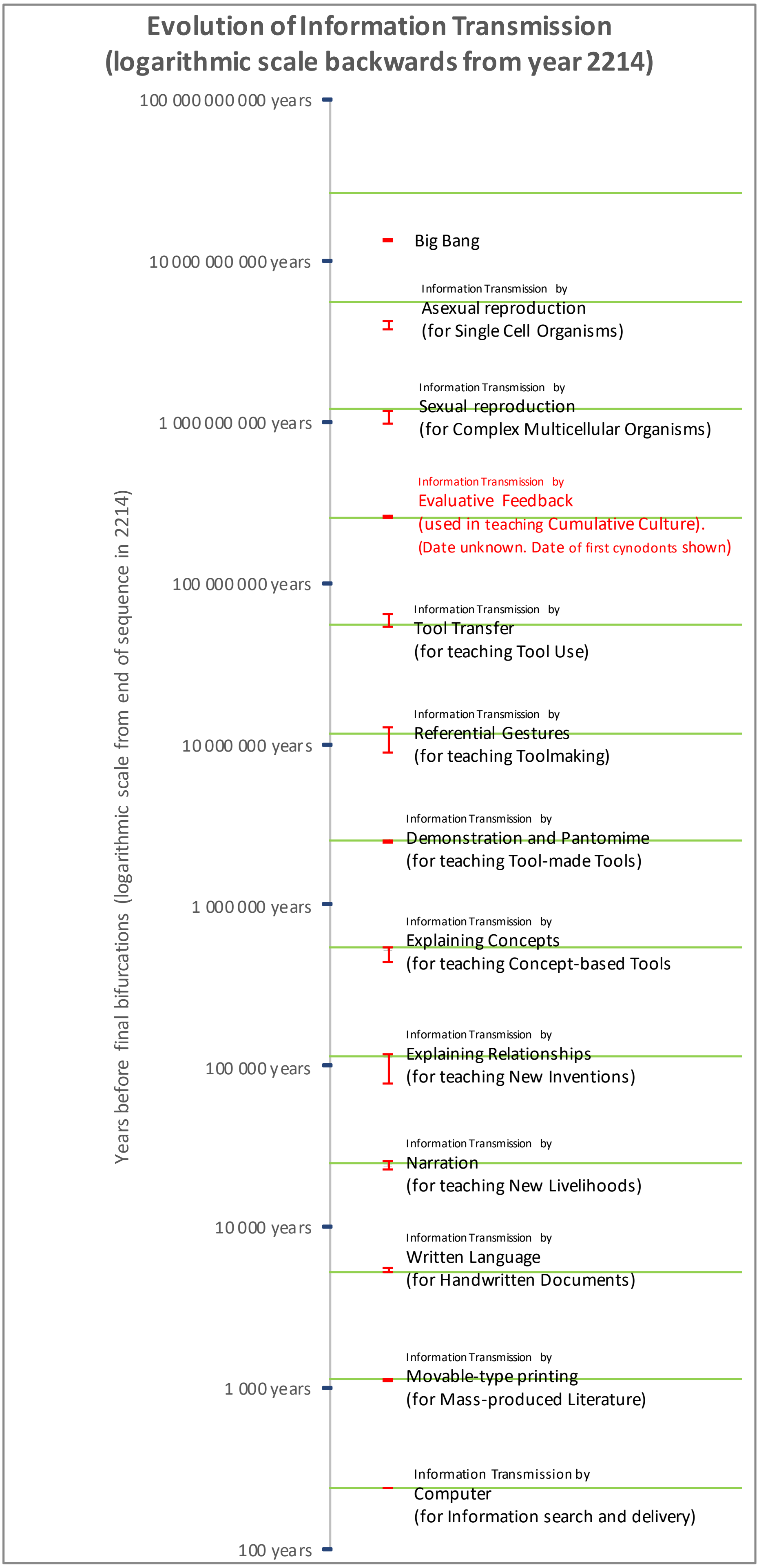

Evolution of Information Transmission
(logarithmic scale backwards from year 2214)
Years before final bifurcations (logarithmic scale from end of sequence in 2214)
100 000 000 000 years
10 000 000 000 years
1 000 000 000 years
100 000 000 years
10 000 000 years
1 000 000 years
100 000 years
10 000 years
1 000 years
100 years
Big Bang
Information Transmission by
Asexual reproduction
(for Single Cell Organisms)
Information Transmission by
Sexual reproduction
(for Complex Multicellular Organisms)
Information Transmission by
Evaluative Feedback
(used in teaching Cumulative Culture).
(Date unknown. Date of first cynodonts shown)
Information Transmission by
Tool Transfer
(for teaching Tool Use)
Information Transmission by
Referential Gestures
(for teaching Toolmaking)
Information Transmission by
Demonstration and Pantomime
(for teaching Tool-made Tools)
Information Transmission by
Explaining Concepts
(for teaching Concept-based Tools
Information Transmission by
Explaining Relationships
(for teaching New Inventions)
Information Transmission by
Narration
(for teaching New Livelihoods)
Information Transmission by
Written Language
(for Handwritten Documents)
Information Transmission by
Movable-type printing
(for Mass-produced Literature)
Information Transmission by
Computer
(for Information search and delivery)

Figure 1. The red error bars are actual dates of new Information Transmission mechanisms. It can be seen that they match the green lines, which are theoretical dates predicted by the Feigenbaum constant. Dates are measured from where the sequence converges around the year 2214. The time scale is logarithmic, so that the intervals between stages appear equidistant on the diagram even though they are actually getting smaller by a constant factor 4.66920…, known as the Feigenbaum constant. **The actual date for the first occurrence of Intentional Evaluative Feedback is not known (red text), but the predicted date suggests that Cynodonts (which later evolved into mammals) were the first animals to use Intentional Evaluative Feedback.** The first two dates do not match the dates predicted by the Feigenbaum ratio. However, in Chaos Theory it is nearly always the case that initial interval ratios do not match the Feigenbaum constant, but rapidly converge to it, which is what we see here.

No.	Information Transmission	Best known date from the historical record (upper and lower limit) (Years before 2000)	Date calculated from Feigenbaum constant 4.66920…* (Years before 2000)	Deviation of known date from Feigenbaum constant**
1	No information transmission (Big Bang)	13.82 to 13.78 billion years [21]	26.6 billion years *	-52% *
2	Asexual Reproduction (used by Single-celled life)	4.28 to 3.77 billion years [22]	5.70 billion years *	-25% *
3	Sexual Reproduction (used by Complex Multicellular Organisms)	1.0 to 1.2 billion years [23] [24]	1.22 billion years	-1.6%
4	Intentional Evaluative Feedback (used in teaching Cumulative Culture)	Date unknown. Near the theoretical date, the mammal precursors known as Cynodonts (260 million years ago[25]) appeared. They were social animals living in burrows.	261 million years	Date unknown
5	Tool Transfer (used in teaching Tool Use)	65 to 55 million years ago [3][4]	56.0 million years	0%
6	Referential Gestures (used in teaching Tool-making	13 to 9 million years[5][6]	12.0 million years	0%
7	Demonstration and pantomime (used in teaching Making Tools with Tools)	2.60 to 2.55 million years [7]	2.57 million years	0%
8	Communication of Concepts (used in teaching Making Concept–Tools)	Stone spearhead 550,000 to 450,000 years [8][9][10]	550,000 years	0%
9	Explanation of relationships between concepts (used in teaching New Inventions)	Tools for making clothes 120,000 to 90,000 years[18]. Harpoon 110,000 to 80,000 years[19][20]	118,000 years	0%
10	Narration and structurally complete language (used in teaching New Livelihoods)	Domestication (of the dog) 26,000–23,000 years[11]	25,000 years	0%
11	Written Language (used for Hand-written Documents)	5,500 to 5,200 years (3500 to 3200 BCE) [26][27]	5,210 years	0%
12	Movable-type Printing (used for Mass-produced Literature)	961 to 952 years (1039 to 1048 CE) [28]	961 years	0%
13	Computers (used for Automating Information)	52 years (1948 CE) [29]	52 years	0%

Table I. The data used in figure 1 (Evolution of Information Transmission). * The first two events (Big Bang, and start of life) deviate from the Feigenbaum ratio, but the intervals then converge quickly to the theoretical value at the next stage. This convergence from a different interval is normal for

period-doubling bifurcations.
* 0% deviation means that the date calculated from the Feigenbaum constant is within the error range of the known date.

5. Summary

Studying the time intervals between the levels of intentional teaching in human evolution reveals a pattern of intervals decreasing by a constant factor which is very close to the Feigenbaum constant. This indicates so-called period-doubling bifurcations in the population which is the expected result of an evolving growth rate in a limited-resource ecosystem. Each bifurcation probably represents a step-change in adaptation rate, as would be expected with the step-changes in tool evolution during this period, corresponding to the transitions to higher levels of teaching.

The pattern also appears to extend back in time to cover information transmission events in physical and biological evolution, and forwards to cover future information technology

I speculate that the increase in growth rate is due to the increase in complexity of species, which in turn increases with evolutionary time. Linearity between these parameters is not a requirement as the Feigenbaum constant is universal and is not sensitive to the mathematical relationship between these parameters.

6. Conclusions

The aspect of chaos theory normally applied to evolution is the butterfly effect, whereby small initial changes (for example, Darwinian variation) cause large differences in the result. A different aspect of chaos theory is presented here, namely the universality of the period-doubling cascade, which causes convergence to a common pattern (perhaps in the same way as natural selection also does).

7. Discussion

The results reported in this paper, if confirmed, would represent something of a paradigm shift in evolution theory, from random evolution to the Victorian idea of an evolutionary "ladder". Much has been written about how wrong the ladder paradigm was. The new ladder is somewhat different to the old ladder. Perhaps it is again time for evolutionary biologists to reconsider old facts in the light of new ideas.

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Data Accessibility: All relevant data are within the paper or its Supporting Information files.

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