Fractal Belief Jensen–Shannon Divergence

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Abstract

In this paper, a novel belief divergence measurement method, fractal belief Jensen–Shannon ($FBJS$) divergence is proposed to better measure conflicts between evidences. The proposed $FBJS$ divergence is the first belief divergence that combines the belief divergence theory and the concept of fractal.

Keywords: Dempster–Shafer evidence theory; Belief divergence; Fractal.

1. The proposed method

Definition 1 ($FBJS$ divergence).

Let m_1 and m_2 be two BPAs in the frame of discernment Θ. The fractal belief Jensen–Shannon divergence ($FBJS$ divergence) is defined as

$$
\begin{aligned}
FBJS(m_1||m_2) = {} & \frac{1}{2} \cdot \sum_{i=1}^{2^N-1} \sum_{j=1}^{2^N-1} \tau(A_i, A_j) m_{F_1}(A_i) \log_2 \left[\frac{2m_{F_1}(A_i)}{m_{F_1}(A_i) + m_{F_2}(A_j)} \right] \\
& + \frac{1}{2} \cdot \sum_{i=1}^{2^N-1} \sum_{j=1}^{2^N-1} \tau(A_j, A_i) m_{F_2}(A_j) \log_2 \left[\frac{2m_{F_2}(A_j)}{m_{F_1}(A_i) + m_{F_2}(A_j)} \right].
\end{aligned}
\tag{1}
$$

m_{F_j} is defined as

$$
m_{F_j}(A_i) = \sum_{A_i \subseteq G_i}^{2^N} \frac{m_j(G_i)}{2^{|G_i|} - 1}, \quad j = 1, 2.
\tag{2}
$$

where G_i is any subset of A_i. m_{F_j} represents that BPAs are transferred for one time.

$\tau(A_i, A_j)$ is defined as

$$
\tau(A_i, A_j) = \frac{|A_i \cap A_j|}{|A_i \cup A_j|},
\tag{3}
$$

where A_i are hypotheses of m. $A_i \cap A_j$ is the intersection of A_i and A_j. $A_i \cup A_j$ is the union of A_i and A_j. $|A_i|$ represents the cardinality of A_i.