Does Replication in Cellular Automata Spaces Always Correspond to Broken Symmetries?

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Abstract

The Noether theorem establishes that symmetries in physics correspond to conservation laws, and therefore contra-positively that un-conserved quantities correspond to broken symmetries. This theorem was not established for dynamic systems in discrete space and discrete time, but the author in previous work has observed that replication of published cellular automata (CA) loop structures corresponds to chiral asymmetries. The work described here shows that replication of a different kind of CA replicator (the HighLife replicator of Nathan Thompson, 1994) corresponds to loss and restoration of a line of symmetry in each replication cycle. In addition to my past observations of loop replication chirality, the broken symmetry observed in the achiral replication of the achiral Thompson replicator suggests a hypothetical general principle of a correspondence between replication observed as an un-conserved (increasing) number of replicator instances and a range of broken symmetries. The paper concludes with some speculation about the possible relevance of simple CA replicators to proto-biology.

Keywords: broken symmetry, cellular automata, chirality, Game of Life, HighLife, homochirality, origin of life, replicator

Introduction

The theorem of Emmy Noether (1918) has become a deep principle of modern physics [8]. It shows that differentiable (continuous) symmetries of physics (*i.e.*, invariance with continuous translation) correspond to conserved physical quantities. The theorem was derived from Lagrangian formulations of the dynamics of physical bodies in our universe of apparent continuous space and continuous time. The theorem shows that space translation-, time translation- and rotation-invariances correspond respectively to conservation of linear momentum, mass/energy and angular momentum.

Recognition of the correspondence of symmetry to conservation laws suggests a contrapositive form: does an un-conserved quantity correspond to a broken symmetry? The second law of thermodynamics is a recognition that un-conserved entropy corresponds to the "arrow of time" (broken T-symmetry), but recognizing this, we can ask: is thermodynamics within the scope of the Noether theorem?

Entropy is understood at the scale of *discrete* entities, *e.g.*, ordered to disordered distributions of discrete molecules in fluids, so the answer to the question is of strong recent and continuing interest, *e.g.*, [7]. Continuous entropy is well-defined as an obvious extension of Shannon entropy, but it has flaws which contraindicate its relevance to experimental and observational sciences, except that continuous *relative* entropy is useful for mathematical reasoning in information theory [6].

Subject to the ideal condition of no confounding interactions, we can readily recognize that in a cellular automata (CA) space equipped with a universal state-transition function, spatial translation symmetry applies in a limited discrete sense: the state-transition dynamics of a structure is not changed by moving (copying) the structure to elsewhere on the CA grid, nor does the dynamic behaviour change with any of the finite number of possible discrete rotations within the CA space

(or rotation of the CA space), nor does insertion of a copy of a structure into open CA space at any arbitrary time correspond to any change in the dynamic behaviour.

Recognising these correspondences of symmetry and invariance in CA universes prompts us to ask: are there general principles analogous to the principles of the Noether theorem which apply to dynamics within the discrete space and discrete time of CA universes?

Cellular automata replicators

Cellular automata environments can support replicators, and we may consider that replication in CA environments is of more relevance than the translations which are crucial in dynamics observed in continuous space and time. The computer scientist Gregory Chaitin states it thus:

"… in this CA world … it is easier for an organism to reproduce itself than it is for it to move (translate itself)." [2]

In considering CA dynamics including replication, we can acknowledge the existence of symmetries and asymmetries. Symmetries of importance in CA replication are the structural symmetries/asymmetries of replicating structures, and the symmetries or asymmetries of the state transition functions which facilitate replication.

Under the ideal conditions of one initial replicator instance in an otherwise quiescent/dead CA universe, replication cycles repeat indefinitely over time, so the number of replicator instances increases, *i.e.*, the number of replicator instances and corresponding Shannon entropy are not conserved. The apparent arrow of time observed in our physical reality in which entropy is not conserved is also characteristic of CA universes: the iterative dynamic behaviour of CA structures in general is irreversible.

CA loop replicators

Self-replicating loops [1][3][5] are chiral in structure, and their replication process is chiral in the sense that there is no single comprehensive cell state-transition function which supports coexistence of left- and right-handed replication [9][10]. However, some particular permutations of the cell state set applied to a chiral replicator, but not applied to its mirror complement allow a functional heterochirality of replication [11][12]. In these arrangements, the left-handed replicator is not the mirror form of its right-handed complement due to one-sided reassignment of state-labels (*e.g.*, a swapping of states 2 and 4 applied to only one of the two Byl replicator chiralities [11]), but with the chiral reassignment of states and applying the appropriate state-transition rules (*e.g.*, Moore rules), left- and right-handed replication can coexist under a common state-transition function.

HighLife: a variation of John Conway's original Game of Life (GoL), and Nathan Thompson's HighLife replicator

We have noted that replication of Langton's loop [5] and later simplifications of it [1][3] is chiral. However, there are structures in CA spaces which are achiral replicators. A simple example supported by a variation of Conway's Game of Life (GoL) was discovered by Nathan Thompson in 1994. A description can be found in [4], p. 386, Figure 12.2.

The GoL variation ("HighLife", rule string B36/S23) includes the rules of the original GoL, but adds the rule that a dead cell with exactly six live neighbours comes to life. The state-transition function is achiral (each explicit state-transition rule coexists with its corresponding mirror-complement in the state-transition function, so both left- and right-handed replication of any replicating structure is supported). The replicating structure discovered by Thompson is very small and simple – twelve live cells within a square region of 5x5 cells (Figure 1 below, Time = 0). The structure itself is achiral – it is its own mirror-complement, so a left- and right-handed distinction in the replication of this structure does not exist.

The 26 panels (Time = 0 through to 24, and Time = 36) of **Figure 1** below show replication of the Nathan Thompson replicator in detail:
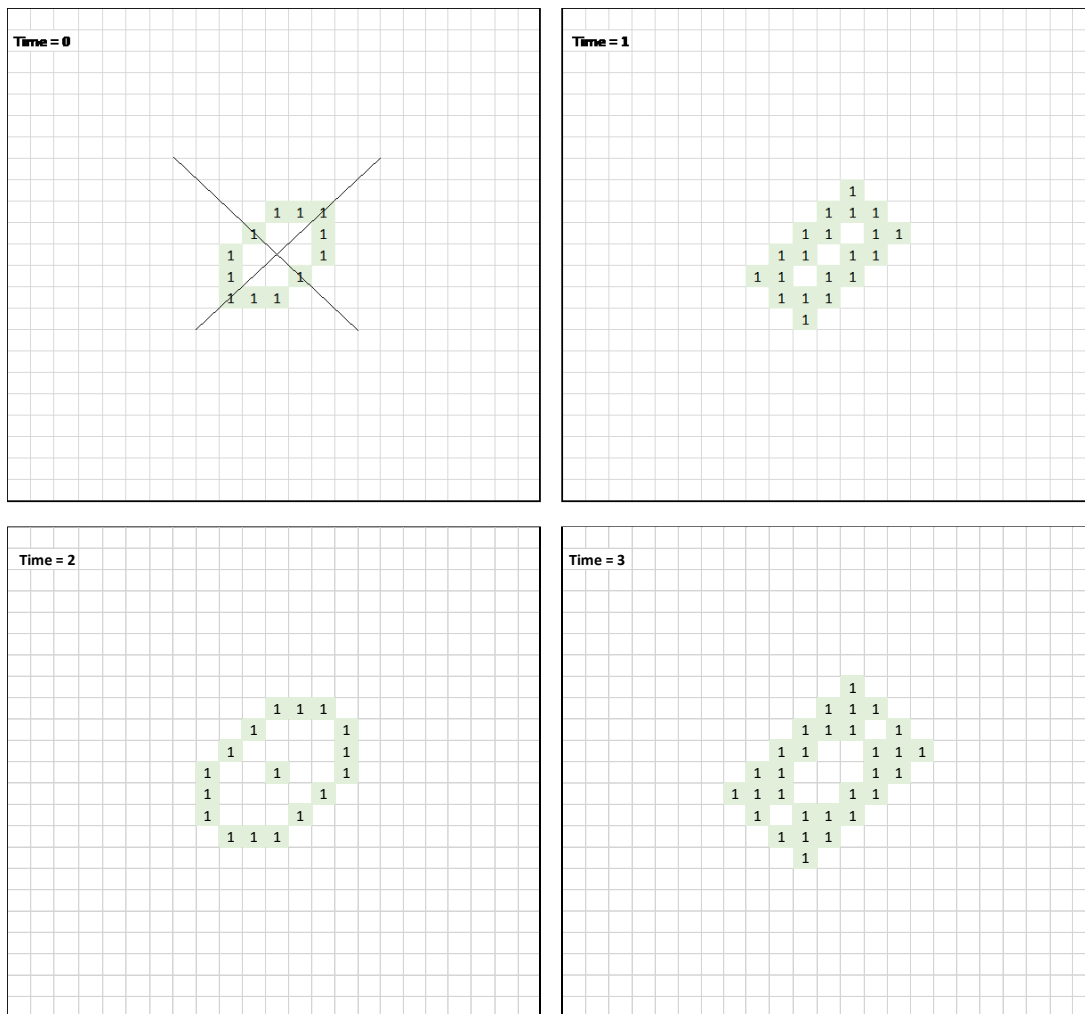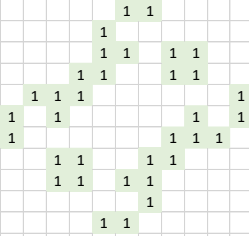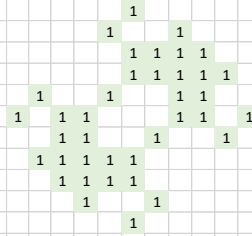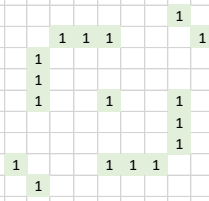
Time = 4
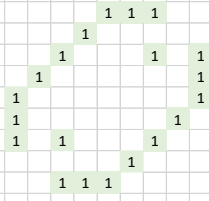
Time = 5
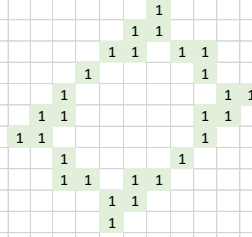
Time = 6

Time = 7

Time = 8

Time = 9

Time = 10
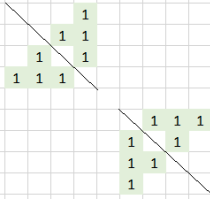
Time = 11
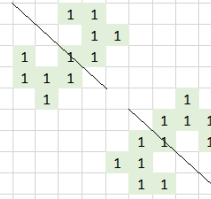
Time = 12
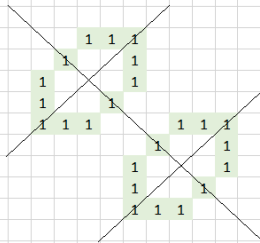
Time = 13
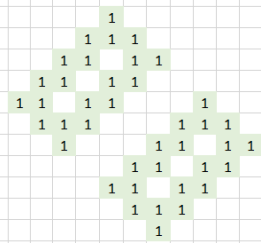
Time = 14
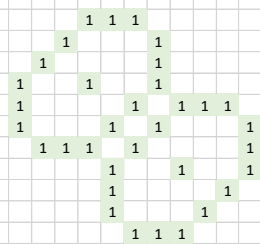
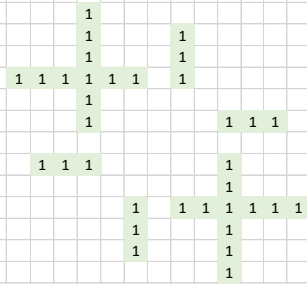**Time = 15**
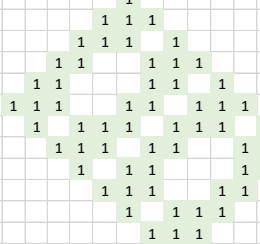
Time = 16
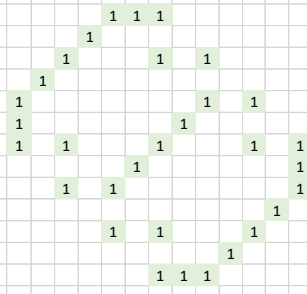
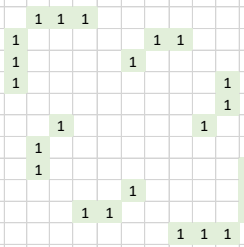**Time = 17**
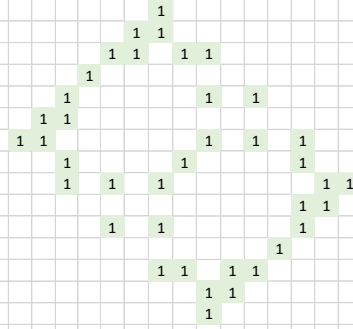
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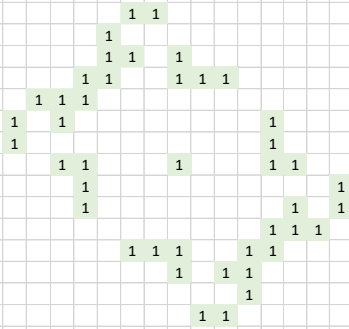
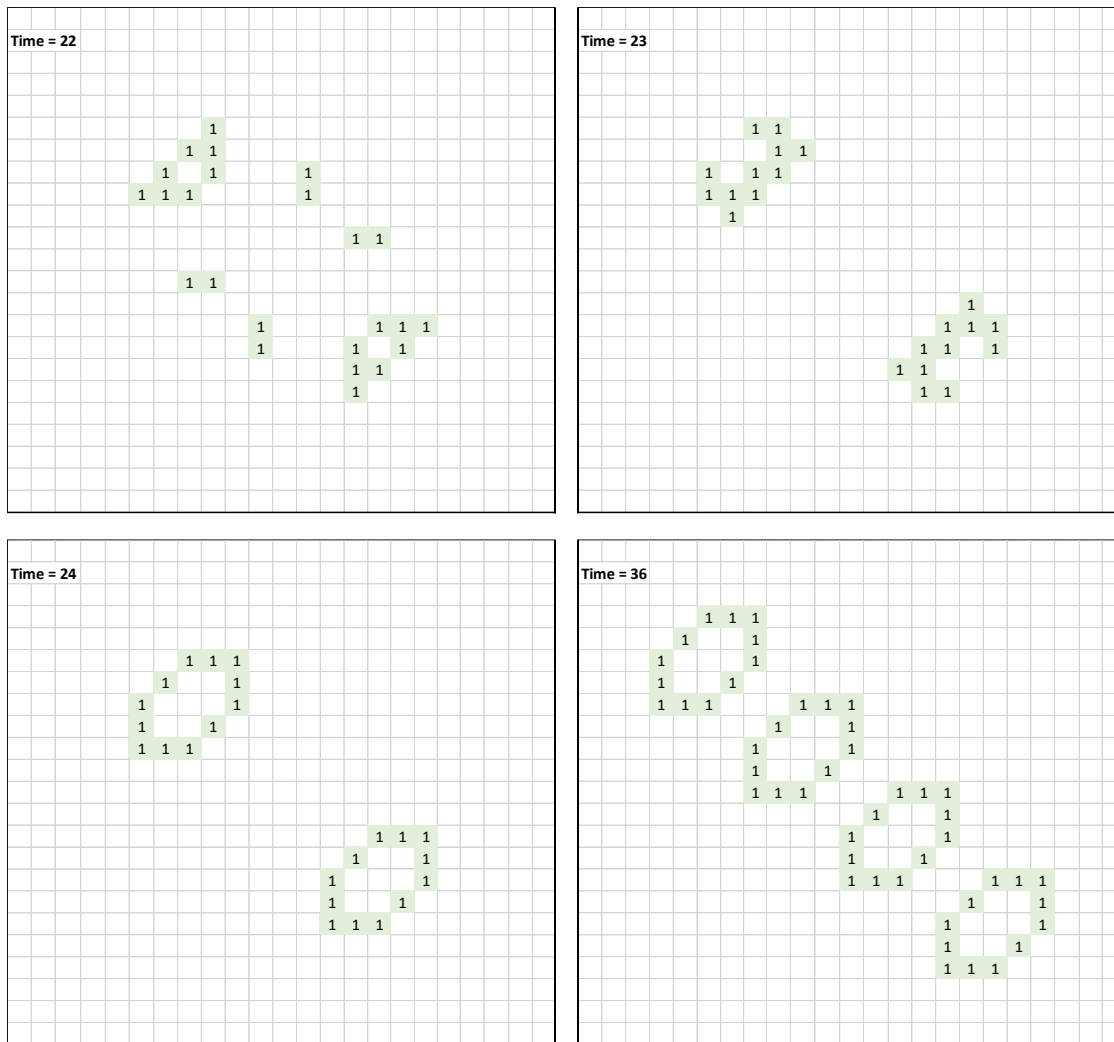**Time = 19**
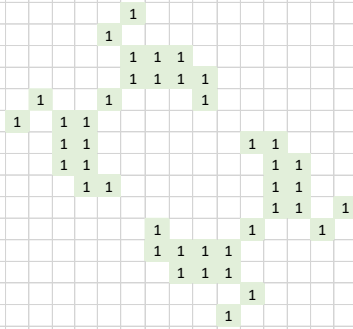
Time = 20

Time = 21

Figure 1. Replication of the Nathan Thompson structure in the HighLife CA environment from one instance at Time = 0 to two instances sufficiently spatially separated for unimpeded replication of each to yield a total of four instances from Time = 24 to 36, shown in the last two frames. Superimposed oblique lines shown at Times 0, 10, 11 and 12 are lines of structural symmetry (see text for discussion).

A replication cycle of the Nathan Thompson replicator incorporates transient loss of one of two lines of structural symmetry during each replication cycle – a different kind of broken symmetry from the chirality observed in CA loop replication.

Figure 1 shows replication cycles from Time = 0 to 36, with one cycle completed by Time = 12. The live-state (state 1) configurations at Times 10 and 11 have lost the line of symmetry perpendicular to the line of symmetry along which replication propagates. The two perpendicular lines of symmetry are restored at Time = 12 when there are two completed instances of the initial Time = 0 replicator.

By Time = 10, it is unambiguous that there are two identical structures which separately develop into instances of the initial Time = 0 replicator by Time = 12. However, these two instances of the replicator are too close together to continue a direct development to four instances – the two child instances which otherwise could develop between the two parent instances established at Time = 12 mutually annihilate so that by Time = 24 there are still only two instances, but they are at this time sufficiently separated for successful replication to two children each by repeats of the replication dynamic shown in complete detail from Time = 0 to Time = 12.

Discussion

While dynamic behaviour in CA universes breaks T-symmetry (*i.e.*, is irreversible), observations of a narrow range of small, simple CA replicators indicate that replication requires additional corresponding broken symmetries – chiral asymmetries in the cases of replicating loops, or loss of structural lines of symmetry in cases of achiral replication.

The hypothesis that replication is not possible without breaking of symmetries is falsifiable by identification of replication without broken or lost symmetries. The author is not aware of any such examples (but note the discussion below about Game of Life (GoL) glider dynamics).

Interpretation of GoL glider dynamics as no-growth replication

John Conway's GoL and HighLife both support a structure of five live cells which advances in the grid by one cell as it iteratively cycles through four different structural configurations. This structure is well known to GoL enthusiasts as a *glider* [4].

As an alternative to recognizing glider dynamics as spatial translation, we might think of a glider walking across a GoL CA space as a *replication* in which production of one child corresponds to annihilation of the preceding parent structure, so replication with conservation of the number of replicator instances (*i.e.*, never exceeding one instance) is equivalent to spatial translation. The cycle of four iterations per one-cell translation comprises two chiral live-state configurations with mirror-complements, *i.e.*, both left- and complement right-handed chiral structures coexist in the cycle. Additionally, the GoL state-transition function is achiral, therefore there are no chiral asymmetries in the dynamics of a GoL glider. There are also no lines of structural symmetry in any of the four structural configurations per glider cycle, so the number of lines of structural symmetry is conserved at zero. Aside from being irreversible, there are no asymmetries in the dynamics of a GoL glider.

A general hypothesis

With consideration of the observations of CA loop replication and the HighLife achiral replicator, we can consider a general hypothesis:

A replication process generating an increasing population of replicators corresponds to one or more broken or lost symmetries in the replication cycle. The corresponding contrapositive form is that absence of asymmetries corresponds to no replication growth.

The absence of asymmetries in the four-step GoL glider cycle coincides with its spatial translation. If we alternatively interpret the spatial translation of the GoL glider as no-growth replication as discussed above, we can interpret glider dynamics as consistent with the contrapositive form of the hypothesis.

Relating replicator abstractions to prospective biological processes of the past

If there is a credible correspondence of simple CA dynamics to ancestral biological phenomena, what might cell states correspond to, and how might the spatial distribution of cell states be interpreted?

The simplicity of a small CA replicator may correspond to a small and simple proto-biological replicator, or to consider a wider range of possibilities, cell states may plausibly correspond to identifiably-distinct chemical networks, in which case a CA replicator may correspond to a more complex bio-dynamic system.

With this prospective correspondence in mind, the rigid spatial arrangement of CA cells in a neighbourhood may then map logically to a pattern of local interactions between chemical networks, with a pattern of spatial arrangement less rigid than indicated by the CA cell neighbourhoods. A CA state-transition rule would then indicate the dynamics of local interactions between the chemical networks corresponding to the CA cell neighbourhood specified in the rule.

As we have seen in this paper and past work, asymmetries corresponding to replication can be either achiral or chiral [9][10][11][12]. Given that we observe homochirality in contemporary extant biology, the chirality of CA loop replication [1][3][5] may be of particular relevance to thinking about the specific problem of the origin of handedness in biology.

I conclude here with acknowledgement of "the elephant in the room". It is obvious that replication of these CA structures is perfect, *i.e.*, there is no evolutionary development. The prospective correspondence between cell states and more-complex chemical networks may provide a way forward for integration of evolutionary change.

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