# A note on rings in which each element is a sum of two idempotents

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**ABSTRACT**

In this paper we consider a result on rings in which each element is a sum of two idempotents appeared in [1] and we improve the result by providing a counterexample.

**Key-words:** idempotent, Boolean ring.

**MSC 2020:** 16U40, 16E50

## Introduction

Rings in which each element is a sum of two idempotents have been studied in [1-2]. In this note we consider an important result appeared in [1] and we provide an important observation on this result. We improve this result by providing a counterexample.

As per [1, Proposition 6.1] the following are equivalent for a ring $R$.

(1) Every element of $R$ is a sum of two idempotents.

(2) $R \cong R_1 \times R_2$, here $ch(R_1) = 2$ and every element of $R_1$ is a sum of two idempotents, and $R_2$ is zero or a subdirect product of $Z_3$'s.

In this note each ring $R$ is a unital and associative ring. It may be noted that an element $a \in R$ is called idempotent if $a^2 = a$ and $R$ is called Boolean if $a^2 = a$ for each $a \in R$ [1-2].

In the next section we provide an example which serves as a counterexample for the above result of [1].

**2. Observation**

Let $R = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \right\}$ .

One may verify that $R$ is a commutative ring of characteristic three under addition and multiplication of matrices modulo three.

We note that

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \text{ etc.}$$

Thus each element of $R$ is a sum of two idempotents.

Let $R \cong R_1 \times R_2$. It may be noted that since $R$ is a ring of order nine and its characteristic is three and therefore the characteristic of $R_1$ can never be two.

Therefore this example serves as a counterexample for the above result of [1].

## References

[1] Zhiling Ying, Tamer Kosan, Yiqiang Zhou, Rings in which every element is a sum of two tripotents, Canad. Math. Bull., 59 (3), 661-672, 2016.

[2] Y. Hirano, H. Tominaga, Rings in which every element is the sum of two idempotents, Bull. Austral. Math. Soc., 37(2), 161-164), 1988.

## Statements and Declaration:

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