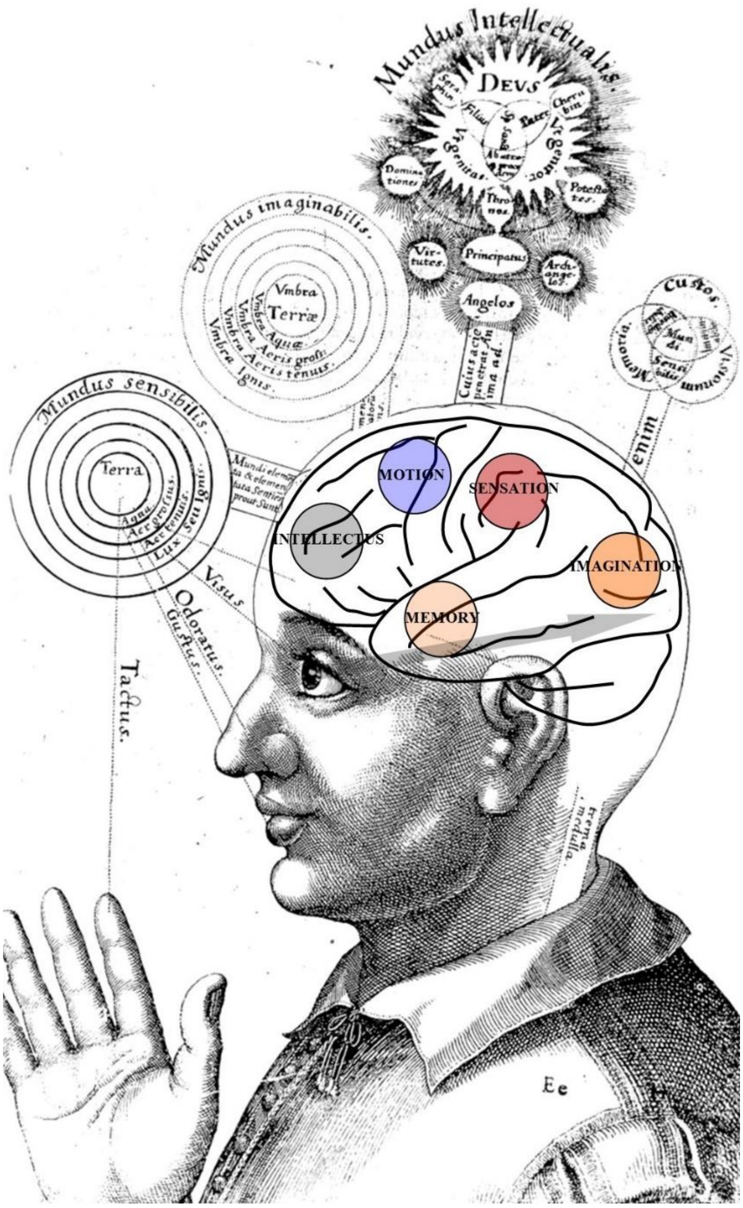


# TOPOLOGY OF THE BRAIN WAVES

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Novel methodological approaches to assess topological mappings and projections between biophysical phase spaces have been recently introduced in neuroscience. In this brief survey of our published manuscripts, we discuss how the “unreasonable power” of algebraic topology permits an experimentally testable top-down inquiry of the brain activity.

## THE BORSUK-ULAM THEOREM IN NEUROSCIENCE

Recently introduced versions of the Borsuk-Ulam theorem (BUT) reveal that a feature vector on a n-manifold projects two feature vectors (matching descriptions of a single object) onto an n+1 manifold. Starting from this rather simple, yet far-reaching, computational topology observation, we build a fruitful general framework, able to elucidate disparate "real" physical and biological phenomena, from quantum entanglement to gauge theories. Summarizing this novel topological approach, we consider projections among functional and/or real dimensions. We achieve a system of mappings that fit very well with experimental results, making it possible to assess countless systems in far-flung scientific branches. We highlight the computational character of matching descriptions (arising from descriptively proximal objects) that display the widest range of possible uses. Such observations point to BUT not just from the standpoint of a novel interpretation of almost all the biological and physical phenomena, but also as suitable tools in evaluating the slight (objective and subjective) differences that make our world an astonishing realm of rich heterogeneity.

*Tozzi A, Peters JF. 2017. The multidimensional world. Lambert Academic Publishing, Saarbrücken, Germany. ISBN-13: 978-3-330-03530-0.*

**Topodynamics of the metastable nervous activity: BUT tackles the nonlinear chaotic brain.**

The brain displays both the anatomical features of a vast amount of interconnected topological mappings as well as the functional features of a nonlinear, metastable system at the edge of chaos, equipped with a phase space where mental random walks tend towards lower energetic basins. Nevertheless, except for some advanced neuro-anatomic descriptions and present-day connectomic research, very few studies have been addressing the topological path of a brain embedded or embodied in its external and internal environment. Herein, by using new formal tools derived from algebraic topology, we provide an account of the metastable brain, based on the neuro-scientific model of Operational Architectonics of brain-mind functioning. We introduce a "topodynamic" description that shows how the relationships among the countless intertwined spatio-temporal levels of brain functioning can be assessed in terms of projections and mappings that take place on abstract structures, equipped with different dimensions, curvatures and energetic constraints. Such a topodynamical approach, apart from providing a biologically plausible model of brain function that can be operationalized, is also able to tackle the issue of a long-standing dichotomy: it throws indeed a bridge between the subjective, immediate datum of the naïve complex of sensations and mentations and the objective, quantitative, data extracted from experimental neuro-scientific procedures. Importantly, it opens the door to a series of new predictions and future directions of advancement for neuroscientific research.

*Tozzi A, Peters JF, Fingelkurts AA, Fingelkurts AA, Marijuán PC. 2017. Topodynamics of metastable brains. Physics of Life Reviews, 21, 1-20. http://dx.doi.org/10.1016/j.plrev.2017.03.001.*

**BUT as a general principle of nervous symmetry breaks.**

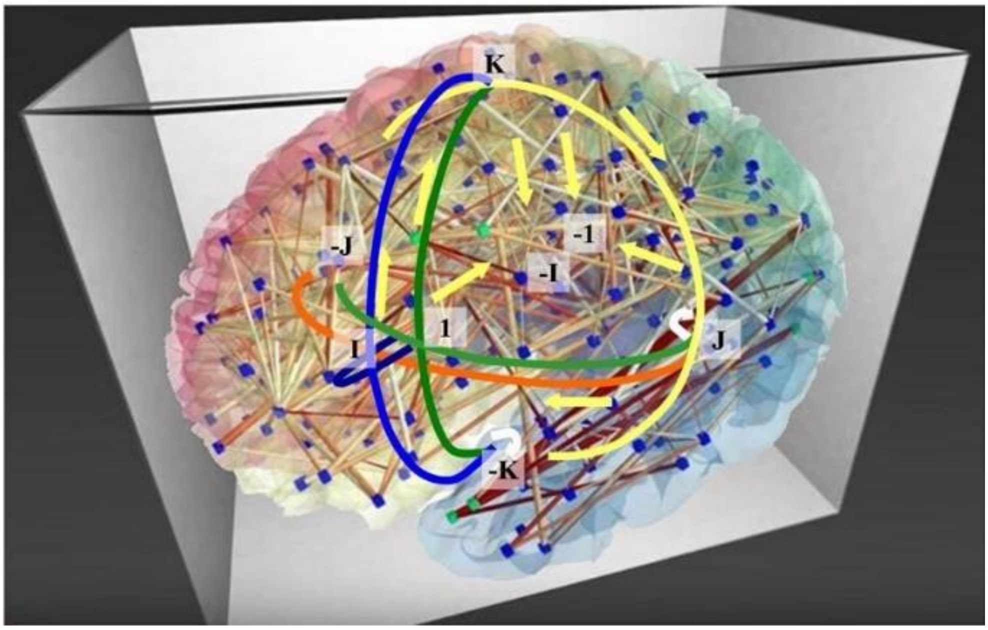
Symmetries are widespread invariances underlining countless systems. A symmetry break occurs when the symmetry is present at one level of observation, but "hidden" at another level. In such a general framework, a concept from algebraic topology, namely BUT, comes into play and sheds new light on the general mechanisms of nervous symmetries. BUT tells us that we can find, on an n-dimensional sphere, a pair of opposite points that have same encoding on an n-1 sphere. This mapping makes it possible to describe both antipodal points with a single real-valued vector on a lower dimensional sphere. Here we argue that this topological approach is useful in the evaluation of hidden biophysical symmetries. This means that symmetries can be found when evaluating the system in a proper dimension, while they disappear (are hidden or broken) when we evaluate the same system in just one dimension lower.

*Tozzi A, Peters JF. 2016. A Topological Approach Unveils System Invariances and Broken Symmetries in the Brain. Journal of Neuroscience Research 94 (5): 351–65. doi:10.1002/jnr.23720.*

**The brain activity takes place in higher dimensions: not just a figure of speech!**

Brain activity takes place in three spatial-plus time dimensions. This rather obvious claim has been recently questioned by papers that, considering the big data outburst and novel available computational tools, are starting to unveil a more intricate state of affairs. Indeed, various brain activities and their correlated mental functions can be assessed in terms of trajectories embedded in phase spaces of dimensions higher than the canonical ones. Further dimensions may not just represent a convenient methodological tool that allows a better mathematical treatment of otherwise elusive cortical activities, rather may also reflect genuine functional or anatomical relationships among real nervous functions. I describe how to extract hidden multidimensional information from real or artificial neurodata series, and make clear how our mind dilutes, rather than concentrates as currently believed, inputs coming from the environment. Finally, I argue that the principle "the higher the dimension, the greater the information" may explain the occurrence of mental activities and elucidate the mechanisms of human diseases associated with dimensionality reduction.

*Tozzi A, Peters JF. 2016. Towards a Fourth Spatial Dimension of Brain Activity. Cognitive Neurodynamics 10 (3): 189–199. doi:10.1007/s11571-016-9379-z.*




**Neural thermodynamics and information: a topological account of the brain entropies.**

Recent approaches to brain phase spaces reinforce the foremost role of symmetries and energy requirements in the assessment of nervous activity. Changes in thermodynamic parameters and dimensions occur in the brain during symmetry breakings and transitions from one functional state to another. Based on topological results and string-like trajectories into nervous energy landscapes, we provide a novel method for the evaluation of energetic features and constraints in different brain functional activities. We show how abstract approaches, namely BUT and its variants, may display real, energetic physical counterparts. When topology meets the physics of the brain, we arrive at a general model of neuronal activity, in terms of multidimensional manifolds and computational geometry, that has the potential to be operationalized.

*Tozzi A, Peters JF. 2017. From abstract topology to real thermodynamic brain activity. Cognitive Neurodynamics, 11(3) 283–292. Doi:10.1007/s11571-017-9431-7.*

**Evolutional architecture of multidimensional visual perception: the winner takes all.**

A novel demon-based architecture is introduced to elucidate brain functions such as pattern recognition during human perception and mental interpretation of visual scenes. Starting from the topological concepts of invariance and persistence, we introduce a Selfridge pandemonium variant of brain activity that takes into account a novel feature, namely, demons that recognize short straight-line segments, curved lines and scene shapes, such as shape interior, density and texture. Low-level representations of objects can be mapped to higher-level views (our mental interpretations): a series of transformations can be gradually applied to a pattern in a visual scene, without affecting its invariant properties. This makes it possible to construct a symbolic multi-dimensional representation of the environment. These representations can be projected continuously to an object that we have seen and continue to see, thanks to the mapping from shapes in our memory to shapes in Euclidean space. Although perceived shapes are 3-dimensional (plus time), the evaluation of shape features (volume, colour, contour, closeness, texture, and so on) leads to n-dimensional brain landscapes. Here we discuss the advantages of our parallel, hierarchical model in pattern recognition, computer vision and biological nervous system's evolution.

*Tozzi A, Peters JF. 2018. Multidimensional brain activity dictated by winner-take-all mechanisms. Neuroscience Letters, 678 (21):83-89. https://doi.org/10.1016/j.neulet.2018.05.014.*

**A neuroscientific account of (lower-dimensional) syntax and (higher-dimensional) semantics suggests that the Wittgenstein's Tractatus was right.**

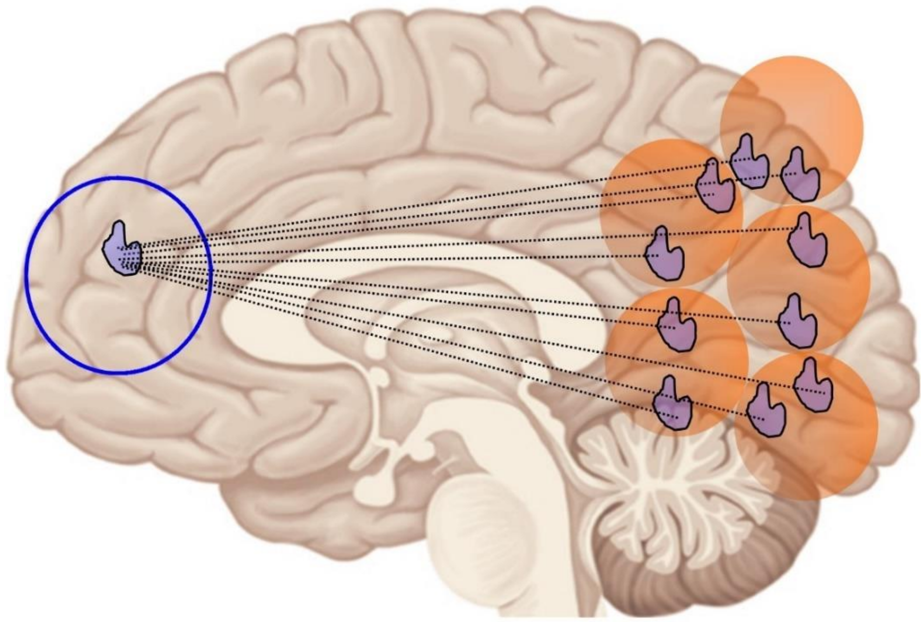
The discrepancy between syntax and semantics is a painstaking issue that hinders a better comprehension of the underlying neuronal processes in the human brain. To tackle the issue, we at first describe a striking correlation between Wittgenstein's Tractatus, that assesses the syntactic relationships between language and world, and Perlovsky's joint language-cognitive computational model, that assesses the semantic relationships between emotions and "knowledge instinct". Once established a correlation between a purely logical approach to the language and computable psychological activities, we aim to find the neural correlates of syntax and semantics in the human brain. Starting from topological arguments, we suggest that the semantic properties of a proposition are processed in higher brain's functional dimensions than the syntactic ones. In a fully reversible process, the syntactic elements embedded in Broca's area project into multiple scattered semantic cortical zones. The presence of higher functional dimensions gives rise to the increase in informational content that takes place in semantic expressions. Therefore, diverse features of human language and cognitive world can be assessed in terms of both the logic armor described by the Tractatus, and the neurocomputational techniques at hand. One of our motivations is to build a neuro-computational framework able to provide a feasible explanation for brain's semantic processing, in preparation for novel computers with nodes built into higher dimensions.

*Tozzi A, Peters JF, Fingelkurts A, Fingelkurts A, Perlovsky L. 2018. Syntax meets semantics during brain logical computations. Progr Biophys Mol Biol, 140: 133-141. https://doi.org/10.1016/j.pbiomolbio.2018.05.010.*

**A topologically framed, multidimensional theory of knowledge. Starring: Richard Avenarius, Ernst Mach and their phenomenal account of the "mental".**

Experience is a process of awareness and mastery of facts or events, gained through actual observation or second-hand knowledge. Recent findings reinforce the idea that a naturalized epistemological approach is needed to further advance our understanding of the nervous mechanisms underlying experience. This essay is an effort to build a coherent topological-based framework able to elucidate aspects of experience, e.g., how it is acquired by a single individual, transmitted to others and collectively stored in form of general ideas. Considering concepts from neuroscience, algebraic topology and Richard Avenarius' philosophical analytical approach, we provide a scheme which is cast in an empirically testable fashion. We emphasize the foremost role of variants of BUT, which tells us that, when a pair of opposite (antipodal) points on a sphere are mapped onto a single point in Euclidean space, the projection provides a description of both antipodal points. These antipodes stand for nervous functions and activities of the brain correlated with the mechanisms of acquisition and transmission of experience.

*Tozzi A, Peters JF. 2017. Towards Topological Mechanisms Underlying Experience Acquisition and Transmission in the Human Brain. Integr Psychol Behav Sci. 51(2), 303–323. doi: 10.1007/s12124-017-9380-z.*



**Multisensory integration & BUT: a Machian complex of sensations between visual and auditory cues.**

Recent advances in neuronal multisensory integration suggest that the five senses do not exist in isolation of each other. Perception, cognition and action are integrated at very early levels of central processing, in a densely coupled system equipped with multisensory interactions occurring at all temporal and spatial stages. In such a novel framework, a concept from the far-flung branch of topology, namely BUT, comes into play. The theorem states that when two opposite points on a sphere are projected onto a circumference, they give rise to a single point containing their matching description. Here we show that the theorem applies also to multisensory integration: two environmental stimuli from different sensory modalities display similar features when mapped into cortical neurons. Topological tools not only shed new light on questions concerning the functional architecture of mind and the nature of mental states, but also provide an empirically assessable methodology. We argue that BUT is a general principle underlying nervous multisensory integration, resulting in a framework that has the potential to be operationalized. Going beyond Ernst Mach, we suggest the occurrence of complexes of multi-sensations.

*Tozzi A, Peters JF. 2017. A Symmetric Approach Elucidates Multisensory Information Integration. Information 8,1. doi: 10.3390/info8010004.*

## TOPOLOGICAL HOLES IN THE BRAIN

The brain, rather than being homogeneous, displays an almost infinite topological genus, since it is punctured with a high number of "cavities". We might think to the brain as a sponge equipped with countless, uniformly placed, holes. Here we show how these holes, termed topological vortexes, stand for nesting, non-concentric brain signal cycles resulting from the activity of inhibitory neurons. Such inhibitory spike activity is inversely correlated with its counterpart, i.e., the excitatory spike activity propagating throughout the whole brain tissue. We illustrate how Pascal's triangles and linear and nonlinear arithmetic octahedrons can describe the three-dimensional random walks generated by the inhbitory/excitatory activity of the nervous tissue. In case of nonlinear 3D paths, the trajectories of excitatory spiking oscillations can be depicted as the operation of filling the numbers of octahedrons in the form of "islands of numbers": this leads to excitatory neuronal assemblies, spaced out by empty areas of inhibitory neuronal assemblies. These mathematical procedures allow us to describe the topology of a brain of infinite genus, to represent inhibitory neurons in terms of Betti numbers and to highlight how spike diffusion in neural tissues is generated by the random activation of tiny groups of excitatory neurons. Our approach suggests the existence of a strong mathematical background subtending the intricate oscillatory activity of the central nervous system.

*Tozzi A, Yurkin A, Peters JF. 2021. A Geometric Milieu Inside the Brain. Found Sci. https://doi.org/10.1007/s10699-021-09798-x.*

**The occurrence of Betti numbers in neurodata.**

Spatio-temporal brain activities with variable delay detectable in resting-state functional magnetic resonance imaging (rs-fMRI) give rise to highly reproducible structures, termed cortical lag threads, that can propagate from one brain region to another. Using a computational topology of data approach, we found that Betti numbers that are cycle counts and the areas of vortex cycles covering brain activation regions in triangulated rs-fMRI video frames make it possible to track persistent, recurring blood oxygen level dependent (BOLD) signals. Our findings have been codified and visualized in what are known as persistent barcodes. Importantly, a topology of data offers a practical approach in coping with and sidestepping massive noise in neuro data, such as unwanted dark (low intensity) regions in the neighbourhood of non-zero BOLD signals. A natural outcome of a topology of data approach is the tracking of persistent, non-trivial BOLD signals that appear intermittently in a sequence of rs-fMRI video frames. The result of this tracking of changing lag structures is a persistent barcode, which is a pictograph that offers a convenient visual means of exhibiting, comparing and classifying brain activation patterns.

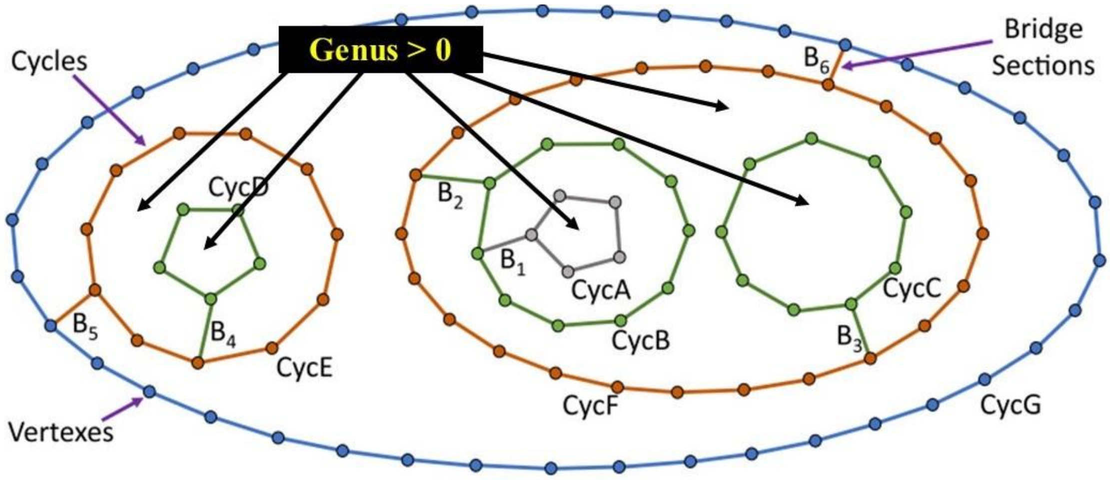
*Don AP, Peters JF, Ramanna S, Tozzi A. 2020. Topological View of Flows inside the BOLD Spontaneous Activity of the Human Brain. Front. Comput. Neurosci. DOI: 10.3389/fncom.2020.00034.*

**Cell complexes & vortex shapes in BOLD signals**.

Quaternionic views of multi-level brain activation region intensities in resting-state functional magnetic resonance imaging (rs-fMRI) videos are introduced. Quaternions make it possible to explore rs-fMRI brain activation regions in a

4D space in which there are varying brain activation intensities in spiraling activation cycles (each with its own intensity). As a result, there is a natural formation of multi-level cycles that form pyramidal vortex shapes with varying diameters. These pyramidal vortexes reflect the fractality (self-similarity) of clusters of similar multilevel brain activation region cycles. Using a computational topology of data approach, we have found that persistent, recurring clusters of spiraling cycles resulting from blood oxygen level dependent (BOLD) signals in triangulated rs-fMRI video frames. Each brain activation region cycle is a cell complex, which is a collection path-connected vertexes that has no end vertex. Measurement of persistence of spiraling vortex shapes in BOLD signal propagation regions is carried out in terms of Betti numbers (counts of distinguished cycle vertexes called generators) that rise and fall over time during spontaneous activity of the brain. A main result given here is that every quaternionic brain activation region vortex has a free group presentation. In addition, we introduce 3D barcodes of brain activation videos that help visualize and quantify the fractality of clusters of multilevel vortexes arising naturally from triangulated brain activation regions in rs-fMRI video frames. We have made freely available downloadable archives of videos that exhibit the resulting clusters of spiraling brain activation cycles.

*Don APH, Peters JF, Ramanna S, Tozzi A. 2021. Quaternionic views of rs-fMRI hierarchical brain activation regions. Discovery of multilevel brain activation region intensities in rs-fMRI video frames. Chaos, Solitons & Fractals. Vol 152, 111351. https://doi.org/10.1016/j.chaos.2021.111351.*




**Detection of topological holes in cognitive processes**.

Neuroscientists draw lines of separation among structures and functions that they judge different, arbitrarily excluding or including issues in our description, to achieve positive demarcations that permits a pragmatic treatment of the nervous activity based on regularity and uniformity. However, uncertainty due to disconnectedness, lack of information and absence of objects' sharp boundaries is a troubling issue that prevents these scientists to select the required proper sets/subsets during their experimental assessment of natural and artificial neural networks. Starting from the detection of metamorphoses of shapes inside a Euclidean manifold, we propose a technique to detect the topological changes that occur during their reciprocal interactions and shape morphing. This method, that allows the detection of topological holes development and disappearance, makes it possible to solve the problem of uncertainty in the assessment of countless dynamical phenomena, such as cognitive processes, protein homeostasis deterioration, fire propagation, wireless sensor networks, migration flows, and cosmic bodies analysis.

*Tozzi A, Peters JF. 2020 Removing uncertainty in neural networks. Cognitive Neurodynamics, 14:339–345. https://doi.org/10.1007/s11571-020-09574-w.*

## OTHER TOPOLOGICAL THEOREMS: APPLICATIONS IN NEUROSCIENCE

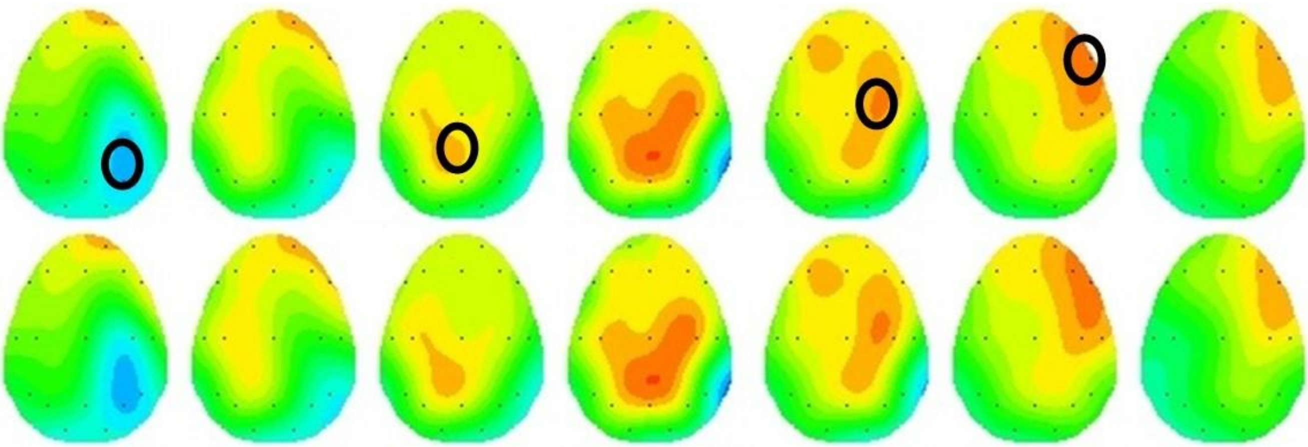
Observable facts involving changes in system shape, dimension and symmetry may elude simple cause and effect inductive explanations. We argue that numerous physical and biological phenomena such as chaotic dynamics, symmetry breaking, long-range collisionless neural interactions, zero-valued energy singularities, and particle/wave duality can be accounted for in terms of purely topological mechanisms devoid of causality. We illustrate how simple topological theorems, seemingly far away from scientific inquiry (e.g., "given at least some wind on Earth, there must at all times be a cyclone or anticyclone somewhere"; "if one stirs to dissolve a lump of sugar in a cup of coffee, it appears there is always a point without motion"; "at any moment, there is always a pair of antipodal points on the Earth's surface with equal temperatures and barometric pressures") reflect the action of non-causal topological rules. To do so, we introduce some fundamental topological tools and illustrate how phenomena such as double slit experiments, cellular mechanisms and some aspects of brain function can be explained in terms of geometric projections and mappings, rather than local physical effects. We conclude that unavoidable, passive, spontaneous topological modifications may lead to novel functional biophysical features, independent of exerted physical forces, thermodynamic constraints, temporal correlations and probabilistic a priori knowledge of previous cases.

*Tozzi A, Papo D. 2020. Projective mechanisms subtending real world phenomena wipe away cause effect relationships. Progress in Biophysics and Molecular Biology. 151:1-13. DOI: 10.1016/j.pbiomolbio.2019.12.002.*

**The hairy ball theorem and the electric wave fronts in the brain.**

Whenever one attempts to comb a hairy ball flat, there will always be at least one tuft of hair at one point on the ball. This seemingly worthless sentence is an informal description of the hairy ball theorem, an invaluable mathematical weapon that has been proven useful to describe a variety of physical/biological processes/phenomena in terms of topology, rather than classical cause/effect relationships. In this paper we will focus on the electrical brain field—electroencephalogram (EEG). As a starting point we consider the recently raised observation that, when electromagnetic oscillations propagate with a spherical wave front, there must be at least one point of the tangential components of the vector fields where the electromagnetic field vanishes. We show how this description holds also for the electric waves produced by the brain and detectable by EEG. Once located these zero-points in EEG traces, we confirm that they can modify the electric wave fronts detectable in the brain. This sheds new light on the functional features of a nonlinear, metastable nervous system at the edge of chaos, based on the neuroscientific model of Operational Architectonics of brain-mind functioning. As an example of practical application of this theorem, we provide testable previsions, suggesting the proper location of transcranial magnetic stimulation's coils to improve the clinical outcomes of drug-resistant epilepsy.

*Tozzi A, Bormashenko E, Jausovec N. 2021. Topology of EEG wave fronts. Cognit Neurodyn, 15, 887–896 https://doi.org/10.1007/s11571-021-09668-z.*



**The Lusternik–Schnirelmann theorem describes the topological horizons of the Gibson's ecological theory of vision.**

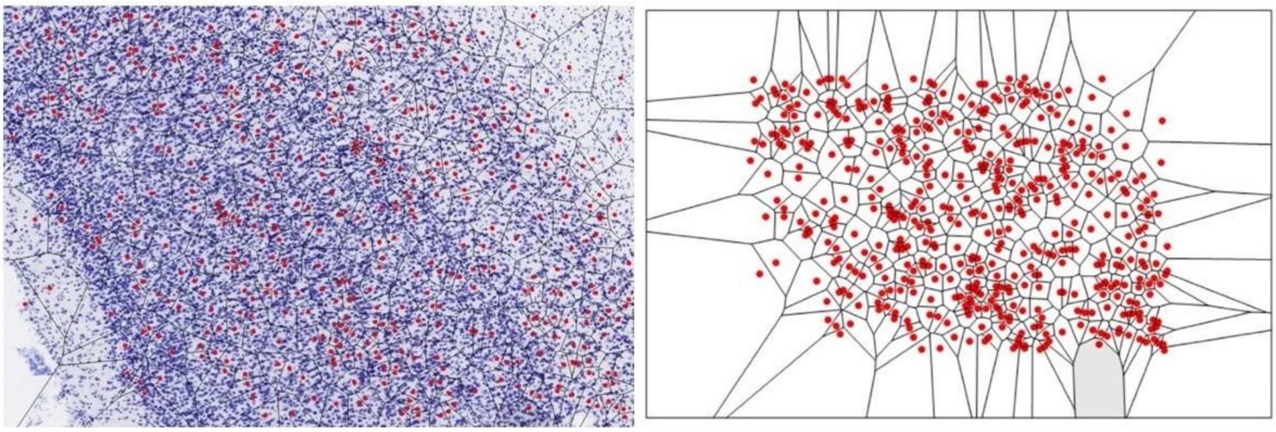
During the exploration of the surrounding environment, the brain links together external inputs, giving rise to perception of a persisting object. During imaginative processes, the same object can be recalled in mind even if it is out of sight. Here, topological theory of shape provides a mathematical foundation for the notion of persistence perception. We focus on ecological theories of perception, that account for our knowledge of world objects by borrowing a concept of invariance in topology. We show how a series of transformations can be gradually applied to a pattern (e.g., the shape of an object) without affecting its invariant properties such as boundedness of parts of a visual scene. High-level representations of objects in our environment are mapped to simplified views (our interpretations) of the objects, to construct a symbolic representation of the environment. The representations can be projected continuously to an environmental object that we have seen and continue to see, thanks to the mapping from shapes in our memory to shapes in Euclidean space.

*Tozzi A, Peters JF. 2019. Topology of Human Perception. Preprints, 2019030235.*

**Voronoi tessellations in histological brain tissues: every neuron is different from another.**

We provide a novel, fast and cheap method for the morphological evaluation of simple 2-D images taken from histological samples. This method, based on computational geometry, leads to a novel kind of "tessellation" of every type of biological picture, in order to locate the zones equipped with the highest functional activity. As an example, we apply the technique to the evaluation of histological images from brain sections and demonstrate that the cortical layers, rather than being a canonical assembly of homogeneous cells as usually believed, display scattered neuronal micro-clusters equipped with higher activity than the surrounding ones.

*Peters JF, Tozzi A. Ramanna S. 2016. Brain Tissue Tessellation Shows Absence of Canonical Microcircuits. Neuroscience Letters 626: 99–105. doi:10.1016/j.neulet.2016.03.052.*



**Further proofs of hidden nervous dimensions. Introducing 4D maximal nucleus cluster from algebraic topology of digital images.**

We introduce a novel method for the measurement of information level in fMRI (functional Magnetic Resonance Imaging) neural data sets, based on image subdivision in small polygons equipped with different entropic content. We show how this method, called maximal nucleus clustering (MNC), is a novel, fast and inexpensive image-analysis technique, independent from the standard blood-oxygen-level dependent signals. MNC facilitates the objective detection of hidden temporal patterns of entropy/information in zones of fMRI images generally not considered by the subjective standpoint of the observer. This approach befits the geometric character of fMRIs. The main purpose of this study is to provide a computable framework for fMRI that not only facilitates analyses, but also provides an easily decipherable visualization of structures. This framework commands attention because it is easily implemented using conventional software systems. To evaluate the potential applications of MNC, we looked for the presence of a fourth dimension's distinctive hallmarks in a temporal sequence of 2D images taken during spontaneous brain activity. Indeed, recent findings suggest that several brain activities, such as mind-wandering and memory retrieval, might take place in the functional space of a four-dimensional hypersphere, which is a double donut-like structure undetectable in the usual three dimensions. We found that the Rényi entropy is higher in MNC areas than in the surrounding ones, and that these temporal patterns closely resemble the trajectories predicted by the possible presence of a hypersphere in the brain.

*Peters JF, Ramanna S, Tozzi A, İnan E. 2017. Bold-Independent Computational Entropy Assesses Functional Donut-Like Structures in Brain fMRI Images. Front Hum Neurosci. 2017 Feb 1;11:38. doi: 10.3389/fnhum.2017.00038. eCollection 2017.*

**Topological manifolds and duality of the brain functions.**

The term "brain activity" refers to a wide range of mental faculties that can be assessed either on anatomical/functional micro-, meso- and macro- spatiotemporal scales of observation, or on intertwined cortical levels with mutual interactions. Our aim is to show that every brain activity encompassed in a given anatomical or functional level necessarily displays a counterpart in others, i.e., they are "dual". "Duality" refers to the case where two seemingly different physical systems turn out to be equivalent. We describe a method, based on novel topological findings, that makes different manifolds (standing for different brain activities) able to scatter, collide and combine, in order that they merge, condense and stitch together in a quantifiable way. We develop a computational tool which explains how, despite their local cortical functional differences, all mental processes, from perception to emotions, from cognition to mind wandering, may be reduced to a single, general brain activity that takes place in dimensions higher than the classical three-dimensional plus time. This framework permits a topological duality among different brain activities and neuro-techniques since it holds for all the types of spatio-temporal nervous functions, independent of their cortical location, inter- and intra-level relationships, strength, magnitude and boundaries.

*Tozzi A, Peters JF. 2019. The common features of different brain activities. Neurosci Lett, 692 (23): 41-46. https://doi.org/10.1016/j.neulet.2018.10.054.*