

# Literature Review of recent advancements in Hypergraph Learning as it relates to optimizer

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## **Abstract**

Hypergraphs are a generalization of a graph in which an edge can join any number of vertices. In contrast[1], in an ordinary graph, an edge connects exactly two vertices. The applications of hypergraphs can range from analogical explanations such as social networks to hard generalities in the case of collaborative game theory where they are known as simple games. The more abstract applications can be used in localized and global optimizations of radial function under computational geometry , and the optimizers generated could also be used to solve linear scheduling problems.[2] The theoretical approach developed under these categories can be used in embedding . clustering and classification which can be solved through the application of spectral hypergraph clustering too.[3]

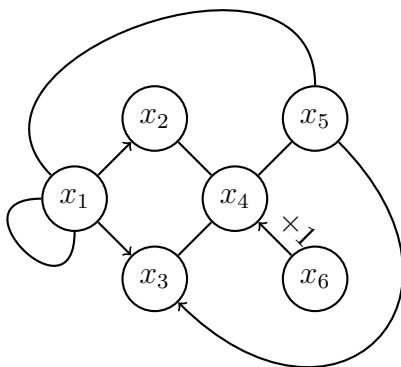
## **1 Introduction**

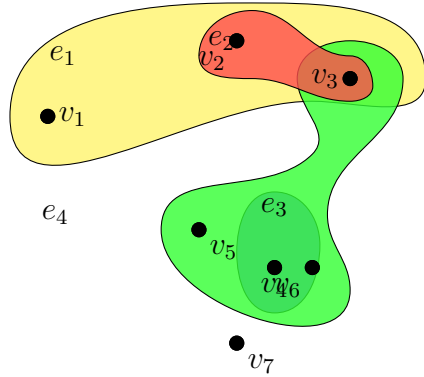
Hypergraphs constitute a set of multi-variate relations among discrete in a simple range system when considering computational geometry, on the other

hand when talking about levi systems we can deploy laplace transformations of the differential equation which is obtained by a linear transformation of a cooperative game theory object object which is called a simple game.[4]

Generally when talking about machine learning problems we arrange systems in accordance to the dual relation structure of independent objects or features depending on the problem statement. When considering this we also take in the assumption of the finite set structure for optimization and loss local minimization [5], while the set is also a part of a Euclidean space associated with the kernel matrix is also a general example of bi-linear optimization with bi linear gradiential learning.

However, despite the easy general approach indicated above might and does work pretty well for simple datasets pointing to euclidean system with linearly independent relations, they do demonstrate lesser accuracy in more complex real world problems which may or may not have linear dependencies. representing a set of complex relational objects as undirected or directed graphs is not complete. For illustrating this point of view[4], let us consider a problem of grouping a collection of articles into different topics. Given an article, assume the only information that we have is who wrote this article. One may construct an undirected graph in which two vertices are joined together by an edge if there is at least one common author of their corresponding articles , and then an undirected graph based clustering approach is applied, e.g. spectral graph techniques.[6]





## 2 Algorithms

The Algorithmics for this learning algorithm is a derivation from various Hypergraph theorems and their application to back propagation based ANNs through[4] , analytical differentiation of the case values and extending the set of binary independent relationships between objects (features) of the system(dataset) , to non euclidean multi-collinearity between the new objects.

### 2.1 Hall-Type Theorems

Hall type theorems are essentially generalizations of the Hall-Marriage theorems to hypergraph systems. Hall's marriage theorem provides a condition guaranteeing that a bipartite graph  $(X+Y, E)$  admits a perfect matching, or - more generally - a matching that saturates all vertices of  $Y$ . The condition involves the number of neighbors of subsets of  $Y$ . [4] Generalizing Hall's theorem to hypergraphs requires a generalization of the concepts of bipartiteness, perfect matching, and neighbors.

#### 2.1.1 Bipartiteness

Bipartiteness is the condition that let's say a hypergraph  $(H,E) = \{1,3,5,7,...\}$  Here  $E = \{1,3,..\} , \{1,5,..\} , \{3,7,..\}$  is bipartite with  $X = \{1,2\}$   $Y = \{2,3.. \}$  which is multi-linearly distributed over the bi linear relation known as A1.

$$f: H \rightarrow V, V = \{1, 2, 3, \dots\}, \{1, 2, \dots\}, \{4, 5, \dots\}, \{n - 1, n, \dots\}$$

$$f: H_1 \rightarrow V_1, V_1 = \{1, 2, \dots\}, \{1, 3, \dots\}, \{4, 7, \dots\} \{3, 5, 7\}$$

$$f_n: H_n \rightarrow V_n, V_{(n-1)} = \{n+1, n+2, N+3, \dots\}, \{n+1, n+2, \dots\}, \{n+4, n+5, \dots\}, \{n, n-2, \dots\}$$

$$\begin{aligned} & \bigcup_{i=1}^{\infty} V_i = V_{(i-1)} \bigcup E_{(i_1)} \\ & = \bigcup_{i=1}^{\infty} V_i = V_{(i-1)} \bigcup E_{(i_2)} \bigcup Z_{(i_1)} \\ & = \bigcup_{i=1}^n V_i = V_{(i-1)} \bigcap E_{(i)} \\ & = \bigcup_{i=1}^{n-1} V_i = V_{(i-1)} \bigcap E_{(i_1)} \bigcap Z_{(i)} \end{aligned}$$

### 2.1.2 Perfect Matching

matching in hypergraph systems let's say  $V=(2,4)$  constructing a  $H=(1,3)$  bipartite relation between two non-euclidean objects say  $X$  and  $Y$ [6]. matching can be described as schedule and size optimization of the modulated by  $X$  and  $Y$  which appears on the hyper vertex of edge  $M$ . which can be considered a  $Y$ -perfect matching definition.[5]

$$f(H_i) \bigcap_{n=1}^{\infty} F(V_i) = \sum_{n=1}^{\infty} n^{-v_i}$$

$$f(H_i) \bigcap_{n=1}^{\infty} F(V_i) = \sum_{i=1}^{\infty} n^{-v_i} \sum_{j=1}^{\infty} n^{v_j}$$

$$f(H_i \bigcap_{n=1}^{\infty} V_i) = \sum_{i=1}^{\infty} n^{-v_i} \sum_{j=1}^{\infty} n^{v(j-1)}$$

$$\nu(H_H(Y_0)) \geq (r - 1) \cdot (|Y_0| - 1)$$

$$\nu^*(H_H(Y_0)) \geq (r - 1) \cdot (|Y_0| - 1) + 1$$

$$\nu(N_H(Y_0)) \geq (r - 1) \cdot (|Y_0| - 1) + 1$$

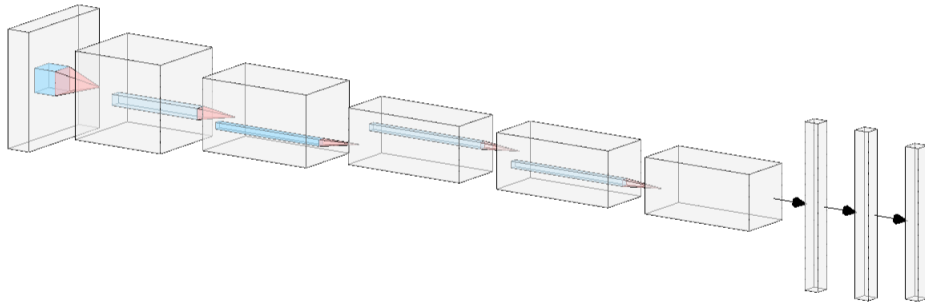
$$\tau(H_H(Y_0)) \geq (2r - 3) \cdot (|Y_0| - 1)$$

$$\tau(H_H(Y_0)) \geq (2r - 3 + \epsilon) \cdot (|Y_0| - 1) + 1$$

$$mw(N_H(Y_0)) \geq |Y_0| \text{ or } w(N_H(Y_0)) \geq 2|Y_0| - 1 \text{ or } w(N_H(Y_0)) \geq 2|Y_0| - 1$$

$$mw(N_H(Y_0)) \geq |Y_0|$$

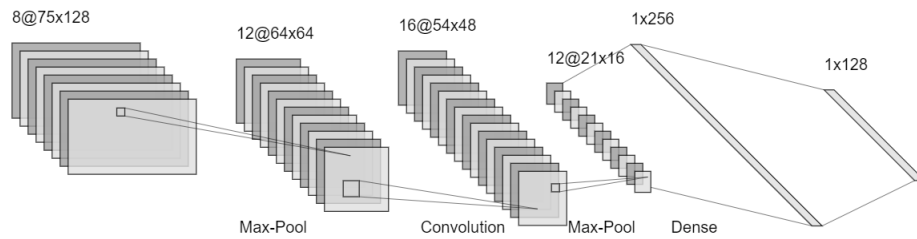
Figure 1: Alexnet Rep of The NN plot  $(\sin(x), \cos(x), x)$



### 3 Neural Network Topology

The below is 3-d representation developed under the category of Hall Hypergraphed NNs.

Figure 2: LeNet representation



Model	Accuracy Score	f1-score	Loss(Sparse)
Alexnet	0.903	0.77	0.0981
U-Net	0.87	0.73	0.105
CS-Net	0.88	0.82	0.107
YOLO	0.83	0.79	0.119
HypergraphNN	0.86	0.81	0.097

Table 1: Accuracy and loss function table

## 4 Evaluation

The evaluation of the algorithm is done with general metrics we find out that p-score and accuracy scores as well as different loss functions as the output may or may not be binary in nature [1], looking from this direction we realize that the hypergraph algorithm although may not show astute results in the field of medical imaging but they can be incredibly effective in spatial transformation for generating non euclidean GAN objects.[4]

## 5 Conclusion

In this paper, we tried to review the recent developments in the application of hypergraph theory to computer vision and object detection oriented neural networks.

## References

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