Bayes Theorem

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Abstract A simple explanation of the use of Bayes Theorem with AIDS testing results.

P(H) is the probability of not having AIDS. It's used to calculate the probability of having AIDS.
P(H) = .999 => P(A) = .001 Sample Size 100,000 with #H = 99,900 and #A = 100

Our test will produce a negative result for people without AIDS 98% of the time. This allows us to calculate the percentage of positive and negative test results in people with AIDS. It also allows us to calculate the percentage of false positive results in people without AIDS.

P(- | H) = .98 => P(- | A) = .02, P(+ | A) = .98, P(+ | H) = .02

This allows us to calculate the number of persons in each of the four categories and the percentage of persons getting a positive and a negative test result, which is necessary to apply Bayes Theorem.

#(- | H) = (99,900)(.98) = 97,902 => #(+ | H) = 1,998,
#(-) = #(- | H) + #(- | A) = 97,902 + 2 = 97,904 P(-) = .97904 => P(+) = .02096

We calculate the percentage of people having AIDS, who get a positive test result in two different ways: (1) using the number of persons with AIDS and without AIDS, who get a positive test result and (2) using Bayes Theorem. The percentages agree.

P(A | +) = (#(+ | A) / (#(+ | A) + #(+ | H)) 98 / (98 + 1998) = .046756
P(A | +) = P(+ | A) * P(A) / P(+) (.98)(.001) / (.02096) = .046756

We calculate the percentage of people without AIDS, who get a positive test result in two different ways: (1) using the number of persons with AIDS and without AIDS, who get a positive test result and (2) using Bayes Theorem. Note that P(A | +) + P(H | +) = 100%

P(H | +) = (#(+ | H) / (#(+ | A) + #(+ | H)) 1,998 / (98 + 1998) = .953244
P(H | +) = P(+ | H) * P(H) / P(+) (.02)(.999) / (.02096) = .953244

We calculate the percentage of people having AIDS, who get a negative test result in two different ways: (1) using the number of persons with AIDS and without AIDS, who get a negative test result and (2) using Bayes Theorem. The percentages agree.

P(A | -) = (#(- | A) / (#(- | A) + #(- | H)) 2 / (2 + 97,902) = .00002
P(A | -) = P(- | A) * P(A) / P(-) (.02)(.001) / (.97904) = .00002

We calculate the percentage of people without AIDS, who get a negative test result in two different ways: (1) using the number of persons with AIDS and without AIDS, who get a negative test result and (2) using Bayes Theorem. Note that P(A | -) + P(H | -) = 100%

P(H | -) = (#(- | H) / (#(- | A) + #(- | H)) 97,902 / (2 + 97,902) = .99998
P(H | -) = P(- | H) * P(H) / P(-) (.98)(.999) / .97904 = .99998