

Exact mathematical formula that connect 6 dimensionless physical constants

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Abstract

In this paper we will present a new exact formula for the fine-structure constant α in terms of the golden angle, the relativity factor and the fifth power of the golden mean. A new interpretation and a very accurate value of the fine-structure constant has been discovered in terms of the golden ratio. We propose the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers and two other exact mathematical expressions for the proton to electron mass ratio. We present the exact mathematical expressions that connect the proton to electron mass ratio and the fine-structure constant. Also we will find a new formula for the Planck length and a new formula for the Avogadro number. Eleven exact mathematical formulas that connect six dimensionless physical constants. The six dimensionless physical constants are the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A , the gravitational coupling constant α_G for the electron and the gravitational coupling constant $\alpha_{G(p)}$ of the proton. Finally we will find a new formula for the gravitational constant G .

Keywords

Fine-structure constant, Proton to electron mass ratio, Dimensionless physical constants, Gravitational constant, Avogadro's number, Gravitational constant, Ratio of electric force to gravitational force, Gravitational coupling constant

1. Introduction

In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

In the 1.920s and 1.930s, Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories, later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas, seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron, muon and tau has been discovered by physicist Yoshio Koide, but this formula remains unexplained.

Dimensionless physical constants cannot be derived and have to be measured. Developments in physics may lead to either a reduction or an extension of their number: discovery of new particles, or new relationships between physical phenomena, would introduce new constants, while the development of a more fundamental theory might allow the derivation of several constants from a more fundamental constant. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure, along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters.

2. Fine-structure constant

One of the most important numbers in physics is the fine-structure constant α which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass, length, time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why α itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. The fine-structure constant α is defined as:

$$\alpha = \frac{q_e^2}{4 \cdot \pi \cdot \epsilon_0 \cdot \hbar \cdot c}$$

The 2.018 CODATA recommended value of α is:

$$\alpha = 0.0072973525693(11)$$

With standard uncertainty $0,000000011 \times 10^{-3}$ and relative standard uncertainty $1,5 \times 10^{-10}$. For reasons of convenience, historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1} = 137,035999084(21)$$

With standard uncertainty $0,000000021 \times 10^{-3}$ and relative standard uncertainty $1,5 \times 10^{-10}$. There is general agreement for the value of α , as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of α obtained experimentally (as of 2.012) is based on a measurement of g using a one-electron so-called "quantum cyclotron" apparatus, together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

$$\alpha^{-1} = 137,035999174(35)$$

This measurement of α has a relative standard uncertainty of $2,5 \times 10^{-10}$. This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020, giving the value:

$$\alpha^{-1} = 137,035999206(11)$$

with a relative accuracy of 81 parts per trillion. We propose in [8] the exact formula for the fine-structure constant α with the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \varphi^{-2} - 2 \cdot \varphi^{-3} + (3 \cdot \varphi)^{-5} \quad (1)$$

with numerical value:

$$\alpha^{-1} = 137,035999164\dots$$

3. Proton to electron mass ratio

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron, from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of m_e and m_p , and the equilibrium between them, govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars, leading to the formation of basic biochemical elements, including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons, two constant particles that make up about 95% of the visible Universe, may be related to the total computational value of the Universe. Thus, as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity.

The proton-to-electron mass ratio μ is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897, and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges. The 2.018 CODATA recommended value of the proton to electron mass ratio μ is:

$$\mu = m_p/m_e = 1.836,15267343$$

With standard Uncertainty $0,00000011$ and relative standard uncertainty $6,0 \times 10^{-11}$. The value of μ is known at about 0,1 parts per billion. The value of μ is a solution of the equation:

$$3 \cdot \mu^4 - 5.508 \cdot \mu^3 - 841 \cdot \mu^2 + 10 \cdot \mu - 2.111 = 0$$

The 2.018 CODATA recommended value of μ^{-1} is:

$$\mu^{-1} = m_e/m_p = 0,000544617021487$$

With standard uncertainty $0,000000000000033$ and relative standard uncertainty $6,0 \times 10^{-11}$. We propose in [9] the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32} = \varphi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \quad (2)$$

with numerical value:

$$\mu = 1.836,15267343\dots$$

Also we propose in [9] the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^3 = 7^{-1} \cdot (5 \cdot 13)^3 \cdot [\ln(2.5)]^{11} \quad (3)$$

with numerical value:

$$\mu = 1836,15267392\dots$$

Also other exact mathematical expression in [9] for the proton to electron mass ratio is:

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15} \quad (4)$$

with numerical value:

$$\mu = 1.836,15267343\dots$$

Finally in [9] we present the exact mathematical expression that connect the proton to electron mass ratio μ and the fine-structure constant α :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\phi + 42) \quad (5)$$

4. Gravitational coupling constant for the electron

In physics, the gravitational coupling constant α_G is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by α_G . The choice of units of measurement, but only with the choice of particles. The gravitational coupling constant α_G is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale, atomic scale, and cosmological scale). The gravitational coupling constant can be used for comparison of length, range and force values. The gravitational coupling constant α_G is defined as:

$$\alpha_G = G \cdot m_e^2 / \hbar \cdot c$$

There is so far no known way to measure α_G directly. The value of the constant gravitational coupling α_G is only known in four significant digits. The approximate value of the constant gravitational coupling α_G is:

$$\alpha_G = 1,751809945 \times 10^{-45}$$

5. Gravitational coupling constant for the proton

The gravitational coupling constant for the proton is produced similar to the electron, but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton $\alpha_{G(p)}$ is defined as:

$$\alpha_{G(p)} = G \cdot m_p^2 / \hbar \cdot c$$

The approximate value of the constant gravitational coupling of the proton $\alpha_{G(p)}$ is:

$$\alpha_{G(p)} = 5,906151273 \times 10^{-39}$$

6. Ratio of electric force to gravitational force between electron and proton

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1.904. But Weyl and Eddington suggested that the number was about 10^{40} and was related to cosmological quantities. The electric force F_c between electron and proton is defined as:

$$F_c = q_e^2 / 4 \cdot \pi \cdot \epsilon_0 \cdot r^2$$

The gravitational force F_g between electron and proton is defined as:

$$F_g = G \cdot m_e \cdot m_p / r^2$$

So from these expressions we have:

$$N_1 = F_c / F_g$$

$$N_1 = q_e^2 / 4 \cdot \pi \cdot \epsilon_0 \cdot G \cdot m_e \cdot m_p$$

$$N_1 = k_e \cdot q_e^2 / G \cdot m_p \cdot m_e$$

$$N_1 = a \cdot \hbar \cdot c / G \cdot m_p \cdot m_e$$

The value of the ratio N_1 of electric force to gravitational force between electron and proton is:

$$N_1 = 2,26866072471 \times 10^{39}$$

The approximate value of the ratio N_1 of electric force to gravitational force between electron and proton is:

$$N_1 = (5/3) \cdot 2^{130} = 2,26854911 \times 10^{39}$$

7. Avogadro number

Avogadro's number N_A is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. Avogadro's number N_A is the fundamental physical constant that links the macroscopic physical world of objects that we can see and feel with the submicroscopic, invisible world of atoms. The name honors the Italian mathematical physicist Amedeo Avogadro, who proposed that equal volumes of all gasses at the same temperature and pressure contain the same number of molecules. The most accurate definition of the Avogadro's number N_A value involves the change in molecular quantities and, in particular, the change in the value of an elementary charge. The exact value of the Avogadro's number N_A is:

$$N_A = 6,02214076 \times 10^{23}$$

The value of the Avogadro's number N_A can also be written in numerical expressions:

$$N_A = 84.446.885^3 = 6,02214076 \times 10^{23}$$

$$N_A = 2^{79} = 6,04462909 \times 10^{23}$$

8. New formula for the planck length

A Planck length ℓ_{pl} is about 10^{-20} times the diameter of a proton, meaning it is so small that immediate observation at this scale would be impossible in the near future. The length Planck ℓ_{pl} has dimension [L]. The length Planck ℓ_{pl} can be defined by three fundamental natural constants, the speed of light at vacuum c , the reduced Planck constant \hbar and the gravity constant G as:

$$\ell_{pl} = (\hbar \cdot G / c^3)^{1/2}$$

The Bohr radius a_0 is a physical constant, approximately equal to the most probable distance between the nucleus and the electron in a hydrogen atom in its ground state. The Bohr radius a_0 is defined as:

$$a_0 = \hbar / a \cdot m_e \cdot c$$

The Planck constant, or Planck's constant, is a fundamental physical constant of foundational importance in quantum mechanics. The constant gives the relationship between the energy of a photon and its frequency, and by the mass-energy equivalence, the relationship between mass and frequency. Specifically, a photon's energy is equal to its frequency multiplied by the Planck constant. The constant is generally denoted by h . The reduced Planck constant, equal to the constant divided by 2π , is denoted by \hbar . The reduced Planck constant \hbar is defined as:

$$\hbar = a \cdot m_e \cdot a_0 \cdot c$$

So from these expressions we have:

$$\hbar^2 = a^2 \cdot m_e^2 \cdot a_0^2 \cdot c^2$$

$$(\hbar \cdot G / c^3) = a^2 \cdot m_e^2 \cdot a_0^2 \cdot (G / \hbar \cdot c)$$

$$(\hbar \cdot G / c^3) = a^2 \cdot a_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

$$\ell_{pl}^2 = a^2 \cdot a_0 \cdot a_0^2$$

So the new formula for the Planck length ℓ_{pl} is:

$$\ell_{pl} = a \cdot a_0^{1/2} \cdot a_0 \quad (6)$$

9. New formula for the Avogadro number

Jeff Yee proposed in [10] that the mole and charge are related by deriving Avogadro's number from three constants, the Bohr radius, the Planck length and Euler's number. The smallest components of spacetime will never be seen with the human eye as it is orders of magnitudes smaller than an atom. If an atom was the size of the Milky Way galaxy, a granule of Planck length radius would be roughly the size of a grain of sand on Earth. Thus, it will never be directly observed but it can be deduced by mathematics. These units are referred to as the Planck units. The fundamental unit of length in this unit system is the Planck length ℓ_{pt} . Spacetime is proposed to be a lattice structure, in which its unit cells have sides of length a , marked below in the next figure. The lattice contains repeating cells with this structure, so it can be simplified to model a single unit cell of this repeating structure. These types of structures are commonly found in molecules. The center point of wave convergence is referred to here as a wave center. The separation length between granules in the unit cell is the diameter of a granule ($2 \cdot \ell_{pt}$) multiplied by Euler's number (e), which is the base of the natural logarithm. There are exactly Avogadro's number of unit cells in the radius of hydrogen. The Avogadro number N_A can be calculated from the Planck length ℓ_{pt} , the Bohr radius a_0 and Euler's number e :

$$N_A = a_0 / 2 \cdot e \cdot \ell_{pt}$$

From this expression we have:

$$N_A = a_0 / 2 \cdot e \cdot a \cdot a_0 \cdot a_G^{1/2}$$

$$N_A = 1/2 \cdot e \cdot a \cdot a_G^{1/2}$$

So the new formula for the Avogadro number N_A is:

$$N_A = (2 \cdot e \cdot a \cdot a_G^{1/2})^{-1} \quad (7)$$

From this expression resulting the beautiful formula:

$$2 \cdot e \cdot N_A \cdot a \cdot a_G^{1/2} = 1 \quad (8)$$

10. Mathematical formulas that connect 3 dimensionless physical constants

The exact mathematical formula that connect the mass ratio of proton to electron, the fine-structure constant α and the proton-proton gravitational coupling constant $a_{G(pp)}$ is:

$$\alpha^7 = \mu^7 \cdot [a_{G(pp)} \cdot \log_2(2 \cdot n)]$$

The exact mathematical formula that connect the mass ratio of proton to electron, the fine-structure constant α and the proton-electron gravitational coupling constant $a_{G(pe)}$ is:

$$\alpha^7 = \mu^8 \cdot [a_{G(pp)} \cdot \log_2(2 \cdot n)]$$

The exact mathematical formula that connect the mass ratio of proton to electron, the fine-structure constant α and the gravitational coupling constant of electrons-electrons $a_{G(ee)}$ is:

$$\alpha^7 = \mu^9 \cdot [a_{G(pp)} \cdot \log_2(2 \cdot n)]$$

The exact mathematical formula that connect the proton to electron mass ratio μ , the gravitational coupling constant a_G for the electron and the gravitational coupling constant of the proton $a_{G(p)}$ is:

$$a_{G(p)} = \mu^2 \cdot a_G$$

The mathematical formula that connect the fine-structure constant α , the gravitational coupling constant a_G for the electron and the Avogadro number N_A is:

$$2 \cdot e \cdot N_A \cdot a \cdot a_G^{1/2} = 1 \quad (9)$$

11. Mathematical formulas that connects 4 dimensionless physical constants

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton and the gravitational coupling constant a_G for the electron is:

$$\alpha = \mu \cdot N_1 \cdot a_G \quad (10)$$

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton and the gravitational coupling constant of proton $a_{G(p)}$ is:

$$\alpha \cdot \mu = N_1 \cdot a_{G(p)} \quad (11)$$

The exact mathematical formula that connect the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the gravitational coupling constant a_G for the electron and the gravitational coupling constant of proton $a_{G(p)}$ is:

$$\alpha^2 = N_1^2 \cdot a_G \cdot a_{G(p)} \quad (12)$$

The exact mathematical formula that connect the fine-structure constant α , the gravitational coupling constant a_G for the proton $a_{G(p)}$ and the Avogadro number N_A is:

$$\mu = 2 \cdot e \cdot N_A \cdot \alpha \cdot a_{G(p)} \quad (13)$$

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton and the Avogadro number N_A is:

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \quad (14)$$

12. Mathematical formulas that connect 5 dimensionless physical constants

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A and the gravitational coupling constant a_G for the electron is:

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (15)$$

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A and the gravitational coupling constant of proton $a_{G(p)}$ is:

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (16)$$

The exact mathematical formula that connect the proton to electron mass ratio μ , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A , the gravitational coupling constant a_G for the electron and the gravitational coupling constant of proton $a_{G(p)}$ is:

$$\mu = 2 \cdot e \cdot a_G^{1/2} \cdot a_{G(p)} \cdot N_A \cdot N_1 \quad (17)$$

13. Mathematical formulas that connect 6 dimensionless physical constants

The exact mathematical formula that connect the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro number N_A , the gravitational coupling constant a_G for the electron and the gravitational coupling constant of proton $a_{G(p)}$ is:

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (18)$$

We need to find a correction number to have more accurate equations. We propose:

$$2 \cdot e = 2 \cdot e \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13/2} \quad (19)$$

From (18) and (19) resulting the exact mathematical formula that connect 6 dimensionless physical constants:

$$\mu = 4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13} \cdot \alpha \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (20)$$

$$\mu = 2^{15} \cdot 3^{13} \cdot 5^{-26} \cdot 7^{13} \cdot \phi^{13} \cdot e^{-11} \cdot \alpha \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (21)$$

$$\alpha \cdot \mu^{-1} \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 = 2^{-15} \cdot 3^{-13} \cdot 5^{26} \cdot 7^{-13} \cdot \phi^{-13} \cdot e^{11} \quad (22)$$

14. New formula for gravitational constant

The gravitational constant G is an empirical physical constant that participates in the calculation of gravitational force between two bodies and is denoted by the letter G . It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. The physicist Sir Isaac Newton in 1687 published his book "Philosophiæ Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper, Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However, these must be adjusted by introducing the gravity constant G . The gravitational constant G is defined as:

$$G = a_G \cdot \hbar \cdot c / m_e^2$$

The 2.018 CODATA recommended value of gravitational constant G is:

$$G=6,67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$$

With standard uncertainty $0,00015 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$ and relative standard uncertainty $2,2 \times 10^{-5}$. From (8) the gravitational coupling constant α_G can be written in the form:

$$2 \cdot e \cdot N_A \cdot a \cdot \alpha_G^{1/2} = 1$$

$$\alpha_G = (2 \cdot e \cdot a \cdot N_A)^{-2}$$

Therefore the formula for the gravitational constant G is:

$$G = \alpha_G \cdot \hbar \cdot c / m_e^2$$

So the new formula for gravitational constant G is:

$$G = (2 \cdot e \cdot a \cdot N_A)^{-2} \cdot (\hbar \cdot c / m_e^2) \quad (23)$$

15. Conclusions

We presented new exact formula for the fine-structure constant α in terms of the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \varphi^{-2} - 2 \cdot \varphi^{-3} + (3 \cdot \varphi)^{-5}$$

A new interpretation and a very accurate value of the fine-structure constant has been discovered in terms of the golden ratio. The equation is simple, elegant and symmetrical in a great physical meaning.

We propose the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32} = \varphi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19}$$

We propose the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^3 = 7^{-1} \cdot (5 \cdot 13)^3 \cdot [\ln(2 \cdot 5)]^{11}$$

Also other exact mathematical expression in for the proton to electron mass ratio is:

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15}$$

We present the exact mathematical expressions that connect the proton to electron mass ratio and the fine-structure constant:

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42)$$

The new formula for the Planck length ℓ_{Pl} is:

$$\ell_{Pl} = a \cdot a_0 \cdot \alpha_G^{1/2}$$

The new formula for the Avogadro number N_A is:

$$N_A = (2 \cdot e \cdot a \cdot \alpha_G^{1/2})^{-1}$$

Eleven mathematical formulas that connect dimensionless physical constants:

$$\alpha = \mu \cdot N_1 \cdot \alpha_G$$

$$\alpha \cdot \mu = N_1 \cdot \alpha_G(p)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_G(p)$$

$$\mu = 2 \cdot e \cdot N_A \cdot a \cdot \alpha_G(p)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot a^3 \cdot N_A^2$$

$$2 \cdot e \cdot a \cdot N_A \cdot \alpha_G^{1/2} = 1$$

$$4 \cdot e^2 \cdot a \cdot \mu \cdot aG^2 \cdot NA^2 \cdot N1 = 1$$

$$\mu^3 = 4 \cdot e^2 \cdot a \cdot aG(p)^2 \cdot NA^2 \cdot N1$$

$$\mu = 2 \cdot e \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1$$

$$\mu = 4 \cdot e^2 \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1$$

$$a \cdot \mu^{-1} \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 = 2^{-15} \cdot 3^{-13} \cdot 5^{26} \cdot 7^{-13} \cdot \phi^{-13} \cdot e^{11}$$

The new formula for gravitational constant G is:

$$G = (2 \cdot e \cdot a \cdot NA)^{-2} \cdot (\hbar \cdot c / m_e^2)$$

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