Interval quadripartitioned Neutrosophic Sets

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Abstract

Quadripartitioned neutrosophic set is a mathematical tool, which is the extension of neutrosophic set and n-valued neutrosophic refined logic for dealing with real-life problems. A generalization of the notion of quadripartitioned neutrosophic set is introduced. The new notion is called Interval Quadripartitioned Neutrosophic set (IQNS). Quadripartitioned neutrosophic set is developed by combining the Quadripartitioned neutrosophic set and interval neutrosophic set. Several set theoretic operations of IQNSs, namely, inclusion, complement, and intersection are defined. Various properties of set-theoretic operators of IQNS are established.

Keywords: Neutrosophic set, Single valued neutrosophic set, Interval neutrosophic set, quadripartitioned neutrosophic set, Interval quadripartitioned neutrosophic set

1. Introduction

Chatterjee et al. [1] defined Quadripartitioned Single Valued Neutrosophic Set (QSVNS) by utilizing the concept of Single Valued Neutrosophic Set (SVNS) [2], four valued logic [3] and n- valued refined logic [4] that involves degrees of truth, falsity, unknown and contradiction membership. Chatterjee et al. [5] investigated interval-valued possibility quadripartitioned single valued neutrosophic soft sets by generalizing the possibility intuitionistic fuzzy soft set [6].

No investigation regarding Interval Quadripartitioned Neutrosophic Set (IQNS) is reported in the literature. The motivation of the present work comes from the works of Chatterjee et al. [1, 5]. The notion of IQNS is developed by combining the concept of QSVNS and Interval Neutrosophic Set(INS) [7]. The proposed structure is a generalization of existing theories of INS and QSVNS.

The organization of the paper is as follows: Section 2 presents some preliminary results. Section 3 introduces the concept of IQNS and set-theoretic operations over IQNS. Section 4 concludes the paper by presenting the future direction of research.

2. Preliminary

Definition 2.1. Assume that a set W is fixed. An NS [8] H over W is defined as:

$H = \{w, (T_H(w), I_H(w), F_H(w)) : w \in W\}$ where $T_H, I_H, F_H : W \to]^-0, 1^+[$ and

$^-0 \le T_H(w) + I_H(w) + F_H(w) \le 3^+$.

Definition 2.2 Assume that a set W is fixed .An SVNS [2] H over W is defined as:

$H = \{w, (T_H(w), I_H(w), F_H(w)) : w \in W\}$ where $T_D, I_D, F_D : W \to [0,1]$ and

$0 \le T_H(w) + I_H(w) + F_H(w) \le 3$.

Definition 2.3. Let a set W be fixed. An INS [7] H over W is defined as:

$H = \{(w, (T_H(w), I_H(w), F_H(w)) : w \in W\}$

where for each $w \in W$, $T_H(w), I_H(w), F_H(w) \subseteq [0,1]$ are the degrees of membership functions of truth, indeterminacy, and falsity and

$T_H(w) = [\inf T_H(w), \sup T_H(w)], I_H(w) = [\inf I_H(w), \sup I_H(w)]), F_H(w) = [\inf F_H(w), \sup F_H(w)]$ and

$0 \le \sup T_H(w) + \sup I_H(w) + \sup F_H(w) \le 3.$

D can be expressed as:

$H = \{w, ([\inf T_H(w), \sup T_H(w)], [\inf I_H(w), \sup I_H(w)]), [\inf F_H(w), \sup F_H(w)]) : w \in W\}$

2.4. . Let a set W be fixed. A QSVNS [1] H, over W is defined as:

H= $\{(w, T_H(w), C_H(w), U_D(w), F_D(w)): w \in W\}$, where for each point $w \in W$, $T_H(w)$, $C_H(w)$, $U_H(w)$ U_D, $F_D(w)$ $\to [0,1]$ are the degrees membership functions of truth, contradiction, ignorance, and falsity and

$0 \le \sup T_H(w) + \sup C_H(w) + \sup U_H(w) + \sup F_H(w) \le 4.$

3. The Basic Theory of IQNSs

Definition 3.1. IQNS

Let *W* be a fixed set. Then , an IQNS over *W* is denoted by H and defined as follows:

H= $\{(w, T_H(w), C_H(w), U_D(w), F_D(w)): w \in W\}$, where for each point $w \in W$, $T_H(w)$, $C_H(w)$, $U_H(w)$ U_D, $F_D(w)$ $\subseteq [0, 1]$ are the degrees of membership functions of truth, contradiction, ignorance, and falsity and $T_H(w)$= [inf T_H(w, sup $T_H(w)$] , $C_H(w)$ =[inf $C_H(w)$, sup $C_H(w)$] $U_{H,}(w)$ = [inf $U_H(w)$, sup $U_H(w)$], $F_H(w)$ = [inf $F_H(w)$, sup $F_H(w)$] $\subseteq [0, 1]$ and

$0 \le \sup T_H(w) + \sup C_H(w) + \sup U_H(w) + \sup F_H(w) \le 4.$

Example 3.1. Suppose that W = [w_1, w_2, w_3], where w_1, w_2, and w_3 present respectively the capability, trustworthiness, and price. The values of w_1, w_2, and w_3 are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be degree of truth (good), degree of contradiction, degree of ignorance, and degree of false (poor). D_1 is an IQNS of *W* defined by

D_1 = {[0.5, 0.7], [0.15, 0.2], [0.2, 0.4], [0.2, 0.3]}/w_1 + {[0.55, 0.85], [0.25, 0.35], [0.15, 0.25], [0.2, 0.3]}/w2+ [0.65, 0.85], [0.2, 0.35], [0.1, 0.25], [0.15, 0.25]}/ w_3

D_2 is an IPNS of *W* defined by

D_2 = {[0.6, 0.8], [0.1, 0.2], [0.1, 0.25], [0.15, 0.3]}/w_1 + {[0.6, 0.9], [0.25, 0.3], [0.1, 0.2], [0.1, 0.3]}/w2+ [0.5, 0.7], [0.1, 0.2], [0.15, 0.25], [0.1; 0.2]}/ w_3

Definition 3.2 An IQNS is said to be empty (null) denoted by $\hat{0}$ iff inf $T_H(w)$= sup $T_H(w)$ = 0, inf $C_H(w)$= sup $C_H(w)$ = 0, , inf $U_H(w)$ = sup $U_H(w)$= 1, inf $F_H(w)$= sup $F_H(w)$ = 1 ,

$$\hat{0} = \{[0,0],[0,0],[1,1],[1,1]\}$$

Definition 3.3 An IQNS is said to be unity *denoted by* $\hat{1}$ iff

inf $T_D(w)$ = sup $T_D(w)$ = 1, inf $C_D(w)$ = sup $C_D(w)$ = 1, inf $U_D(w)$ = sup $U_D(w)$ = 0,
inf $F_D(w)$= sup $F_D(w)$ = 0.

$$\hat{1} = \{[1,1],[1,1],[0,0],[0,0]\}$$

Also, we have $\underline{0} = \langle 0,0,1,1\rangle$ *and* $\underline{1} = \langle 1,1,0,0\rangle$

Definition 3.4. (Containment) Let H_1 and H_2 be any two IQNS over W, H_1 is said to be contained in H_2, denoted by $H_1 \subseteq H_2$ iff
for any $w \in W$.

$$\begin{aligned}
&\inf T_{H_1}(w) \le \inf T_{H_2}(w), \sup T_{H_1}(w) \le \sup T_{H_2}(w),\\
&\inf C_{H_1}(w) \le \inf C_{H_2}(w), \sup C_{H_1}(w) \le \sup C_{H_2}(w),\\
&\inf U_{H_1}(w) \ge \inf U_{H_2}(w), \sup U_{H_1}(w) \ge \sup U_{H_2}(w),\\
&\inf F_{H_1}(w) \ge \inf F_{H_2}(w), \sup F_{H_1}(w) \ge \sup F_{H_2}(w),
\end{aligned}$$

Definition 3.5. Two IQNSs H_1 and H_2 are equal iff $H_1 \subseteq H_2$ and $H_1 \supseteq H_2$

Definition 3.6. (Complement) Let $H = \{(w, T_H(w), C_H(w), U_H(w), F_H(w)) : w \in W\}$ be an IQNS.
The complement of H is denoted by H' and defined as:

$$T_{H'}(w) = F_H(w),\ C_{H'}(w) = U_H(w),\ U_{H'}(w) = C_H(w), F_{H'}(w) = T_H(w)$$
$$H' = \{(w,[\inf F_H(w), \sup F_H(w)],[\inf U_H(w), \sup U_H(w)],[\inf C_H(w), \sup C_H(w)], [\inf T_H(w), \sup T_H(w)]) : w \in W\}$$

Example 3.2 . Consider an IQNS H of the form:
D = {[0.35, 0.75], [0.2, 0.25], [0.2, 0.3], [0.2, 0.4]}/w_1 + {[0.55, 0.85], [0.2, 0.3], [0.15, 0.25], [0.2, 0.35]}/w2+ [0.75, 0.85], [0.15, 0.25], [0.15, 0.25], [0.1, 0.25]}/ w_3
Then, complement of
D' == {[0.2, 0.4] , [0.2, 0.3], [0.2, 0.25], [0.35, 0.75]}/w_1 + {[0.2, 0.35], [0.15, 0.25], [0.2, 0.3], [0.55, 0.85]}/w2+ [0.1, 0.25], [0.15, 0.25], [0.15, 0.25], [0.75, 0.85]}/ w_3

Definition 3.7.(Intersection)

The intersection of any two IQNSs H_1 and H_2 is an IQNS H_3, denoted as H_3 and presented as:
$H_3 = H_1 \cap H_2$

$$\{(w,[\inf T_{H_3}(w), \sup T_{H_3}(w)],[\inf C_{H_3}(w), \sup C_{H_3}(w)],[\inf U_{H_3}(w), \sup U_{H_3} w)],$$
$$[\inf F_{H_3}(w), \sup F_{H_3}(w)]) : w \in W\}.$$

$$= \begin{aligned}
&\{(w,[\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))],\\
&[\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))],\\
&[\max(\inf U_{H_1}(w), \inf U_{H_2}(w))], \max(\sup U_{H_1}(w), \sup U_{H_2}(w))],\\
&[\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}
\end{aligned}$$

Example 3.3. Let H_1 and H_2 be the IQNSs defined in Example 1.

Then, $D_1 \cap D_2 = \{[0.5, 0.7], [0.1, 0.2], [0.2, 0.4], [0.2, 0.3]\}/w_1 + \{[0.55, 0.85], [0.25, 0.3], [0.15, 0.25], [0.2, 0.3]\}/w2 + [0.5, 0.7], [0.1, 0.2], [0.15, 0.25], [0.15, 0.25]\}/ w_3$

Definition 3.8. (**Union**) The union of any two IQNSs H_1 and H_2 is an IQNS denoted as H_3, and presented as:

$$H_3 = H_1 \cup H_2$$
$$\{(w,[\inf T_{H_3}(w), \sup T_{H_3}(w)],[\inf C_{H_3}(w), \sup C_{H_3}(w)],[\inf U_{H_3}(w), \sup U_{H_3} w)],$$
$$[\inf F_{H_3}(w), \sup F_{H_3}(w)]) : w \in W\}.\}.$$
$$= \{(w,[\max(\inf T_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))],[\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\min(\inf U_{D_1}(w), \inf U_{D_2}(w)), \min(\sup U_{D_1}(w), \sup U_{D_2}(w))],[\min(\inf F_{D_1}(w), \inf F_{D_2}(w)), \min(\sup F_{D_1}(w), \sup F_{D_2}(w))]) : w \in W\}$$

Example 3. 4. Let H_1 and H_2 be the IQNSs in example 1. Then

$H_1 \cup H_2 = \{[0.6, 0.8], [0.15, 0.2], [0.1, 0.25], [0.15, 0.3]\}/w_1 + \{[0.6, 0.9], [0.25, 0.35], [0.1, 0.2], [0.1, 0.3]\}/w2 + [0.65, 0.85], [0.2, 0.35], [0.1, 0.25], [0.1, 0.2]\}/ w_3$

Theorem 3.1 Let H_1 and D_2 be any two IQNSs over W *defined by*
$H_i = \{(w, T_{H_i}(w), C_{H_i}(w), G_{H_i}(w), U_{H_i}(w), F_{H_i}(w))]) : w \in W\}, i = 1, 2,$ and
$T_{H_i}(w), C_{H_i}(w), G_{H_i}(w), U_{H_i}(w), F_{H_i}(w))] \subseteq [0,1], i = 1, 2.$

Then
(a) $H_1 \cup H_2 = H_2 \cup H_1$
(b) $H_1 \cap H_2 = H_2 \cap H_1$

Proof. (a):

$$H_1 \cup H_2 = \{(w,[\max(\inf T_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))],$$
$$[\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\min(\inf U_{H_1}(w), \inf U_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\min(\inf F_{H_1}(w), \inf F_{H_2}(w)), \min(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}$$
$$= \{w,[\max(\inf T_{H_2}(w), \inf T_{D_1}(w)), \max(\sup T_{D_2}(w), \sup T_{D_1}(w))],[\max(\inf C_{D_2}(w), \inf C_{D_1}(w)), \max(\sup C_{D_2}(w), \sup C_{D_1}(w))],$$
$$[\min(\inf U_{H_2}(w), \inf U_{H_1}(w)), \min(\sup U_{H_2}(w), \sup U_{H_1}(w))],[\min(\inf F_{H_2}(w), \inf F_{H_1}(w)), \min(\sup F_{H_2}(w), \sup F_{H_1}(w))] : w \in W\}$$
$$= H_2 \cup H_1$$

Proof. (b):

$$H_1 \cap H_2 = \{(w,[\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))],$$
$$[\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\max(\inf U_{H_1}(w), \inf U_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}.$$
$$= \{(w,[\min(\inf T_{D_H}(w), \inf T_{H_1}(w)), \min(\sup T_{H_2}(w), \sup T_{H_1}(w))],$$
$$[\min(\inf C_{H_2}(w), \inf C_{H_1}(w)), \min(\sup C_{H_2}(w), \sup C_{H_1}(w))],$$
$$[\max(\inf U_{H_2}(w), \inf U_{H_1}(w)), \max(\sup U_{H_2}(w), \sup U_{H_1}(w))],$$
$$[\max(\inf F_{H_2}(w), \inf F_{H_1}(w)), \max(\sup F_{H_2}(w), \sup F_{H_1}(w))]) : \forall w \in W\}.$$
$$= H_2 \cap H_1$$

Theorem 3.2. For any two IPNS, H_1, and H_2:

(a) $H_1 \cup (H_1 \cap H_2) = H_1$

(b) $H_1 \cap (H_1 \cup H_2) = H_1$

Proof .(a):

$$H_1 \cup (H_1 \cap H_2) =$$
$$\{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [[\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$\cup \{(w, [\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))],$$
$$[\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\max(\inf U_{H_1}(w), \inf U_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}.$$
$$= \{w, ([\max(\inf T_{H_1}(w), \min(\inf T_{H_1}(w), \inf T_{H_2}(w))), \max(\sup T_{H_1}(w), \min(\sup T_{H_1}(w), \sup T_{H_2}(w)))],$$
$$[\max(\inf C_{H_1}(w), \min(\inf C_{H_1}(w), \inf C_{H_2}(w))), \max(\sup C_{H_1}(w), \min(\sup C_{H_1}(w), \sup C_{H_2}(w)))]),$$
$$[\min(\inf (U_{H_1}(w), \max(\inf U_{H_1}(w), \inf U_{H_2}(w))), \min(\sup U_{H_1}(w), \max(\sup U_{H_1}(w), \sup U_{H_2}(w)))],$$
$$[\min(\inf (F_{H_1}(w), \max(\inf F_{H_1}(w), \inf F_{H_2}(w))), \min(\sup F_{H_1}(w), \max(\sup F_{H_1}(w), \sup F_{H_2}(w)))]) : w \in W\}$$
$$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [[\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$= H_1$$

Proof (b):

$$H_1 \cap (H_1 \cup H_2)$$
$$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]),$$
$$[\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\} \cap$$
$$\{(w, [\max(\inf T_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))],$$
$$[\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\min(\inf U_{H_1}(w), \inf U_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\min(\inf F_{H_1}(w), \inf F_{H_2}(w)), \min(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}$$
$$= \{w, [\min(\inf T_{H_1}(w), \max(\inf T_{H_1}(w), \inf T_{H_2}(w))), \min(\sup T_{H_1}(w), \max(\sup T_{H_1}(w), \sup T_{H_2}(w)))],$$
$$[\min(\inf C_{H_1}(w), \max(\inf C_{H_1}(w), \inf C_{H_2}(w))), \min(\sup C_{H_1}(w), \max(\sup C_{H_1}(w), \sup C_{H_2}(w)))],$$
$$[\max(\inf U_{H_1}(w), \min(\inf U_{H_1}(w), \inf U_{H_2}(w))), \max(\sup U_{H_1}(w), \min(\sup U_{H_1}(w), \sup U_{H_2}(w)))],$$
$$[\max(\inf F_{H_1}(w), \min(\inf F_{H_1}(w), \inf F_{H_2}(w))), \max(\sup F_{H_1}(w), \min(\sup F_{H_1}(w), \sup F_{H_2}(w)))]$$
$$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [[\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$= H_1$$

Theorem 3.3. For any IPNS D1**:**

(a) $H_1 \cup H_1 = H_1$

(b) $H_1 \cap H_1 = H_1$

Proof. (a):

$$H_1 \cup H_1 = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)],$$
$$[\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\} \cup \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]),$$
$$[\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$= \{w, ([\max(\inf T_{H_1}(w), \inf T_{H_1}(w)), \max(\sup T_{H_1}(w), \sup T_{H_1}(w))],$$
$$[\max(\inf C_{H_1}(w), (\inf C_{H_1}(w))), \max(\sup T_{H_1}(w), \sup C_{H_1}(w)]),$$
$$[\min(\inf U_{H_1}(w), \inf U_{H_1}(w)), (\min(\sup U_{H_1}(w), (\sup U_{H_1}(w))],$$
$$[\min(\inf F_{H_1}(w), \inf F_{H_1}(w)), \min(\sup F_{H_1}(w), \sup F_{H_1}(w))]) : w \in W\}$$
$$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$= H_1$$

Proof. (b):

$$H_1 \cap H_1 = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)],$$
$$[\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\} \cap \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]),$$
$$[\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$

$$\{(w, [\min(\inf T_{H_1}(w), \inf T_{H_1}(w)), \min(\sup T_{H_1}(w), \sup T_{H_1}(w))],$$
$$[\min(\inf C_{H_1}(w), \inf C_{H_1}(w)), \min(\sup C_{H_1}(w), \sup C_{H_1}(w))],$$
$$[\max(\inf U_{H_1}(w), \inf U_{H_1}(w)), \max(\sup U_{H_1}(w), \sup U_{H_1}(w))],$$
$$[\max(\inf F_{H_1}(w), \inf F_{H_1}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}.$$
$$== \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$= H_1$$

Theorem 3.4 For any IQNS H_1,

$$(a) H_1 \cap \hat{0} = \hat{0}$$
$$(b) H_1 \cup \hat{1} = \hat{1}$$

Proof. (a):

$$H_1 \cap \hat{0}$$
$$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$\cap \{[0,0], [0,0], [1,1], [1,1], [1,1]\}$$
$$= \{(w, [\min(\inf T_{H_1}(w), 0), \min(\sup T_{H_1}(w), 0)],$$
$$[\min(\inf C_{H_1}(w), 0), \min(\sup C_{H_1}(w), 0)],$$
$$[\max(\inf U_{H_1}(w), 1), \max(\sup U_{H_1}(w), 1)],$$
$$[\max(\inf F_{H_1}(w), 1), \max(\sup F_{H_1}(w), 1]) : w \in W\}$$
$$= \{(w, [0, 0)], [0, 0)], [1,1], [1,1]), \forall w \in W\}$$
$$= \hat{0}$$

Proof. (b):

$$H_1 \cup \hat{1}$$
$$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$$
$$\cup \{[1,1], [1,1], [0,0], [0,0], [0,0]\}$$
$$= \{(w, [\max(\inf T_{H_1}(w), 1), \max(\sup T_{H_1}(w), 1))], [\max(\inf C_{H_1}(w), 1), \max(\sup C_{H_1}(w), 1)], [\min(\inf U_{H_1}(w), 0), \min(\sup U_{H_1}(w), 0)],$$
$$[\min(\inf F_{H_1}(w), 0)), \min(\sup F_{H_1}(w), 0)]) : w \in W\}$$
$$= \{w([1,1], [1,1], [0,0], [0,0], [0,0]) : w \in W\}$$
$$= \hat{1}$$

Theorem 3.5. For any IQNS H_1,

$$(a) H_1 \cup \hat{0} = H_1$$
$$(b) H_1 \cap \hat{1} = H_1$$

Proof. (a):

$H_1 \cup \hat{0}$
$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$
$\cup \{[0,0],[0,0],[1,1],[1,1],[1,1]\}$
$= \{(w, [\max(\inf T_{H_1}(w), 0), \max(\sup T_{H_1}(w), 0))], [\max(\inf C_{H_1}(w), 0), \max(\sup C_{H_1}(w), 0)],$
$[\min(\inf U_{H_1}(w), 1), \min(\sup U_{H_1}(w), 1)], [\min(\inf F_{H_1}(w), 1)), \min(\sup F_{H_1}(w), 1)]) : w \in W\}$
$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$
$= H_1$
$H_1 \cap \hat{1}$
$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$
$\cap \{[1,1,[1,1,[0,0],[0,0],[0,0]\}$
$= \{(w, [\min(\inf T_{H_1}(w), 1), \min(\sup T_{H_1}(w), 1)], [\min(\inf C_{H_1}(w), 1), \min(\sup C_{H_1}(w), 1)],$
$[\max(\inf U_{H_1}(w), 0), \max(\sup U_{H_1}(w), 0)])[\max(\inf F_{H_1}(w), 0), \max(\sup F_{H_1}(w), 0)]) : w \in W\}$
$= \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W$
$= H_1$

Theorem 3.6. For any IQNS H_1, $(H_1')' = H_1$

Let $H_1 = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$

$H_1' = \{w, ([\inf F_{H_1}(w), \sup F_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf T_{H_1}(w), \sup T_{H_1}(w)], : w \in W\}$

$\therefore (H_1')' = \{w, ([\inf T_{H_1}(w), \sup T_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf F_{H_1}(w), \sup F_{H_1}(w)]) : w \in W\}$
$= H_1$

Theorem 3.7. For any two IQNSs, H_1 and H_2:

(a) $(H_1 \cup H_2)' = H_1' \cap H_2'$

(b) $(H_1 \cap H_2)' = H_1' \cup H_2'$

Proof. (a):

$H_1 \cup H_2 = \{(w, [\max(\inf T_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))],$
$[\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))],$
$[\min(\inf U_{H_1}(w), \inf U_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))],$
$[\min(\inf F_{H_1}(w), \inf F_{H_2}(w)), \min(\sup F_{H_1}(w), \sup F_{H_2}(w))],$

$\therefore (H_1 \cup H_2)' = \{(w, [\min(\inf F_{H_1}(w), \inf F_{H_2}(w)), \min(\sup F_{H_1}(w), \sup F_{H_2}(w))],$
$[\min(\inf U_{H_1}(w), \inf U_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))],$
$[\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))],$
$[\max(\inf F_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))]) : w \in W\}$ (1)

$$H_1' \cap H_2' = \{w, ([\inf F_{H_1}(w), \sup F_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf T_{H_1}(w), \sup T_{H_1}(w)]) : w \in W\}$$
$$\bigcap \{w, ([\inf F_{H_2}(w), \sup F_{H_2}(w)], [\inf U_{H_2}(w), \sup U_{H_2}(w)], [\inf C_{H_2}(w), \sup C_{H_2}(w)]), [\inf T_{H_2}(w), \sup T_{H_2}(w)]) : w \in W\}$$
$$= \{(w, [\min(\inf F_{H_1}(w), \inf F_{H_2}(w)), \min(\sup F_{H_1}(w), \sup F_{H_2}(w))],$$
$$[\min(\inf U_{H_1}(w), \inf U_{H_2}(w)), \min(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\max(\inf C_{H_1}(w), \inf C_{H_2}(w)), \max(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\max(\inf F_{H_1}(w), \inf T_{H_2}(w)), \max(\sup T_{H_1}(w), \sup T_{H_2}(w))]) : w \in W\} \qquad (2)$$

Therefore from (1) and (2), $(H_1 \cup H_2)' = H_1' \cap H_2'$

Proof. (b):

$$(H_1 \cap H_2) = \{(w, [\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))],$$
$$[\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\max(\inf U_{H_1}(w), \inf U_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))]) : w \in W\}'.$$
$$\therefore (H_1 \cap H_2)' = \{(w, [\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w), \sup F_{H_2}(w))]),$$
$$[\max(\inf U_{H_1}(w), \inf U_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))],$$
$$[\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))]) : w \in W\} \qquad (3)$$

Now

$$H_1' \cup H_2' = \{w, ([\inf F_{H_1}(w), \sup F_{H_1}(w)], [\inf U_{H_1}(w), \sup U_{H_1}(w)], [\inf C_{H_1}(w), \sup C_{H_1}(w)]), [\inf T_{H_1}(w), \sup T_{H_1}(w)]) : w \in W\} \cup$$
$$\{w, ([\inf F_{H_2}(w), \sup F_{H_2}(w)], [\inf U_{H_2}(w), \sup U_{H_2}(w)], [\inf C_{H_2}(w), \sup C_{H_2}(w)]), [\inf T_{H_2}(w), \sup T_{H_2}(w)]) : w \in W\}$$
$$= \{w, ([\max(\inf F_{H_1}(w), \inf F_{H_2}(w)), \max(\sup F_{H_1}(w)], \sup F_{H_2}(w)], [\max(\inf U_{H_1}(w), \inf U_{H_2}(w)), \max(\sup U_{H_1}(w), \sup U_{H_2}(w))],$$
$$[\min(\inf C_{H_1}(w), \inf C_{H_2}(w)), \min(\sup C_{H_1}(w), \sup C_{H_2}(w))], [\min(\inf T_{H_1}(w), \inf T_{H_2}(w)), \min(\sup T_{H_1}(w), \sup T_{H_2}(w))]) : w \in W\} \quad (4)$$

Therefore, from (3) and (4), $(H_1 \cap H_2)' = H_1' \cup H_2'$

Theorem 3.9. For any two IPNS H_1, H_2,

$$H_1 \subseteq H_2 \Leftrightarrow H_2' \subseteq H_1'.$$

Proof.

$$H_1 \subseteq H_2 \Leftrightarrow$$
$$\inf T_{H_1}(w) \le \inf T_{H_2}(w), \sup T_{H_1}(w) \le \sup T_{H_2}(w),$$
$$\inf C_{H_1}(w) \le \inf C_{H_2}(w), \sup C_{H_1}(w) \le \sup C_{H_2}(w),$$
$$\inf U_{H_1}(w) \ge \inf U_{H_2}(w), \sup U_{H_1}(w) \ge \sup U_{H_2}(w),$$
$$\inf F_{H_1}(w) \ge \inf F_{H_2}(w), \sup F_{H_1}(w) \ge \sup F_{H_2}(w),$$
$$\Leftrightarrow$$
$$\inf F_{H_2}(w) \le \inf F_{H_1}(w), \sup F_{H_2}(w) \le \sup F_{H_1}(w),$$
$$\inf U_{H_2}(w) \le \inf U_{H_1}(w), \sup U_{H_2}(w) \le \sup U_{H_1}(w),$$
$$\inf C_{H_2}(w) \le \inf C_{H_1}(w), \sup C_{H_2}(w) \le \sup C_{H_1}(w)$$
$$\inf T_{H_2} \ge \inf T_{H_1}, \ \sup T_{H_2}(w) \ge \sup T_{H_1}(w)$$
$$\Leftrightarrow$$
$$H_2' \subseteq H_1'$$

Note: Proposed IQNS can also be called as Interval Quadripartitioned Single Valued Neutrosophic Set (IQSVNS).

4. Conclusions

In this paper, the notion of IQNS is introduced by combining the QSVNS and INS. The notion of inclusion, complement, intersection, union of IQNSs are defined. Some of the properties of IQNSs, are established. In the future, the logic system based on the truth-value based IQNSs will be investigated and the theory can be used to solve real-life problems in the areas such as information fusion, bioinformatics, web intelligence, etc.

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