An Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution

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Abstract

We consult an Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" by Dr. M. Fazlur Rahman. We draw the natural logarithm of the number of words, normalised, starting with a letter vs the natural logarithm of the rank of the letter. We find that the words underlie a magnetisation curve. The magnetisation curve i.e. the graph of reduced magnetisation vs reduced temperature is the exact Onsager solution of two dimensional Ising model in the absence of external magnetic field.

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I. INTRODUCTION

In this article, we study magnetic field pattern behind the rich Arabic language. We study an Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" by Dr. M. Fazlur Rahman, [1]. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as graphical law.

Then, we moved on to investigate into, [3], dictionaries of five disciplines of knowledge and found existence of a curve magnetisation under each discipline. This was followed by finding of the graphical law behind the bengali language,[4] and the basque language[5]. This was pursued by finding of the graphical law behind the Romanian language, [6], five more disciplines of knowledge, [7], Onsager core of Abor-Miri, Mising languages,[8], Onsager Core of Romanised Bengali language,[9], the graphical law behind the Little Oxford English Dictionary, [10], the Oxford Dictionary of Social Work and Social Care, [11], the Visayan-English Dictionary, [12], Garo to English School Dictionary, [13], Mursi-English-Amharic Dictionary, [14] and Names of Minor Planets, [15], A Dictionary of Tibetan and English, [16], Khasi English Dictionary, [17], Turkmen-English Dictionary, [18], Websters Universal Spanish-English Dictionary, [19], A Dictionary of Modern Italian, [20], Langenscheidt's German-English Dictionary, [21], Essential Dutch dictionary by G. Quist and D. Strik, [22], Swahili-English dictionary by C. W. Rechenbach, [23], Larousse Dictionnaire De Poche for the French, [24], respectively.

The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe the graphical law analysis of the arabic words of the Arabic-Bengali dictionary, [1]. Section IV is Acknowledgment. The last section is the Bibliography.

II. MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[25], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs.

The difference $\triangle E$ of energy if we flip an up spin to down spin is, [26], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [27]. In the Bragg-Williams approximation,[28], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L}=2\frac{\gamma\epsilon L+H}{k_BT}=2\frac{L+\frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}}=2\frac{L+c}{\frac{T}{T_c}} \tag{1}$$

where, $c=\frac{H}{\gamma\epsilon}$, $T_c=\gamma\epsilon/k_B$, [29]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c\neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [26]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [25],[26],[27],[28],[29], due to Bethe-Peierls, [30], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}}-factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c}; factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma=4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

BW	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE I. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours .

C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [30], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

Derivation of this formula Ala [30] is given in the appendix of [7].

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

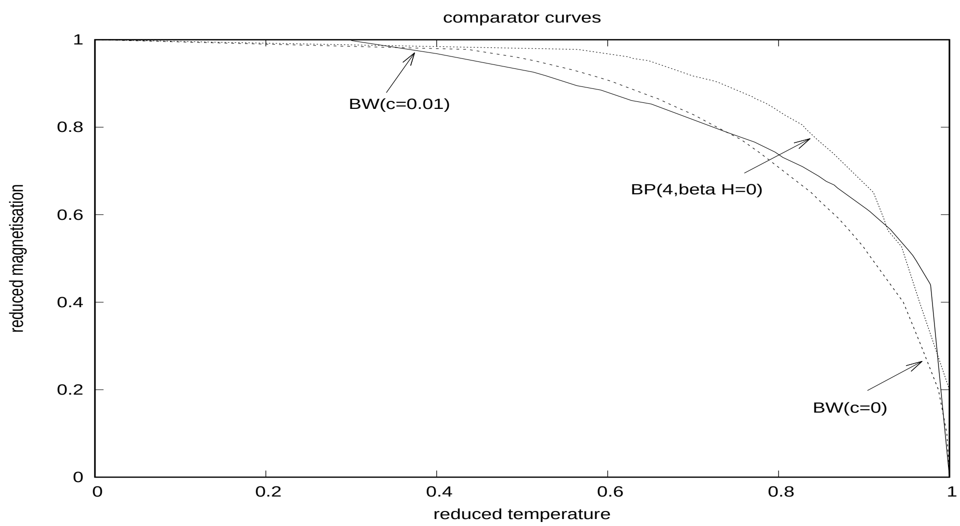

FIG. 1. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.2. Similarly, we plot fig.3. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

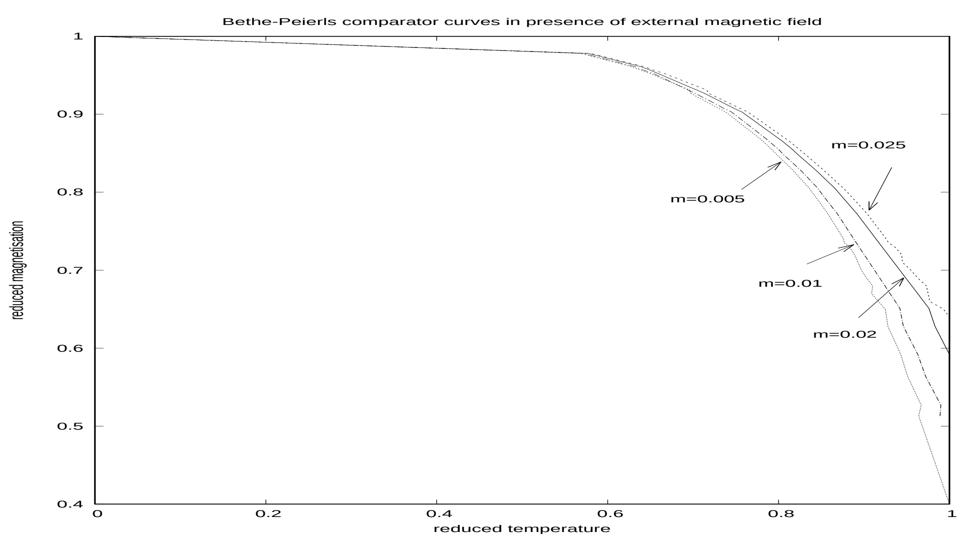

FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

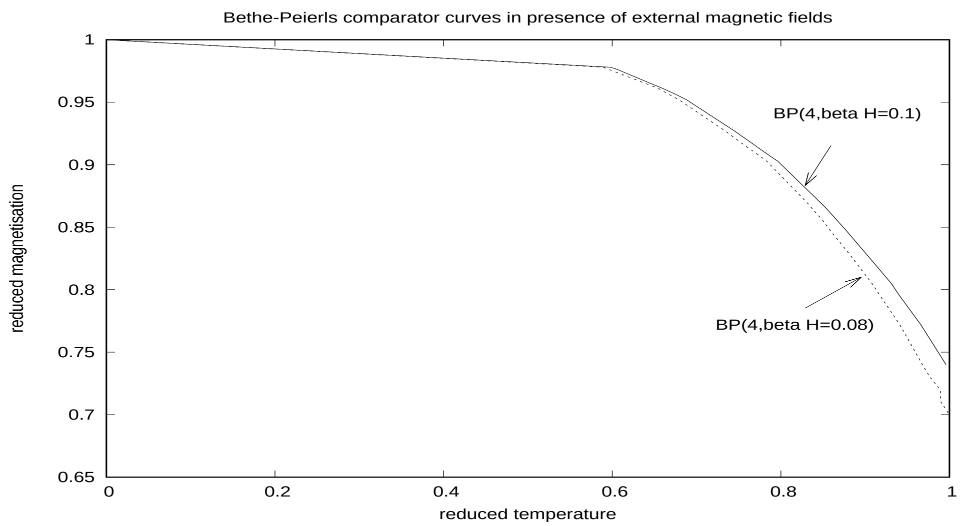

FIG. 3. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

D. Onsager solution

At a temperature T, below a certain temperature called phase transition temperature, T_c, for the two dimensional Ising model in absence of external magnetic field i.e. for H equal to zero, the exact, unapproximated, Onsager solution gives reduced magnetisation as a function of reduced temperature as, [31], [32], [33], [30],

$$\frac{M}{M_{max}} = [1 - (sinh\frac{0.8813736}{\frac{T}{T_c}})^{-4}]^{1/8}.$$

Graphically, the Onsager solution appears as in fig.4.

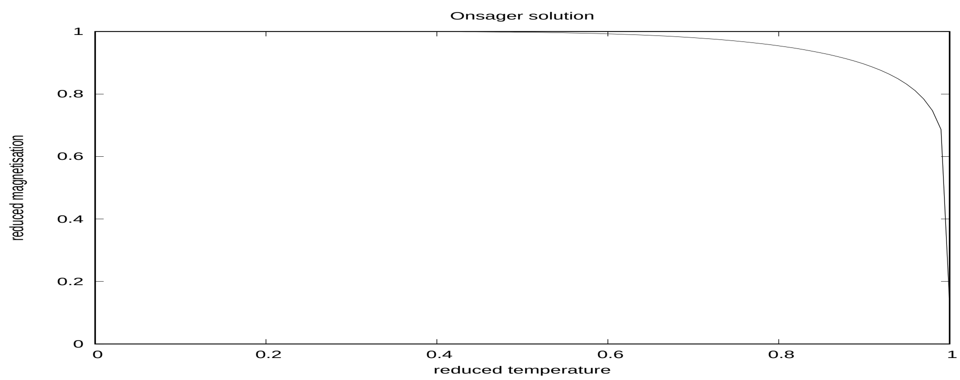

FIG. 4. Reduced magnetisation vs reduced temperature curves for exact solution of two dimensional Ising model, due to Onsager, in absence of external magnetic field

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
6681	1592	3653	297	1263	1852	1204	981	317	1566	615	1669	1241	926	408	677	139	1938	828	1210	1574	1088	937	7314	1861	802	1120	245

TABLE III. Arabic words: 1 stands for aleph

III. ANALYSIS OF WORDS OF THE ARABIC-BENGALI DICTIONARY

The Arabic language alphabet is composed of twenty eight letters. We count all the words, [1], one by one from the beginning to the end, starting with different letters. We have left out the entries which appear starting with different letters in a section belonging to a particular letter. We would like to do it soon, which will improve our result. The result is the table, III.

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by f and the respective rank, denoted by k. Moreover, we attach a limiting rank, k_{lim}, and a limiting number of words. The limiting rank is maximum rank plus one, denoted as k_{lim} or, k_d. Here it is twenty nine and the limiting number of words is one. As a result, k is a positive integer starting from one and both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.5. We then ignore the letter with the highest of words, tabulate in the adjoining table, IV and redo the plot, normalising the lnfs with next-to-maximum $lnf_{nextmax}$, and starting from $k = 2$ in the figure fig.6. This program then we repeat up to $k = 6$, resulting in figures up to fig.11.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{n-max}	lnf/lnf_{2n-max}	lnf/lnf_{3n-max}	lnf/lnf_{4n-max}	lnf/lnf_{5n-max}
1	0	0	7314	8.898	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.205	6681	8.807	0.990	1	Blank	Blank	Blank	Blank
3	1.10	0.326	3653	8.203	0.922	0.931	1	Blank	Blank	Blank
4	1.39	0.412	1938	7.569	0.851	0.859	0.923	1	Blank	Blank
5	1.61	0.478	1861	7.529	0.846	0.855	0.918	0.995	1	Blank
6	1.79	0.531	1852	7.524	0.846	0.854	0.917	0.994	0.999	1
7	1.95	0.579	1669	7.420	0.834	0.843	0.905	0.980	0.986	0.986
8	2.08	0.617	1592	7.373	0.829	0.837	0.899	0.974	0.979	0.980
9	2.20	0.653	1574	7.361	0.827	0.836	0.897	0.973	0.978	0.978
10	2.30	0.682	1566	7.356	0.827	0.835	0.897	0.972	0.977	0.978
11	2.40	0.712	1263	7.141	0.803	0.811	0.871	0.943	0.948	0.949
12	2.48	0.736	1241	7.124	0.801	0.809	0.868	0.941	0.946	0.947
13	2.56	0.760	1210	7.098	0.798	0.806	0.865	0.938	0.943	0.943
14	2.64	0.783	1204	7.093	0.797	0.805	0.865	0.937	0.942	0.943
15	2.71	0.804	1120	7.021	0.789	0.797	0.856	0.928	0.933	0.933
16	2.77	0.822	1088	6.992	0.786	0.794	0.852	0.924	0.929	0.929
17	2.83	0.840	981	6.889	0.774	0.782	0.840	0.910	0.915	0.916
18	2.89	0.858	937	6.843	0.769	0.777	0.834	0.904	0.909	0.909
19	2.94	0.872	926	6.831	0.768	0.776	0.833	0.902	0.907	0.908
20	3.00	0.890	828	6.719	0.755	0.763	0.819	0.888	0.892	0.893
21	3.04	0.902	802	6.687	0.752	0.759	0.815	0.883	0.888	0.889
22	3.09	0.917	677	6.518	0.733	0.740	0.795	0.861	0.866	0.866
23	3.14	0.932	615	6.422	0.722	0.729	0.783	0.848	0.853	0.854
24	3.18	0.944	408	6.011	0.676	0.683	0.733	0.794	0.798	0.799
25	3.22	0.955	317	5.759	0.647	0.654	0.702	0.761	0.765	0.765
26	3.26	0.967	297	5.694	0.640	0.647	0.694	0.752	0.756	0.757
27	3.30	0.979	245	5.501	0.618	0.625	0.671	0.727	0.731	0.731
28	3.33	0.988	139	4.934	0.555	0.560	0.601	0.652	0.655	0.656
29	3.37	1	1	0	0	0	0	0	0	0

TABLE IV. Arabic words: ranking, natural logarithm, normalisations

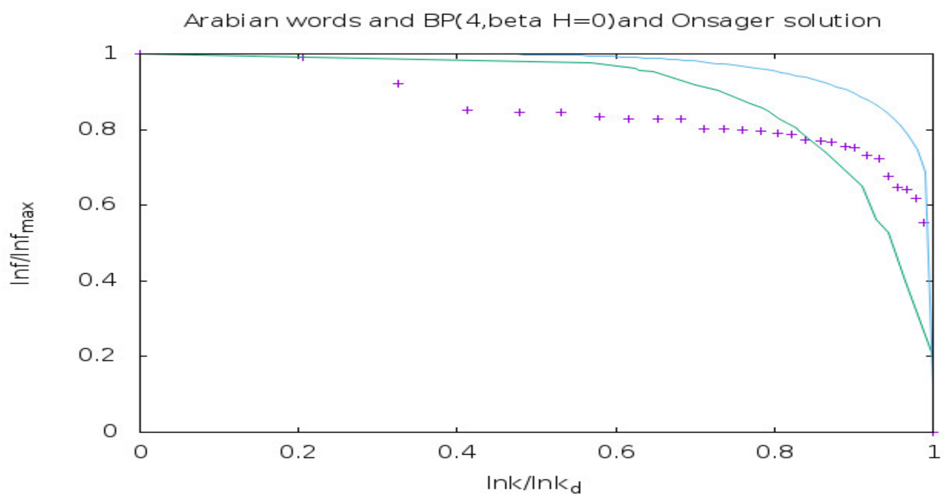

FIG. 5. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the lower line being the Bethe-Peierls curve, BP(4, $\beta H = 0$), with four nearest neighbours, in the absence of external magnetic field. The uppermost curve is the Onsager solution.

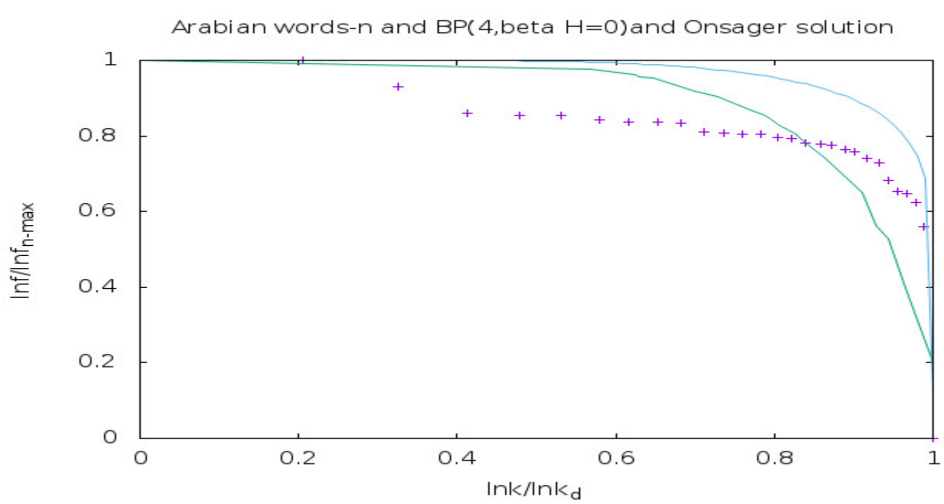

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{next-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the lower line being the Bethe-Peierls curve, BP(4, $\beta H = 0$), with four nearest neighbours, in the absence of external magnetic field. The uppermost curve is the Onsager solution.

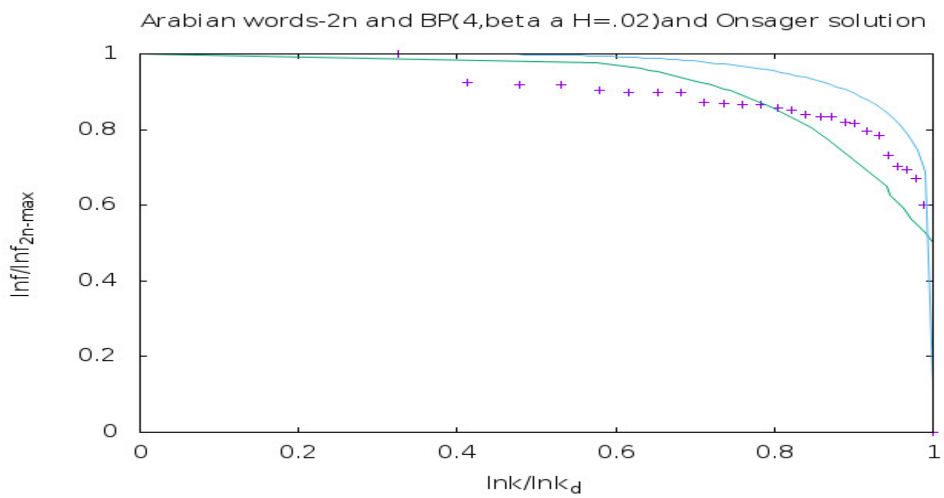

FIG. 7. The vertical axis is $\frac{lnf}{lnf_{nextnext-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the lower line being the Bethe-Peierls curve, BP(4, $\beta H = 0.02$), with four nearest neighbours, in the presence of little external magnetic field. The uppermost curve is the Onsager solution.

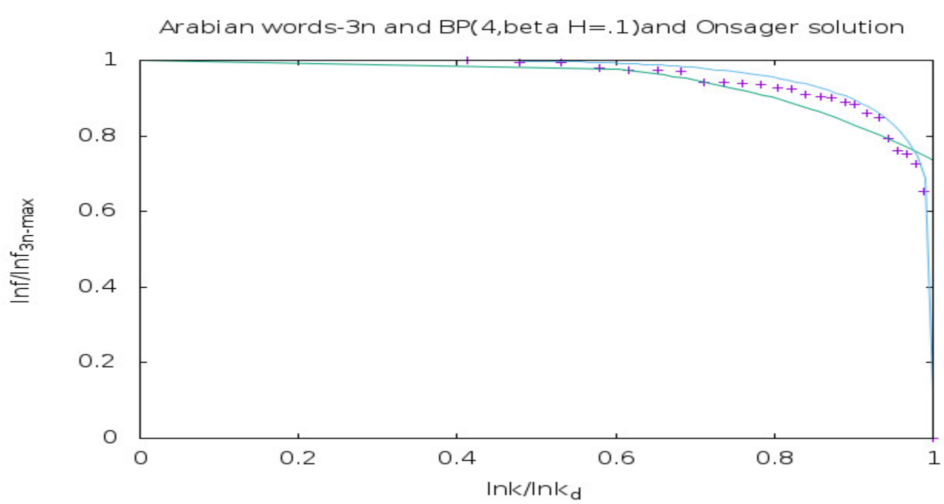

FIG. 8. The vertical axis is $\frac{lnf}{lnf_{nextnextnext-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the lower line being the Bethe-Peierls curve, BP(4, $\beta H = 0.1$), with four nearest neighbours, in the presence of little magnetic field, m=0.05 or, $\beta H = 0.1$. The uppermost curve is the Onsager solution.

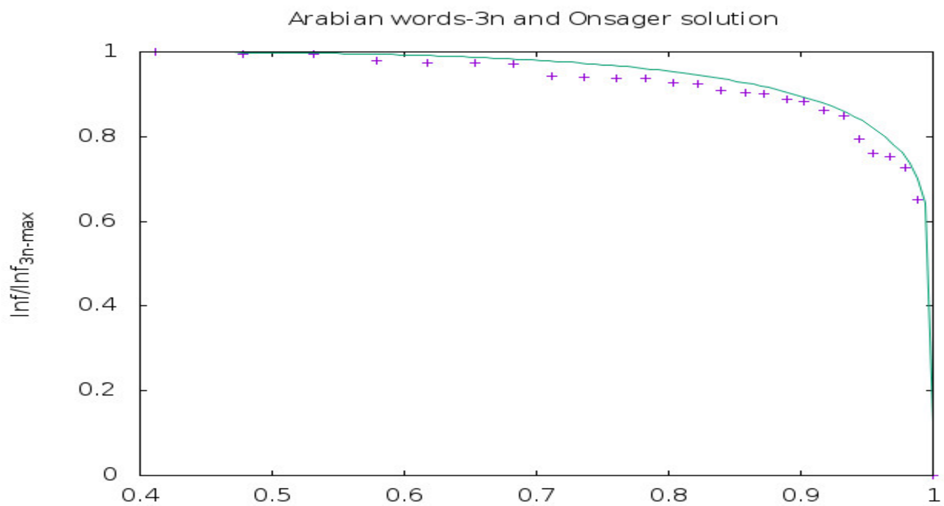

FIG. 9. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the fit curve being the Onsager solution.

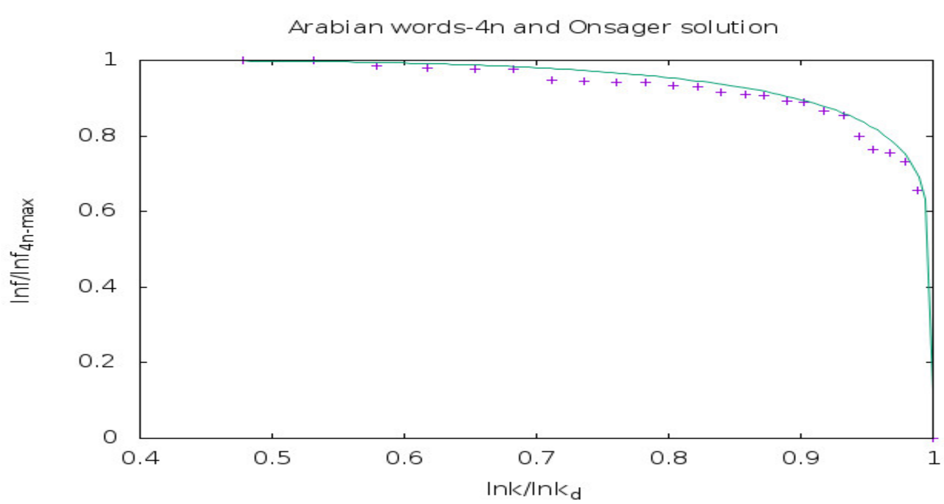

FIG. 10. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the fit curve being the Onsager solution.

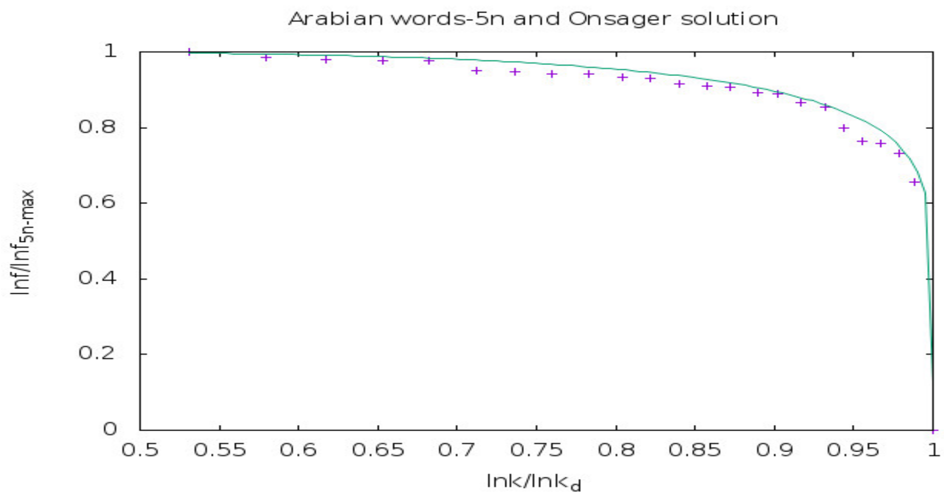

FIG. 11. The vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the words of the Arabic language with the fit curve being the Onsager solution.

A. conclusion

From the figures (fig.5-fig.11), we observe that the words of the Arabic language, [1], underlies the Onsager solution.

Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{3n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [35].

IV. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper.

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