Radii of Electron, Proton, Neutron, Deuteron and Helium Nucleus

Gang Chen†, Tianman Chen, Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China

7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

†Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

In our previous papers, we once gave formulas and value for the classical electron radius (r_e=2.81794032658(43) fm) and the proton charge radius. In this paper, we give new and more reasonable formulas and values for the proton charge radius, and the values should be different according to the three different measurement methods, so we give three values, i.e., $r_{p/H}$=0.8330977868 fm, $r_{p/H\text{-}\mu}$=0.8419605292 fm and $r_{p/e}$=0.8311047299 fm. In addition, we also give formulas and values for the neutron charge radius ($r_{n/e}$=0.3312876729 fm), the deuteron charge radius ($r_{d/D}$=2.142299805 fm, $r_{d/D\text{-}\mu}$=2.125297426 fm and $r_{d/e}$=2.127921954), the neutron equivalent radius in deuteron ($r_{n/D}$=1.309202018 fm) and the charge radius of helium nucleus (α particle) ($r_{\alpha/He}$= 1.688564465 fm, $r_{\alpha/He\text{-}\mu}$= 1.678205173 fm and $r_{\alpha/e}$=1.681409530).

Keywords: radius; electron; proton; neutron; deuteron; helium nucleus.

1. Introduction

In our previous papers[1-10], we gave formulas of the fine structure constant and their applications or relevant developments. Some typical formulas and definitions of the fine-structure constant are as follows.

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0},\quad \alpha_2 = \frac{2\pi r_e}{\lambda_e},\quad \alpha_c = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{v_e}{c}$$

$$\alpha_1 = \frac{36}{7\cdot(2\pi)_{Chen-112}} \frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{13\cdot(2\pi)_{Chen-278}}{100} \frac{1}{112-\frac{1}{64\cdot 3\cdot 29}} = 1/137.035999111818$$

$2\pi - e$ formula: $(2\pi)_{\ Chen\text{-}k} = (\frac{e}{e^{\gamma_{c-k}}})^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{\frac{2\pi a_0}{\lambda_e} \frac{\lambda_e}{2\pi r_e}} = \sqrt{\frac{a_0}{r_e}}$$

$$= \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}$$

$$= \sqrt{137.035999037435 \times 137.035999111818} = 137.035999074626$$

With these definitions and formulas, we calculated the classical electron radius r_e as follows[1].

$$\frac{a_0}{r_e} = \frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = 112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)}) = 18788.865042381$$

$$r_e = \alpha_c^2 a_0 = \alpha_1 \alpha_2 a_0 = \frac{5.29177210903(80) \times 10^{-11} m}{18788.865042381} = 2.81794032658(43)\ fm$$

The above calculated r_e=2.81794032658(43) was more precise (with one more effective digit) than current CODATA recommended value r_e=2.8179403262(13) fm.

In the same paper[1], we also proposed the similar formulas for the proton charge radius r_p as follows.

$$\frac{a_0}{r_p} = \frac{1}{\alpha_{p/c}^2} = \frac{1}{\alpha_{p/1} \alpha_{p/2}} = 225 \cdot (282 + \frac{1}{3} - \frac{1}{12 \cdot 47} + \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79/47}) = 63524.60147736$$

$$= 247 \cdot (257 + \frac{1}{5} - \frac{1}{5 \cdot 13} + \frac{1}{30 \cdot (28 \cdot (2 \cdot 100 - 1) + 1) + \frac{8}{45}})$$

$$= (252 + \frac{1}{24} - \frac{1}{2 \cdot 17 \cdot 37} + \frac{1}{11 \cdot 13 \cdot 19 \cdot (2 \cdot 11 \cdot 19 + 1) + \frac{11}{20}})^2 = 252.040872632515^2$$

$$r_p = \alpha_{p/c}^2 a_0 = \alpha_{p/1} \alpha_{p/2} a_0 = \frac{5.29177210903(80) \times 10^{-11}\ m}{63524.60147736} = 0.833027202999(13)\ \ fm$$

$\alpha_{p/c} \approx \alpha_{p/1} \approx \alpha_{p/2} \approx 252.04$, α_p could be called the second fine-structure constant.
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However, the above formulas for r_p should not be satisfyingly correct and need corrections. In this paper, we try to find more reasonable formulas and values for r_p. In addition, we also give formulas and values for neutron, deuteron and helium nucleus.

2. The Proton-radius Puzzle

Scientists developed two approaches to measure the proton charge radius, one is hydrogen spectroscopy (Lamb shift measurement), the other is elastic electron-proton scattering (p/e method). And the first approach was divided into two sub-branches which are ordinary-hydrogen spectroscopy (p/H method) and muonic hydrogen spectroscopy (p/H-μ method). There were always discrepancies among the results of the measurements with these three methods as shown in the following table and figure (**Table 1** and **Fig. 1**). This strange phenomenon is called the proton-radius puzzle.

Table 1. Values for the proton radius measured by the three methods

Year	Method	r_p	Value (fm)	Ref.
2010	Electronic-proton scattering	$r_{p/e}$	0.879(8)	11
2010	Muonic-hydrogen spectroscopy	$r_{p/H\text{-}\mu}$	0.84184(67)	12
2013	Muonic-hydrogen spectroscopy	$r_{p/H\text{-}\mu}$	0.84087(39)	13
2017	Ordinary-hydrogen spectroscopy	$r_{p/H}$	0.8335(95)	14
2018	Ordinary-hydrogen spectroscopy	$r_{p/H}$	0.877(13)	15
2019	Ordinary-hydrogen spectroscopy	$r_{p/H}$	0.833(10)	16
2019	Electronic-proton scattering	$r_{p/e}$	0.831(14)	17

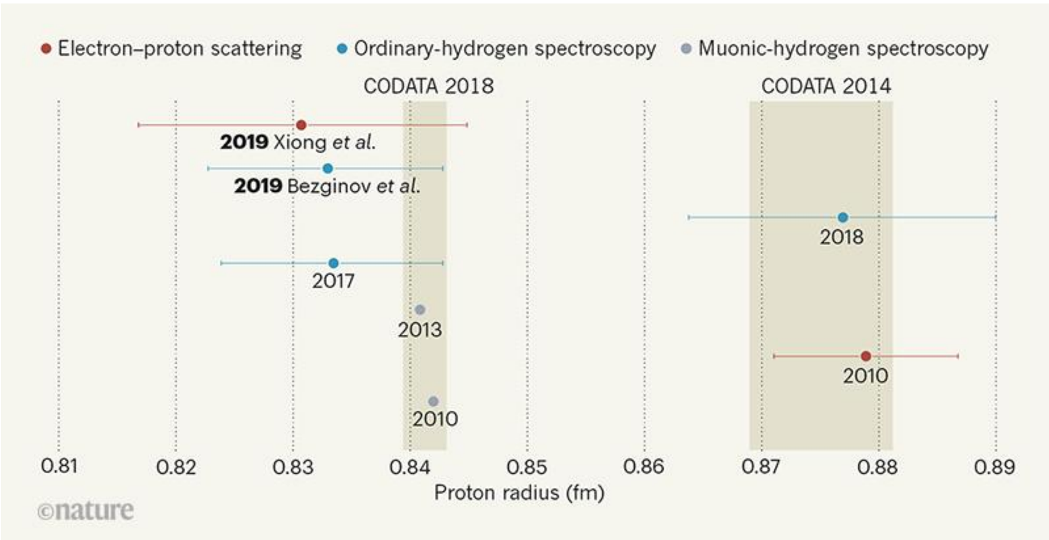

Figure 1. Values for the proton charge radius (figure from Nature[18]).

According to our points of view, these discrepancies are natural and should be exist reasonably. Proton should be just like an elastic football which should give different feelings (radii) to an adult or a child, by hand and by foot, so we suppose there should be three kinds of the proton charge radius which are denoted as $r_{p/H}$, $r_{p/H\text{-}\mu}$ and $r_{p/e}$.

3. Formulas for the Proton Charge Radius

In our previous formulas for the classical electron radius, we employed the fine-structure constant α as follows.

$$r_e = \alpha_c^2 a_0 = \alpha_1 \alpha_2 a_0 = \frac{5.29177210903(80)\times 10^{-11} m}{18788.865042381} = 2.81794032658(43)\ fm$$

$$\alpha_1 = \frac{1}{56+81+\cfrac{1}{28-\cfrac{13\cdot(2\cdot 56\cdot 11-1)}{3\cdot 5\cdot(2\cdot 56\cdot 43+1)}}} = 1/137.035999037435$$

$$\alpha_2 = \frac{1}{56+81+\cfrac{1}{28-\cfrac{2\cdot(16\cdot 27-1)}{3\cdot(16\cdot 81+1)}}} = 1/137.035999111818$$

$$\alpha_c = \frac{1}{56+81+\cfrac{1}{28-\cfrac{5\cdot(4\cdot 3\cdot 7\cdot 17-1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}}} = \frac{1}{56+81+\cfrac{1}{27+\cfrac{4\cdot(4\cdot(2\cdot 27\cdot 29+1)+1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}}}$$

$$= 1/137.035999074626$$

the key element and its isotopes: ${}^{136,137,138}_{56}Ba_{80,81,82}$ (Note: $136 = 8\cdot 17,\ 138 = 6\cdot 23$)

It is supposed that the situation should be similar for the proton charge radius, and the critical point is to find the key element and its isotopes. And hence we construct the following formulas for the proton charge radius to give reasonable and precise values.

$$r_{p/H} = \frac{a_o}{(252+\delta)^2} = \frac{52917.72109\ fm}{(99+153+\cfrac{1}{33+\cfrac{2}{17}})^2} = 0.8330977868\ fm$$

$$r_{p/H-\mu} = r_{p/H}\times(1+\frac{1}{2\cdot 47}) = \frac{a_o}{(251-\frac{1}{3}+\frac{1}{29}-\cfrac{1}{2\cdot 13\cdot 41+\cfrac{4}{31}})^2} = 0.8419605292\ fm$$

$$r_{p/e} = r_{p/H}\times(1-\frac{1}{2\cdot 11\cdot 19}) = \frac{a_o}{(2\cdot 126+\frac{1}{3}-\cfrac{1}{7\cdot(126+1)+\cfrac{2\cdot 7}{41}})^2} = 0.8311047299\ fm$$

the key element and its isotope: ${}^{252}_{99}Es^{*}_{153}$ (Note: $153 = 9\cdot 17$，$252 = 2\cdot 126 = 9\cdot 28$)

$$\frac{r_{p/H}}{r_e} = \frac{2\cdot 17}{5\cdot 23} - \cfrac{1}{2\cdot 3\cdot 7\cdot 19\cdot 109+\cfrac{107}{278}} = 0.295640677332428$$

$$\frac{r_{p/H}}{r_e} = \left(\frac{56+81+\cfrac{1}{28-\cfrac{5\cdot(4\cdot 3\cdot 7\cdot 17-1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}}}{99+153+\cfrac{1}{33+\cfrac{2}{17}}}\right)^2 = 0.29564067732429$$

$$\frac{r_{p/H}}{r_e} == \frac{112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{14\cdot 112\cdot(2\cdot 173+1)}}{(99+153+\frac{1}{33+\frac{2}{17}})^2} = 0.29564067732429$$

$$\frac{r_e}{r_{p/H}} = 3+\frac{1}{2}-\frac{1}{8}+\frac{1}{7\cdot 19}-\frac{1}{27\cdot 13\cdot 83-\frac{23}{30}} = 3.3824844707536$$

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${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{32,33,34}_{16}S_{16,17,18}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{60,61,62}_{28}Ni_{32,33,34}$ ${}^{57,58}_{26}Fe_{31,32}$

${}^{61,62}_{28}Ni_{33,34}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{76,80,82}_{34}Se_{42,46,48}$ ${}^{93}_{41}Nb_{52}$ ${}^{99,100,101,102}_{44}Ru_{55,56,57,58}$

${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{124,125,126}_{52}Te_{72,73,74}$ ${}^{133}_{55}Cs_{78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$

${}^{140}_{58}Ce_{82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{142,143,144}_{60}Nd_{82,83,84}$ ${}^{149,150,152}_{62}Sm_{87,88,90}$ ${}^{153}_{63}Eu_{90}$ ${}^{166,167,168,170}_{68}Er_{98,99,100,102}$

${}^{169}_{69}Tm_{100}$ ${}^{4\cdot 47}_{76}Os_{112}$ ${}^{206,207,208}_{82}Pt_{124,125,126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{251}_{98}Cf^{*}_{153}$

${}^{252}_{99}Es^{*}_{153}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{4\cdot 47}$ ${}^{3\cdot 112}_{133}Ch^{ie}_{203}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{348}_{139}Ch^{ie}_{209}$ ${}^{384}_{153}Ch^{ie}_{7\cdot 33}$ ${}^{418}_{166}Ch^{ie}_{252}$

These formulas and values should not be absolutely correct, but it seems that they are reasonable and should be relatively precise.

4. Formulas for the Neutron Charge Radius

With the same method, we construct the following formulas for the neutron charge radius and give reasonable and precise value of it.

$$r_{n/e} = \frac{a_o}{(400-\delta)^2} = \frac{52917.72109\ fm}{(157+243-\frac{1}{3})^2} = 0.3312876729\ fm$$

$$r_{n/e}{}^2 = 0.1097515222\ fm^2$$

the key element and its isotope: ${}^{400}_{157}Ch^{ie}_{243}$ (Note: $243 = 3^5$)

${}^{157}_{64}Gd_{93}$ ${}^{200}_{80}Hg_{120}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{4\cdot 47}$ ${}^{400}_{157}Ch^{ie}_{243}$

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$$\frac{r_{n/e}}{r_e} = \frac{1}{17}-\frac{1}{2\cdot 9\cdot 23\cdot 29-\frac{13}{2\cdot 17}} = 0.1175637645$$

$$\frac{r_e}{r_{n/e}} = 9-\frac{1}{2}+\frac{1}{2\cdot 83}-\frac{1}{81\cdot 7\cdot(4\cdot 243-1)} = 8.506022280$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{58}_{26}Fe_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80}_{34}Se_{46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{104}_{46}Pd_{58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$

${}^{140,142}_{58}Ce_{82,84}$ ${}^{166,168,170}_{68}Er_{98,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{225}_{87}Fr^{*}_{138}$ ${}^{243}_{95}Am^{*}_{148}$ ${}^{257}_{100}Fm^{*}_{157}$

${}^{2\cdot 157}_{126}Ch^{ie}_{188}$ ${}^{344,346,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{418}_{166}Ch^{ie}_{252}$

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$$\frac{r_{n/e}}{r_{p/H}} = \left(\frac{99+153+\cfrac{1}{33+\cfrac{2}{17}}}{(157+243-\frac{1}{3})}\right)^2 = 0.39765760767979$$

$$\frac{r_{n/e}}{r_{p/H}} = \frac{4\cdot 17}{9\cdot 19} - \frac{1}{2\cdot 31\cdot(2\cdot 81\cdot 31+1) \ or \ 2\cdot 31\cdot(32\cdot 157-1)+\cfrac{11}{8\cdot 7}} = 0.39765760767979$$

$$\frac{r_{p/H}}{r_{n/e}} = \frac{5}{2} + \frac{1}{4\cdot 17} + \frac{1}{11^3\cdot 37-\cfrac{17}{59}} = 2.5147261882772$$

$^{35,37}_{17}Cl_{18,20}$ $^{56,57}_{26}Fe_{30,31}$ $^{62}_{28}Ni_{34}$ $^{64,67,68}_{30}Zn_{34,37,38}$ $^{69,71}_{31}Ga_{38,40}$ $^{85,87}_{37}Rb_{48,50}$ $^{99,100,101}_{44}Ru_{55,56,57}$

$^{12\cdot 59}_{50}Sn_{68}$ $^{121,123}_{51}Sb_{70,72}$ $^{138,139}_{57}La_{81,82}$ $^{136,137}_{56}Ba_{80,81}$ $^{147,148}_{62}Sm_{85,86}$ $^{157}_{64}Gd_{93}$ $^{168}_{68}Er_{100}$ $^{9\cdot 19}_{70}Yb_{101}$

$^{201,204}_{80}Hg_{121,124}$ $^{203,205}_{81}Tl_{122,124}$ $^{257}_{100}Fm^{*}_{157}$ $^{302}_{121}Ch^{ie}_{181}$ $^{310}_{124}Ch^{ie}_{186}$ $^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{400}_{157}Ch^{ie}_{243}$

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A recent paper[19] explained: "Despite the neutron zero-net electric charge, the asymmetric distribution of the positively- (up) and negatively-charged (down) quarks, a result of the complex quark-gluon dynamics, lead to a negative value for its squared charge radius, r_n^2." And it gave a relatively precise result of $r_n^2 = -0.110(8)$ fm^2. By referring to this result, we give the above formulas and value for the neutron charge radius, and they seem to be still reasonable and much more precise.

5. Different Difficulties to Determine the Radii of Electron, Proton and Neutron

We noticed that the difficulties to determine the radii of electron, proton and neutron are quite different by experimental measurements or theoretical calculations. But why? Here we try to propose an explanation.

In our previous paper[3], we developed Mass Model of Elementary Particles. The main picture of the model is shown as follows (**Fig. 2**).

According to the model's stipulations, electron is a Yang particle, we can assume it should be hard, round and smooth like a steel ball, so its radius could be determined easily and precisely. However, a proton contains two up quarks (u) which are Yin and one down quark (d) which is Yang, so a proton is a net Yin particle, and we can assume it should be soft, not very round, elastic and rough like a football, so it is difficult to determine its radius and the determined values should depend on the measurement

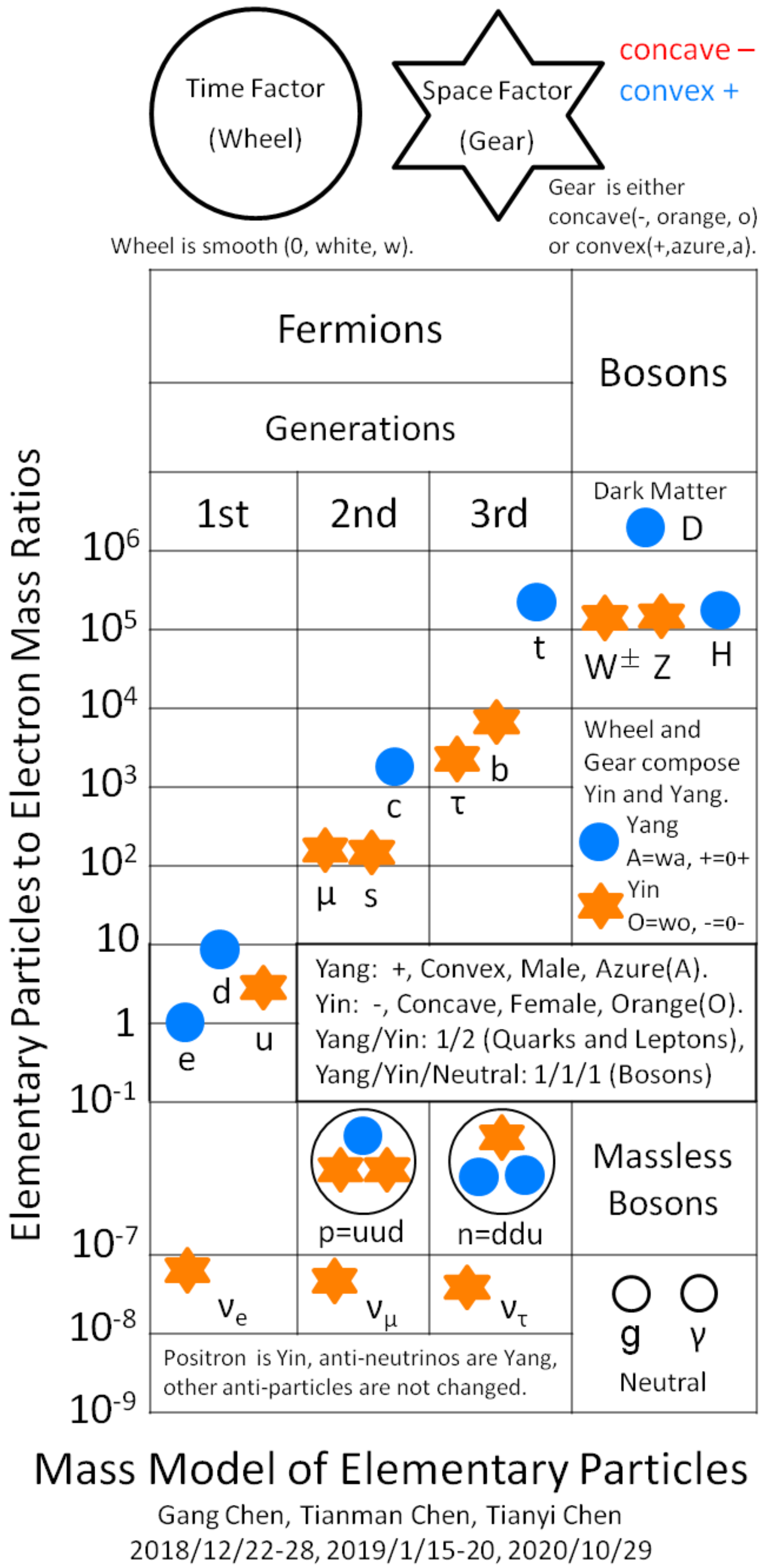

Figure 2.

methods. A neutron contains two down quarks (d) which are Yang and one up quark (u) which is Yin, so a neutron is a net Yang particle, it should be not very difficult to determine its radius, however, it is neutral in net electric charge, so it is still difficult to determine its electric charge radius which should be resulted from asymmetric distribution of its component quarks and gluons. According to this explanation, we could also predict that it should be relatively easy to determine the radii of Yang particles such as Higgs boson (H) and the particle of dark matter (D), and it should be relatively difficult to determine the radii of Yin particles such as muon, tauon, W boson, Z boson and the neutrino particles.

6. Formulas for the Deuteron Charge Radius

Deuteron is the simplest compound nucleus consisting of a proton and a neutron, it should one of typical and important nuclei. Although it should not be in spheric shape, its root-mean square radius (or equivalent radius) could be measured by experiments. And there are also three methods of measurement, i.e., ordinary-deuterium spectroscopy, muonic-deuterium spectroscopy and elastic electron-deuteron scattering, corresponding to three kinds of deuteron charge radius $r_{d/D}$, $r_{d/D\text{-}\mu}$ and $r_{d/e}$ respectively. The experimental values of these three kinds of r_d are listed as follows (**Table 2**).

Table 2. Values for the deuteron charge radius measured by the three methods

Year	Method	r_d	Value (fm)	Ref.
1996	Electronic-deuteron scattering	$r_{d/e}$	2.128(11)	20
2017	Ordinary-deuterium spectroscopy	$r_{d/D}$	2.1415(45)	21
2016	Muonic-deuterium spectroscopy	$r_{d/D\text{-}\mu}$	2.12562(78)	22

Here we construct the following formulas for the deuteron charge radius to give reasonable and precise values.

$$r_{d/D} = \frac{a_0}{(157+\delta)^2} = \frac{52917.72109\ fm}{(64+93+\frac{1}{6})^2} = 2.142299805\ fm$$

$$r_{d/D-\mu} = r_{d/D} \times (1-\frac{1}{126}) = \frac{52917.72109\ fm}{(64+93+\frac{1}{6})^2} \times (1-\frac{1}{126}) = 2.125297426\ fm$$

$$r_{d/e} = r_{d/D} \times (1-\frac{1}{149}) = \frac{52917.72109\ fm}{(64+93+\frac{1}{6})^2} \times (1-\frac{1}{149}) = 2.127921954\ fm$$

the key element and its isotope: ${}^{157}_{64}Gd_{93}$

${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{93}_{41}Nb_{52}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{110,112}_{48}Cd_{62,64}$ ${}^{126}_{52}Te_{74}$ ${}^{149}_{62}Sm_{87}$ ${}^{157}_{64}Gd_{93}$ ${}^{4\cdot 47}_{76}Os_{112}$

${}^{208}_{82}Pt_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{4\cdot 47}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$ ${}^{400}_{157}Ch^{ie}_{243}$

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$$\frac{r_{d/D}}{r_e}=\frac{2.142299805\,fm}{2.81794032656\,fm}=\frac{19}{25}+\frac{1}{4\cdot 3\cdot(32\cdot 11+1)+\frac{4\cdot 11}{3\cdot 29}}=0.7602360436$$

$^{23}_{11}Na_{12}$ $^{55}_{25}Mn_{30}$ $^{63,64}_{29}Cu_{34,36}$ $^{85,87}_{37}Rb_{48,50}$ $^{87,88}_{38}Sr_{49,50}$ $^{99,101,102}_{44}Ru_{55,57,58}$ $^{161,162}_{66}Dy_{95,96}$
$^{223}_{87}Fr^{*}_{136}$ $^{226}_{88}Ra^{*}_{138}$ $^{243}_{95}Am^{*}_{148}$

$$\frac{r_{p/H}}{r_{d/D}}=\frac{0.8330977868\,fm}{2.142299805\,fm}=\frac{7}{2\cdot 9}+\frac{1}{8\cdot 3\cdot 47\cdot 101}=0.3888801114$$

$^{100,101}_{44}Ru_{56,57}$ $^{171}_{70}Yb_{101}$ $^{4\cdot 47}_{76}Os_{112}$ $^{209}_{83}Bi^{*}_{126}$ $^{257}_{100}Fm^{*}_{157}$ $^{258}_{101}Md^{*}_{157}$ $^{2\cdot 157}_{126}Ch^{ie}_{4\cdot 47}$ $^{400}_{157}Ch^{ie}_{243}$

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It could be supposed that $r_{d/D}$ should be composed of two parts which are $r_{p/H}$ and $r_{n/D}$ (the proton charge radius in hydrogen and the neutron equivalent radius in deuteron) as follows.

$$r_{d/D}=r_{p/H}+r_{n/D}=0.8330977868+1.309202018=2.142299805\,fm$$

$$r_{n/D}=\frac{a_0}{(201+\delta)^2}=\frac{52917.72109\,fm}{(80+121+\frac{1}{2\cdot 11}+\frac{1}{36\cdot 11+\frac{3}{2\cdot 11}})^2}=1.309202018\,fm$$

the key element and its isotope: $^{201}_{80}Hg_{121}$

$^{23}_{11}Na_{12}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{66}_{30}Zn_{36}$ $^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ $^{120,122}_{50}Sn_{70,72}$ $^{121,123}_{51}Sb_{70,72}$
$^{136,137,138}_{56}Ba_{80,81,82}$ $^{200,201}_{80}Hg_{120,121}$ $^{302}_{121}Ch^{ie}_{181}$

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$$\frac{r_{n/D}}{r_e}=\frac{1.309202018\,fm}{2.81794032656\,fm}=\frac{13}{28}+\frac{1}{2\cdot 5\cdot 17\cdot 19-\frac{2\cdot 23}{81}}=0.4645953663$$

$$\frac{r_{p/H}}{r_{n/D}}=\frac{0.8330977868\,fm}{1.309202018\,fm}=\frac{7}{11}+\frac{1}{157\cdot(16\cdot 17-1)+\frac{23}{29}}=0.6363401334$$

$^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{35,37}_{17}Cl_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{50,51}_{23}V_{27,28}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$
$^{77,78,80}_{34}Se_{43,44,46}$ $^{79,81}_{35}Br_{44,46}$ $^{100}_{44}Ru_{56}$ $^{104}_{46}Pd_{58}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{157}_{64}Cd_{93}$ $^{168}_{68}Er_{100}$ $^{169}_{69}Tm_{100}$
$^{191,193}_{77}Ir_{114,116}$ $^{223}_{87}Fr^{*}_{136}$ $^{226}_{88}Ra^{*}_{138}$ $^{257}_{100}Fm^{*}_{157}$ $^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{400}_{157}Ch^{ie}_{243}$

2021/8/11

7. Formulas for the Charge Radius of the Helium Nucleus

After hydrogen, helium is the second most abundant element in the universe. Around one-fourth of the atomic nuclei that formed in the first few minutes after the Big Bang were helium nuclei. A helium nucleus, which is especially stable, consists of two protons and two neutrons. For fundamental physics, it is crucial to know the properties of the helium nucleus[24]. One of the properties is the charge radius of it. There are also three methods to measure its charge radius, i.e., ordinary-helium spectroscopy, muonic-helium spectroscopy and elastic electron scattering, corresponding to three kinds of the charge radius of the helium nucleus (also called α particle) $r_{\alpha/He}$, $r_{\alpha/He\text{-}\mu}$ and $r_{\alpha/e}$ respectively. The experimental values of these three kinds of r_α are listed as follows (**Table 3**).

Table 3. Values for the charge radius of helium nucleus measured by the three methods

Year	Method	r_d	Value (fm)	Ref.
2008	Elastic electron scattering	$r_{\alpha/e}$	1.681(4)	23
	Ordinary-deuterium spectroscopy	$r_{\alpha/He}$	so far no determination	
2021	Muonic-deuterium spectroscopy	$r_{\alpha/He\text{-}\mu}$	1.67824(83)	24

Here we construct the following formulas for the charge radius of the helium nucleus to give reasonable and precise values.

$$r_{\alpha/He}=\frac{a_0}{(177+\delta)^2}=\frac{a_0}{(3\cdot59+\delta)^2}=\frac{52917.72109\ fm}{(72+105+\cfrac{1}{36-\cfrac{6}{35}})^2}=1.688564465\ fm$$

$$r_{\alpha/He-\mu}=\frac{a_0}{(177+\delta)^2}(1-\gamma)=\frac{52917.72109\ fm}{(72+105+\cfrac{1}{36-\cfrac{6}{35}})^2}(1-\frac{1}{163})=1.678205173\ fm$$

$$r_{\alpha/e}=\frac{a_0}{(177+\delta)^2}(1-\gamma)=\frac{52917.72109\ fm}{(72+105+\cfrac{1}{36-\cfrac{6}{35}})^2}(1-\frac{1}{4\cdot59})=1.681409530\ fm$$

the key element and its isotope: ${}^{177}_{72}Hf_{105}$

${}^{31}_{15}P_{16}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{46}_{22}Ti_{24}$ ${}^{59}_{27}Co_{32}$ ${}^{66}_{30}Zn_{36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{82,84,86}_{36}Kr_{46,48,50}$ ${}^{97}_{42}Mo_{55}$

${}^{105}_{46}Pd_{59}$ ${}^{141}_{59}Pr_{82}$ ${}^{144}_{60}Nd_{84}$ ${}^{162,163}_{66}Dy_{96,97}$ ${}^{177,180}_{72}Hf_{105,108}$ ${}^{235,4\cdot59}_{92}U^{*}_{143,144}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$

${}^{247}_{97}Bk^{*}_{150}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{293}_{116}Lv^{ie}_{177}$ ${}^{294}_{117}Ts^{ie}_{177}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{408}_{162}Ch^{ie}_{246}$ ${}^{410}_{163}Ch^{ie}_{247}$

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$$\frac{r_{\alpha/He}}{r_e}=\frac{1.688564465\ fm}{2.81794032656\ fm}=\frac{3}{5}+\frac{1}{3\cdot7\cdot61+\cfrac{1}{2\cdot13}}=\frac{3}{5}+\frac{1}{2\cdot(6\cdot107-1)-\cfrac{25}{2\cdot13}}$$

$$=0.5992193833$$

${}^{27}_{13}Al_{14}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{61}_{28}Ni_{33}$ ${}^{106,108}_{46}Pd_{60,62}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{145,146,147}_{61}Pm^{*}_{84,85,86}$ ${}^{183}_{74}W_{109}$ ${}^{306}_{123}Ch^{*}_{183}$

$$\frac{r_{\alpha/He}}{r_e}=\frac{1.688564465\ fm}{2.81794032656\ fm}=\frac{151}{4\cdot 9\cdot 7}+\frac{1}{2\cdot 3\cdot 19\cdot(16\cdot 3\cdot 7+1)}=0.5992193833$$

$${}^{83,84}_{36}Kr_{47,48}\quad {}^{111,112}_{48}Cd_{63,64}\quad {}^{126}_{52}Te_{74}\quad {}^{151,153}_{63}Er_{88,90}\quad {}^{188}_{76}Os_{112}\quad {}^{209}_{83}Bi^{*}_{126}\quad {}^{209}_{84}Po^{*}_{125}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{314}_{126}Ch^{*}_{188}$$

$$\frac{r_{p/H}}{r_{\alpha/He}}=\frac{0.8330977868\ fm}{1.688564465\ fm}=\frac{37}{75}+\frac{1}{2\cdot(4\cdot 5\cdot 7\cdot 83+1)+\frac{7}{47}}=0.4933763586$$

$${}^{35,37}_{17}Cl_{18,20}\quad {}^{47,48}_{22}Ti_{25,26}\quad {}^{83,84}_{36}Kr_{47,48}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{126}_{52}Te_{74}\quad {}^{5\cdot 37,187}_{75}\mathrm{Re}_{110,112}\quad {}^{209}_{83}Bi^{*}_{126}\quad {}^{314}_{126}Ch^{*}_{4\cdot 47}$$

2021/8/12

$$\frac{r_{\alpha/He}}{r_{d/D}}=\frac{1.688564465\ fm}{2.142299805\ fm}=\frac{67}{5\cdot 17}-\frac{1}{5\cdot 67\cdot 89+\frac{6}{17}}=0.7882017544$$

$${}^{35,37}_{17}Cl_{18,20}\quad {}^{67}_{30}Zn_{37}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{89}_{39}Y_{50}\quad {}^{117}_{50}Sn_{67}\quad {}^{151,153}_{63}Eu_{88,90}\quad {}^{165}_{67}Ho_{98}\quad {}^{210}_{85}At^{*}_{125}\quad {}^{227}_{89}Ac^{*}_{138}$$

2021/8/13

According to Krauth's description[24], so far no determination of r_α exists from atomic spectroscopy. For the determination of absolute radii from He atoms (three-body system with two electrons), theory is not yet advanced enough[25]. Sufficiently precise experiments with the H-like He^+ ion, where the two-body theory of H is applicable, will soon be available[26,27].

But here, we have already theoretically calculated the $r_{\alpha/He}$ value which is 1.688564465 fm. If the future experiments give values of $r_{\alpha/He}$ close to our prediction, that will be a strong proof to our theories.

We also notice that the deuteron charge radius is even larger that that of helium nucleus. A deuteron is composed of a proton and a neutron, a helium nucleus consists double of them, so this phenomenon is very strange. Here we also try to give their relationship coefficient ($r_{\alpha/He}/r_{d/D}$).

8. Formulas for Radii of Electron, Proton, Neutron, Deuteron and Helium Nucleus in Atomic Unites

In atomic unites (a.u.), Bohr radius a_0 is equal to 1, so the formulas for the radii of electron, proton, neutron, deuteron and helium nucleus (α particle) in atomic unites will be simplified to the following forms.

$$\text{Atomic Unites }(au)\text{: } \hbar=m=e=1,\ \text{Bohr radius } a_0=1$$

$$r_{e/au}=\frac{1}{112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{14\cdot 112\cdot(2\cdot 173+1)})}$$

$$r_{e/au}=\frac{1}{(137+\delta)^2}=\frac{1}{(56+81+\frac{1}{28-\frac{5\cdot(4\cdot3\cdot7\cdot17-1)}{2\cdot5\cdot(4\cdot5\cdot7\cdot23+1)+1}})^2}$$

the key element and its isotopes: ${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$

Note: 136=8·17, 138=6·23

$$r_{p/H/au}=\frac{1}{(252+\delta)^2}=\frac{1}{(99+153+\frac{1}{33+\frac{2}{17}})^2}$$

$$r_{p/H-\mu/au}=\frac{1+\frac{1}{2\cdot47}}{(99+153+\frac{1}{33+\frac{2}{17}})^2} \qquad r_{p/e/au}=\frac{1-\frac{1}{2\cdot11\cdot19}}{(99+153+\frac{1}{33+\frac{2}{17}})^2}$$

the key element and its isotope: ${}^{252}_{99}Es^{*}_{153}$

Note: $99=9\cdot11=3\cdot33$, $153=9\cdot17$, $252=2\cdot126=9\cdot28$

$$r_{n/e/au}=\frac{1}{(400-\delta)^2}=\frac{1}{(157+243-\frac{1}{3})^2}$$

the key element and its isotope: ${}^{400}_{157}Ch^{ie}_{243}$ (Note: $243=3^5$)

$$r_{d/D/au}=\frac{1}{(157+\delta)^2}=\frac{1}{(64+93+\frac{1}{6})^2}$$

$$r_{d/D-\mu/au}=\frac{1-\frac{1}{126}}{(64+93+\frac{1}{6})^2} \qquad r_{d/e/au}=\frac{1-\frac{1}{149}}{(64+93+\frac{1}{6})^2}$$

the key element and its isotope: ${}^{157}_{64}Gd_{93}$

$$r_{n/D/au}=\frac{1}{(201+\delta)^2}=\frac{1}{(80+121+\frac{1}{2\cdot11}+\frac{1}{36\cdot11+\frac{3}{2\cdot11}})^2}$$

the key element and its isotope: ${}^{201}_{80}Hg_{121}$

$$r_{\alpha/He/au}=\frac{1}{(177+\delta)^2}=\frac{1}{(72+105+\frac{1}{36-\frac{6}{35}})^2}$$

$$r_{\alpha/He-\mu/au} = \frac{1-\frac{1}{163}}{(72+105+\frac{1}{36-\frac{6}{35}})^2} \qquad r_{\alpha/e/au} = \frac{1-\frac{1}{4\cdot 59}}{(72+105+\frac{1}{36-\frac{6}{35}})^2}$$

the key element and its isotope: ${}^{177}_{72}Hf_{105}$ (Note: $177 = 3\cdot 59$)

the key elements and its isotopes:

${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{157}_{64}Gd_{93}$ ${}^{177}_{72}Hf_{105}$ ${}^{201}_{80}Hg_{121}$ ${}^{252}_{99}Es^{*}_{153}$ ${}^{400}_{157}Ch^{ie}_{243}$

Notice the sequences: 56 64 72 80; 93 99 105; 137 157 177; 56 252

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Appendix I: Research History

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Note: Date was recorded according to Beijing Time.			

Appendix II: Version History

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v1	2021/7/30-8/3	9	2021/8/3	viXra:2108:0011v1
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