On the Elementary Function $y = |x|$ and Division by Zero Calculus

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Abstract: In this paper, we will consider the elementary function $y = |x|$ from the viewpoint of the basic relations of the normal solutions (Uchida's hyper exponential functions) of ordinary differential equations and the division by zero calculus. In particular, $y'(0) = 0$ in our sense and this function will show the fundamental identity with the natural sense

$$\frac{0}{0} = 0$$

with the sense

$$\frac{1}{0} = 0$$

that may be considered as 0 as the inversion of 0 through the Uchida's hyper exponential function.

David Hilbert:

The art of doing mathematics consists in finding that special case which contains all the germs of generality.

Oliver Heaviside:

Mathematics is an experimental science, and definitions do not come first, but later on.

Key Words: Division by zero, division by zero calculus, normal solutions of ordinary differential equations, Uchida's hyper exponential functions, isolated singular point, $1/0 = 0/0 = z/0 = \tan\frac{\pi}{2} = \log 0 = 0$, $y = |x|$.

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1 Introduction

K. Uchida ([17]) has a long love for the solutions of the differential equations

$$\frac{d^n y}{dx} = f(x)y$$

and he has been appointing the importance of the solutions. He called the solutions *hyper exponential functions* (*Uchida's hyper exponential functions*). He considered the solutions for some functions $f(x)$ and derived many beautiful computer graphics with their elementary properties ([18]). We see the few concrete solutions from [10] and [18]. Of course, the case $n = 1$ is trivial and the $n > 3$ cases are rare examples and the case $n = 2$ is important.

Meanwhile, we introduced the concept of division by zero calculus in [14] that we can consider analytic functions and their derivatives even at isolated singular points. Therefore, we can consider the Uchida's exponential functions for analytic functions $f(x)$ with singularities. Surprisingly enough, then any analytic functions with any singular points may be considered as the Uchida's hyper exponential functions. As one typical example, we considered the simplest case of

$$f(x) = \frac{1}{(x-a)^m} \tag{1.1}$$

for the general real number m of $m \neq 0$ and for $n = 2$. In [15], we discussed the related ordinary differential equations. However, we know that the elementary function $y = |x|$ is the simplest Uchida's hyper exponential function and we see that the fundamental relations in the natural sense

$$\frac{0}{0} = 0$$

with the sense

$$\frac{1}{0} = 0$$

that may be considered as 0 as the inversion of 0 through the Uchida's hyper exponential function.

2 The function $y = |x|$ is an Uchida's hyper exponential function

Indeed, we will consider the expression

$$y =: \exp\left(\int_1^x \frac{dt}{t}\right) = \exp\left(\log|x|\right) = |x|.$$

Then,

$$y' = |x|\frac{1}{x} = \frac{1}{x}y$$

that shows the desired result.

3 In connection with the division by zero

Now we will consider the above formula at $x = 0$ formally

$$y'(0) = |0|\frac{1}{0} = 0\frac{1}{0}.$$

However, this function is an odd function $f(x) = -f(-x)$ and we see that $f(0) = 0$ should be 0; that is,

$$y'(0) = 0\frac{1}{0} = 0.$$

4 Inversion by 0

Of course, in the above logic, we can derive the identity already

$$0\frac{1}{0} = \frac{0}{0} = 0.$$

Our logic may be considered naturally that the inversion of 0 may be considered and it is zero. Here, the definition of

$$\frac{1}{0}$$

is given as the value of the elementary function $y = 1/x$ at the origin $x = 0$ that is an odd function.

5 The gradient of the y axis is zero

For the sign function y' we see that the derivative at the origin is zero; that is,

$$\tan \frac{\pi}{2} = 0,$$

that is a very important fundamental result on the division by zero calculus. For the division by zero calculus and fundamental results, see the cited papers in the references.

6 Remarks

For the introduced function, we obtain, by setting $x = 0$

$$y(0) = \exp\left(\int_1^0 \frac{dt}{t}\right) = \exp\left(\log 0\right) = |0| = 0.$$

All the terms have their senses, because

$$\int_1^0 \frac{dt}{t} = 0,$$

$$\log 0 = 0,$$

and

$$\exp 0 = 1, 0$$

that has two values ([14]).

7 Conclusion

Dividing by zero is multiplying by zero.

For the elementary function

$$y = |x|$$

we have always

$$y' = \frac{|x|}{x}$$

with

$$y'(0) = 0.$$

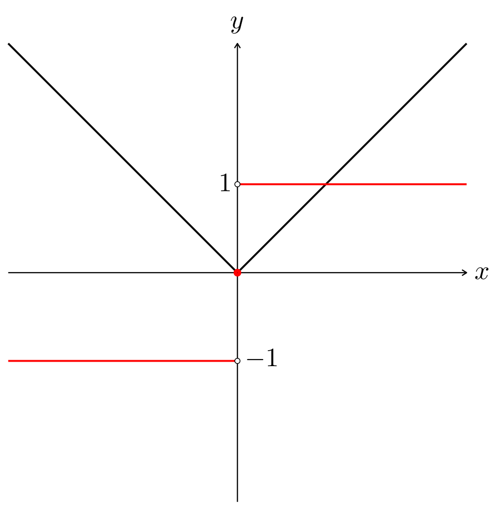

Figure 1 : The graphs of $y = |x|$ (black) and $y = \dfrac{|x|}{x}$ (red).

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