A Better Version of the Mid-Point Rule for Estimating Areas?

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Abstract:

A Modified Midpoint Rule is defined and compared with the Standard Version

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Rather than the standard Mid-Point Rule for estimating areas, construct a modified version using the following:

1. Divide the (finite) interval (a,b) into n equal-sized parts
2. Curve fit a (n-1)th order polynomial through the points (a+(b-a)/2n, f(a+(b-a)/2n)), (a+3(b-a)/2n, f(a+3(b-a)/2n)), …, (b-(b-a)/2n, f(b-(b-a)/2n)) by solving the appropriate simultaneous equations
3. Take the integral of that polynomial from a to b.

   That integral is your new estimate of the area – the Modified Midpoint Rule.

Without loss of generality, just consider areas for f(x) from a=0 to b=1 from now on.

This can be done by transforming any integral with a finite domain by the following transformation:

$$\int_a^b f(x)dx = (b-a)\int_0^1 f((b-a)x+a)dx$$

For n=1 and 2, the modified midpoint rule by the above procedure just gives the standard midpoint rule. But for n=3, the modified midpoint rule becomes:

$$\int_0^1 f(x)dx \approx \frac{3(\min+\max)+2(\text{median})}{8}$$

where: min =minimum (f(1/6), f(3/6), f(5/6))
max =maximum (f(1/6), f(3/6), f(5/6))
median =median (f(1/6), f(3/6), f(5/6))

For n=4 the modified rule is:

$$\int_0^1 f(x)dx \approx \frac{13(\min+\max)+11(2*\text{median})}{48}$$

where: min =minimum (f(1/8), f(3/8), f(5/8), f(7/8))
max =maximum (f(1/8), f(3/8), f(5/8), f(7/8))
median =median (f(1/8), f(3/8), f(5/8), f(7/8))

Modified Midpoint Rules can be constructed for higher n using the above technique.

(For details on how to construct these approximations, see "A Better Type of Sample Mean?" at vixra.org/author/d_williams)

Given the way it was derived you might expect the new approximation to be better for most functions. Some examples (below) suggest this is so.

f(x)	midpoint rule	modified midpoint rule	area
x^(1/3)	0.761685921	0.7576840954	0.75
ln(1+x)	0.3885838638	0.3864737083	0.3862943611
exp(x)	1.7103525248	1.7180564316	1.7182818285
2x^3+7x+3	6.9722222222	7	7
10sin(pi*x/2)	6.4395055086	6.3605602207	6.3661977237
10tan(pi*x/4)	4.3773101598	4.4067069768	4.412712003

Table: Comparison of area approximations for 6 "random-ish" functions (with n=3)

Notice the modified rule is significantly closer for each.

It may be the case that when the modified midpoint rule is closer to the integral for a particular f(x) that the alternative sample mean for the stochastic function f(ran#) (given in "A Better Type of Sample Mean?") is closer to the population mean than the standard sample mean. This needs closer examination.

Still better approximations of areas may be possible using other types of curve fitting, etc.

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